In this segment, we discuss splitting a Poisson process into two different streams.

So we have a Poisson process with some arrival rate lambda, and whenever there is an arrival, we decide to send it either to one or another stream.

And for example, this arrival might be sent to that stream, this arrival might be sent to this stream, this arrival might be sent to this stream, that arrival might be sent to that stream.

How do we decide where to send these arrivals?

We make those decisions using independent coin flips with a coin that has a certain fixed bias equal to q.

So when this arrival comes, there's probability q that it will be sent that direction or probability 1 minus q that it will be sent in the other direction.

All of these coin flips are independent, so the decision on where to send the second arrival is going to be independent from the decision of where to send the first arrival.

And furthermore, we make one more assumption that these coin flips are independent from everything else.

They're independent from the time history of the original Poisson process.

For example, the coin flip that decides the destination of this first arrival can not depend on how long it took for this arrival to occur.

We will argue now that this stream is a Poisson process and by symmetry, therefore, this stream is also a Poisson process.

We need to verify two assumptions.

One has to do with independence.

The argument here is entirely analogous to arguments that we have already carried out in the past, namely disjoint time intervals in the original process are independent.

Coin flips that happened during those disjoint time intervals are also independent of each other, and for this reason, whatever happens during disjoint time intervals in that stream will also be independent.

The other property that we need to verify has to do with probabilities of small intervals.

If we take a little interval here of length delta, what is going to happen during that interval?

Well, we look at what happens during the corresponding interval in the original stream.

In the original stream, the probability of having two or more arrivals-- this probability is order of delta squared, so there's no way of having two or more arrivals during that little interval, or to be more precise, the probability of two or more arrivals here is going to be negligible, order of delta squared.

What is the probability of having one arrival during that little interval?

We will have one arrival here if we've had one arrival in this time interval, which happens with probability lambda times delta, and also the coin flip sent the arrival in this direction, which is something that happens with probability q, and the remaining probability is assigned to the event of having zero arrivals during that interval.

So the probability of two or more arrivals is negligible, and the probability of one arrival is proportional to delta.

And that's what we need in order to have a Poisson process.

The factor of proportionality that multiplies delta is equal to lambda times q.

Therefore, this is a Poisson process with parameter, or arrival rate, equal to lambda times q.

And by a similar argument, this process here is going to be a Poisson process with parameter equal to lambda times 1 minus q.

So by splitting a Poisson process using independent coin flips, we obtain two different Poisson streams.

Are these Poisson streams independent?

For the case of the Bernoulli process, we had seen that the resulting streams were not independent.

The reason for the Bernoulli process was that if I tell you that at a certain slot we had an arrival in this stream, that would tell you that in the corresponding time slot of the other process you could not have an arrival and that was a source of dependence.

It turns out that for the Poisson process, because it runs in continuous time, telling you that we had an arrival at this particular time instant does not give you any substantial or any nontrivial information about the other process, and the two processes remain independent.

This result is surprising in some ways, but it is true.

A mathematical derivation proceeds along a line that's a little different from the intuitive argument that I just outlined.

And we will not go through that derivation, but it's a useful fact to know.