One particular series that shows up in many applications, examples, or problems is the geometric series.

[In] the geometric series, we are given a certain number, alpha, and we want to sum all the powers of alpha, starting from the 0th power, which is equal to 1, the first power, and so on, and this gives us an infinite series.

It's the sum of alpha to the i where i ranges from 0 to infinity.

Now, for this series to converge, we need subsequent terms, the different terms in the series, to become smaller and smaller.

And for this reason, we're going to make the assumption that the number alpha is less than 1 in magnitude, which implies that consecutive terms go to zero.

Let us introduce some notation.

Let us denote the infinite sum by s, and we're going to use that notation shortly.

One way of evaluating this series is to start from an algebraic identity, namely the following.

Let us take 1 minus alpha and multiply it by the terms in the series, but going only up to the term alpha to the n.

So it's a finite series.

We do this multiplication, we get a bunch of terms, we do the cancellations, and what is left at the end is 1 minus alpha to the power n plus 1.

What we do next is we take the limit as n goes to infinity.

On the left hand side, we have the term 1 minus alpha, and then the limit of this finite series is by definition the infinite series, which we're denoting by s.

On the right hand side, we have the term 1.

How about this term?

Since alpha is less than 1 in magnitude, this converges to 0 as alpha goes to infinity, so that term disappears.

We can now solve this relation, and we obtain that s is equal to 1 over 1 minus alpha, and this is the formula for the infinite geometric series.

There's another way of deriving the same result, which is interesting, so let us go through it as well.

The infinite geometric series has one first term and then the remaining terms, which is a sum for i going from 1 to infinity of alpha to the i.

Now, we can take a factor of alpha out of this infinite sum and write it as 1 plus alpha, the sum of alpha to the i, but because we took out one factor of alpha, here, we're going to have smaller powers.

So now the sum starts from 0 and goes up to infinity.

Now, this is just 1 plus alpha times s because here, we have the infinite geometric series.

Therefore, if we subtract alpha s from both sides of this equality, we get s times 1 minus alpha equal to 1.

And now by moving 1 minus alpha to the denominator, we get again the same expression.

So this is an alternative way of deriving the same result.

However, there's one word of caution.

In this step, we subtracted alpha s from both sides of the equation.

And in order to do that, this is only possible if we take for granted that s is a finite number.

So this is taken for granted in order to carry out this derivation.

This is to be contrasted with the first derivation, in which we didn't have to make any such assumption.

So strictly speaking, for this derivation here to be correct, we need to have some independent way of verifying that s is less than infinity.

But other than that, it's an interesting algebraic trick.