Solution 2.1

x(n) is periodic if x(n) = x(n + N) for some integer value of N. For the sequence in (a),

$$x(n+N) = A\cos\left(\frac{3\pi}{7}n + \frac{3\pi}{7}N - \frac{\pi}{8}\right)$$

$x(n + N) = x(n)$ if $\frac{3\pi}{7}N$ is an integer multiple of 2π. The smallest value of N for which this is true is N = 14. Therefore the sequence in (a) is periodic with period 14.

For the sequence in (b),

$$x(n+N) = e^{j\left(\frac{n}{8} + \frac{N}{8} - \pi\right)}$$

$$= e^{j\left(\frac{n}{8} - \pi\right)} e^{j\frac{N}{8}} = x(n)\, e^{j\frac{N}{8}}$$

The factor $e^{j\frac{N}{8}}$ is unity for (N/8) an integer multiple of 2π. This requires that

$$\frac{N}{8} = 2\pi R$$

where N and R are both integers. This is not possible since π is an irrational number. Therefore this sequence is not periodic.

Solution 2.2

$$x(n) = -2\delta(n+3) - \delta(n) + 3\delta(n-1) + 2\delta(n-3)$$

Solution 2.3

Each of the systems given can be tested against the definitions of linearity and time invariance. For example, for

(a), $T[x_1(n)] = 2x_1(n) + 3$

$T[x_2(n)] = 2x_2(n) + 3$

Since $T[ax_1(n) + bx_2(n)] = 2[ax_1(n) + bx_2(n)] + 3$

and $aT[x_1(n)] + bT[x_2(n)] = 2ax_1(n) + 2bx_2(n) + 3(a+b)$

The system is not linear. The system is, however, shift-invariant since $T[x(n-n_o)] = 2x(n-n_o) + 3 = y(n-n_o)$.

In a similar manner we can show that:

(b) is linear but not shift-invariant

(c) is not linear but is shift-invariant

(d) is linear and shift-invariant

Solution 2.4

To determine y(n) we evaluate the convolution sum eq. (2.39) of the text. For part (a), the sequences x(k) and h(n - k) are indicated below as functions of k:

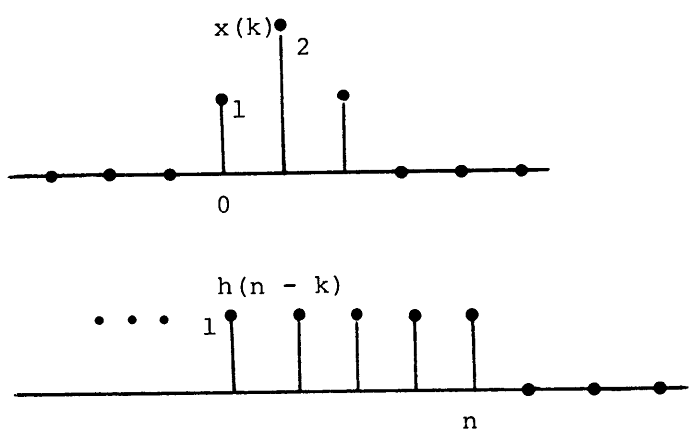


Figure S2.4-1

Since h(n - k) is zero for k > n, and is unity for k < n,

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\ h(n-k) = \sum_{k=-\infty}^{n} x(k)$$

as sketched below:

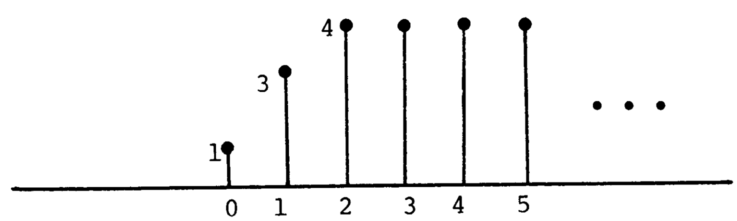


Figure S2.4-2

Part (b) can likewise be done graphically. Alternatively since $h(n) = \delta(n + 2)$,

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k)\ x(n - k)$$

$$= \sum_{k=-\infty}^{+\infty} \delta(k + 2)\ x(n - k)$$

Since $\delta(k + 2) = 0$ except for $k = -2$, and is unity for $k = -2$
$y(n) = x(n + 2)$.

For part (c) $x(k)$ and $h(n - k)$ are as sketched below:

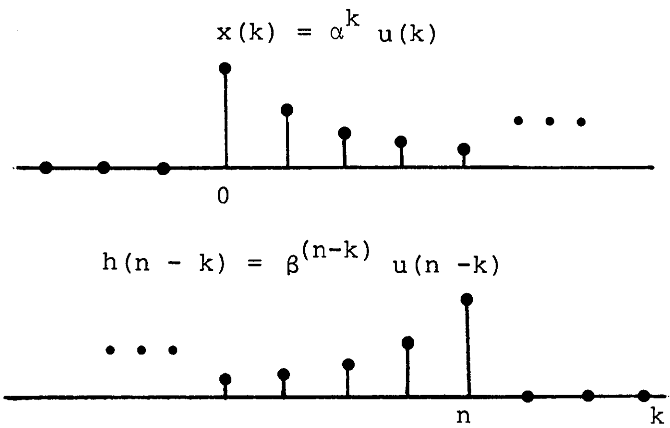


Figure S2.4-3

Graphically we see that for $n < 0$, $x(k)\ h(n - k)$ is zero and consequently $y(n) = 0$, $n < 0$. For $n \geq 0$

$$y(n) = \sum_{k=0}^{n} \alpha^k\ \beta^{n-k} = \beta^n \sum_{k=0}^{n} (\alpha/\beta)^n$$

$$= \beta^n \frac{1-(\alpha/\beta)^{n+1}}{1-(\alpha/\beta)} = \frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}$$

Consequently for all n,

$$y(n) = \left[\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}\right] u(n)$$ which is a decaying exponential for $n \geq 0$.

The answer for part (d) is:

$$y(n) = x(n-2) - x(n-3) = \delta(n-2)$$

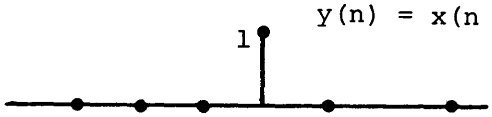
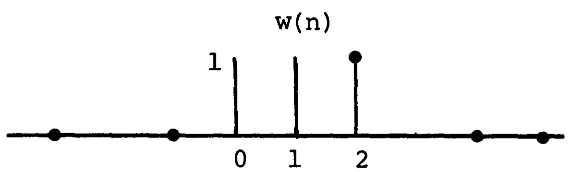


Figure S2.4-4

Solution 2.5*

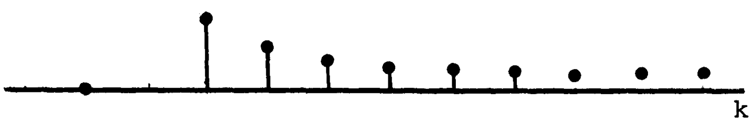
$x(n) * h_1(n) = x(n) * [\delta(n) - \delta(n-3)] = x(n) - x(n-3)$.

Therefore with x(n) as a unit step, $x(n) * h_1(n)$ is:



Convolving w(n) graphically with $h_2(n)$

$$h_2(k) = (.8)^k u(k)$$



W(n - k)

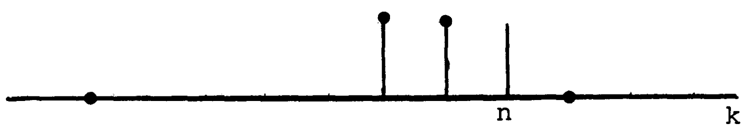


Figure S2.5-1

For $n < 0$ $h_2(k)\, w(n-k) = 0$

For $n = 0$ $y(n) = 1$

For $n = 1$ $y(n) = 1 + (.8)$

For $n \geq 2$ $y(n) = (.8)^{n-2} + (.8)^{n-1} + (.8)^n$

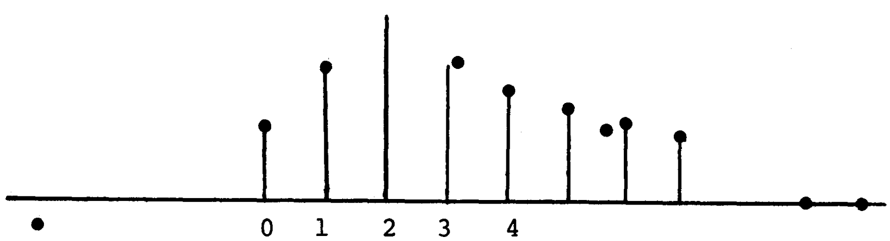


Figure S2.5-2

(b) The convolution of $h_1(n)$ and $h_2(n)$ is:

$$h(n) = h_1(n) * h_2(n) = (.8)^n u(n) - (.8)^{(n-3)} u(n-3)$$

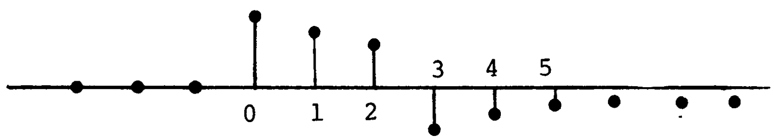


Figure S2.5-3

The convolution of this result with a unit step results in

$$y(n) = \sum_{k=-\infty}^{n} h(k)$$

or

$y(n) = 0 \quad n < 0$

$y(0) = 1$

$y(1) = 1 + .8$

$y(2) = 1 + (.8) + (.8)^2$

$y(3) = 1 + .8 + (.8)^2 + (.8)^3 - 1 = .8 + (.8)^2 + (.8)^3$

$y(4) = 1 + .8 + (.8)^2 + [(.8)^3 - 1] + [(.8)^4 - .8]$

$\quad = (.8)^2 + (.8)^3 + (.8)^4$

etc.

Solution 2.6*

The fact that $x(n) = z^n$ is an eigenfunction follows from the convolution sum. Specifically

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k)\, x(n-k) = \sum_{k=-\infty}^{+\infty} h(k) z^{(n-k)}$$

$$= z^n \sum_{k=-\infty}^{+\infty} h(k) z^{-k} \qquad \text{(S2.6-1)}$$

Since the summation in the equation (S2.6-1) does not depend on n, it is simply a constant for any given z.

While the complex exponential z^n is an eigenfunction of any linear shift-invariant system, $z^n u(n)$ is not. For example, let $h(n) = \delta(n-1)$. Then with $x(n) = z^n u(n)$, $y(n) = z^{n-1} u(n-1)$, which is not a complex constant times $x(n)$.