STUDENT: Let's talk about the Mohr's Circle, an important tool in materials science and mechanical engineering.

What is Mohr's Circle, and what does it do? Assume you have a rod, and you want to break it with your hands. What can you do about it? You can pull it, compress it, or twist it. Each of these will induce a stress in the rod in a different way. And the Mohr's Circle helps you analyze this.

Mohr's Circle is a graphical tool, a visual way to see how to go from one state of stress to another. Usually you are given a state of stress that you calculated from using stress equations. Then you want to find a state of stress at a new angle of orientation, or principal stresses at that orientation, or the maximum in-plane shear stresses. And Mohr's Circle allows you to draw this representative element to visualize the calculations for you.

Now, let's derive the Mohr's Circle. So now we take a look at the piece of material that we cut from the rod that we're trying to break. And originally we had this coordinate system. And we have some stresses, where the signals represent the normal stress and the tiles represent the shear stress. So we draw this representative pair of stresses for convenience.

And then we want to analyze the stresses of this cross-section area. So what we do is, we need to rotate the coordinate system so that x-axis would line up with the unit normal vector to this cross-sectional area.

And then the y-axis would be rotated to this direction, and would be lined up with the n prime unit vector shown here.

So this is done by rotating this coordinate system by an angle theta counterclockwise. Then we can obtain the unit normal vector in matrix form. And that is just cosine theta and sine theta. And similarly, the n prime vector, that's just negative sine theta and cosine theta.

So we can now get the stress tensor by taking the dot product of the stresses in the x-direction with the unit normal vector, and the dot project of stresses in the y-direction with the unit normal vector. So now we can write out the normal stress in the new coordinate system by just taking the dot product of the stress tensor with the unit normal vector. And similarly, we can find the shear stress in the new coordinate system by taking the dot product of the stress tensor with the n prime.

And we can do these complicated calculations on Mathematica, which I will show you right now.

So I've already typed in these vectors into Mathematica. And n represents the unit normal vector, and np is just n prime, and t is the stress tensor. So we want to find the normal stress in the new coordinate system, sigma prime. And that is just dotting t with n. And we get this simplified version. That. So that's our sigma prime.

Then we want to find our tau prime. And that is just t dotted with np. Again, we want to find the simplified version. So that's that.

And don't worry about this thetas and 2 thetas. We'll use trig identities to change them.

So here's our trig identities. And we just plug them back in. And we obtain these two equations for our new stresses.

Then we move all the terms containing 2 theta to the right side of the equations, and all the rest of the terms to the left side of the equations. And we square both sides of the equations to get equation 1 and equation 2, as shown here.

We add equation 1 and equation 2, and some terms cancel. Finally, we get this really clean equation shown here. Now look closely to this equation. What form of equation does it look like?

You might have guessed that it looks like the equation of a circle, and you're absolutely right. This is exactly how Mohr's Circle is derived.

We can compare the equation with the equation of a circle, and we see that Mohr's Circle is centered at the coordinates sigma x plus sigma y over 2, 0. And sigma x plus sigma y over 2 is the average normal stress in the old coordinate system. We can also obtain the radius of Mohr's Circle.

Now let me draw this Mohr's Circle for you. So we go back to our old coordinate system with some calculated stresses. And then we need to construct this new coordinate system to draw this Mohr's Circle on. I take the positive x-direction as the plus sigma and the negative y-direction as plus tau, just by convention.

Then, in order to draw the circle, we take a pair of normal stress and shear stress and we plot their values as the coordinates in our coordinate system. So we find a dot, and it's represented by sigma x and tau xy. Then, by our previous equations that we derived, we can find the center of the circle, which is located at sigma average and 0. And sigma average is just sigma x plus sigma y over 2.

Then we find its values, and we plot that on our sigma axes. That's just our center of the circle. Then we connect these two dots that we drew, and that's just the radius of the circle. Now we can draw this circle, since we know its center and a dot on its circumference.

We can now confirm the equation for the radius of the circle by doing some geometry here. So, this length is just sigma average. And then we have another length that's just sigma x. And the difference between these lengths is just sigma x minus sigma average.

Then this line right here is simply tau xy.

And by looking at this right triangle and using Pythagoras' Theorem, we can obtain the radius of the circle, which is written here under the square root. Actually, it confirms our original equation for obtaining the radius r of Mohr's Circle.

So Mohr's Circle is very useful for visualizing the stresses on the material. So we can, just by doing simple geometry, find the values for its principal stresses.

So in green, you can see on the circle, there is the maximum normal stress and minimum normal stress. And now, in red, I'm labeling the minimum and maximum shear stresses.