# Glossary of Symbols

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# Glossary of Roman Symbols

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| $\| \ \|_2$ | The Euclidean norm or "two-norm." For a vector $\underline{a}$ $\|\underline{a}\|_2 = \sqrt{\sum_k (a_k)^2}$ |
| $\sim$ | When used above a symbol, denotes "in the rotated coordinate system." |
| $a_k, b_k$ | Cross-sectional dimensions of a beam at nodal point *k*. |
| ${}^tA$ | Cross-sectional area at time *t*. |
| $\underline{A}^{(i)}$ | A square matrix used in the BFGS method. |
| $\underline{B}_L$ | Linear strain-displacement matrix used in linear or M.N.O. analysis. |
| ${}_0^t\underline{B}_L$ | Linear strain-displacement matrix used in the T.L. formulation. |
| ${}_t^t\underline{B}_L$ | Linear strain-displacement matrix used in the U.L. formulation. |
| ${}_0^t\underline{B}_{L0}$, ${}_0^t\underline{B}_{L1}$ | Intermediate matrices used to compute ${}_0^t\underline{B}_L$; ${}_0^t\underline{B}_{L1}$ contains the "initial displacement effect." |
| ${}_0^t\underline{B}_{NL}$ | Nonlinear strain-displacement matrix used in the T.L. formulation. |
| ${}_t^t\underline{B}_{NL}$ | Nonlinear strain-displacement matrix used in the U.L. formulation. |
| $c$ | The wave speed of a stress wave (dynamic analysis). |
| $c_{ii}$ | Diagonal element corresponding to the *i*th degree of freedom in the damping matrix (dynamic analysis). |
| $\underline{C}$ | The damping matrix (dynamic analysis). |
| $C_1$, $C_2$ | The Mooney-Rivlin material constants (for rubberlike materials). |
| ${}_0^tC_{ij}$ | Components of the Cauchy-Green deformation tensor (basic concepts of Lagrangian continuum mechanics). |
| $\underline{C}_\ell$ | Matrix containing components of the constitutive tensor referred to a local coordinate system. |
| $\underline{C}$ | Matrix containing components of the constitutive tensor, used in linear and M.N.O. analysis. |
| ${}_0\underline{C}$ | Matrix containing components of the constitutive tensor ${}_0C_{ijrs}$, used in the T.L. formulation. |
| ${}_t\underline{C}$ | Matrix containing components of the constitutive tensor ${}_tC_{ijrs}$, used in the U.L. formulation. |
| $C^E_{ijrs}$ | Components of elastic constitutive tensor relating $d\sigma_{ij}$ to $de^E_{rs}$ |
| $C^{EP}_{ijrs}$ | Components of elasto-plastic constitutive tensor relating $d\sigma_{ij}$ to $de_{rs}$ |
| ${}_0C_{ijrs}$ | Components of tangent constitutive tensor relating $d_0S_{ij}$ to $d_0\varepsilon_{rs}$ |
| ${}_tC_{ijrs}$ | Components of tangent constitutive tensor relating $d_tS_{ij}$ to $d_t\varepsilon_{rs}$ |
| DNORM | Reference displacement used with displacement convergence tolerance DTOL (solution of nonlinear equations). |
| DMNORM | DMNORM is the reference rotation used when rotational degrees of freedom are present. |
| DTOL | Convergence tolerance used to measure convergence of the displacements and rotations (solution of nonlinear equations). |

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| det | The determinant function, for example, $\det {}_0^t\underline{X}$ . |
| ${}^t dV$ | A differential element of volume evaluated at time *t*. |
| ${}^0 dV$ | A differential element of volume evaluated at time 0. |
| $d^t\underline{x}$ | Vector describing the orientation and length of a differential material fiber at time *t* (basic concepts of Lagrangian continuum mechanics). |
| $d^0\underline{x}$ | Vector describing the orientation and length of a differential material fiber at time 0 (basic concepts of Lagrangian continuum mechanics). |
| ${}^t\bar{e}^C$ | Effective creep strain, evaluated at time *t* (creep analysis). |
| $e_{ij}$ | Components of infinitesimal strain tensor (linear and M.N.O. analysis). |
| ${}_0e_{ij}$ | Linear (in the incremental displacements) part of ${}_0\varepsilon_{ij}$ (T.L. formulation) |
| ${}_te_{ij}$ | Linear (in the incremental displacements) part of ${}_t\varepsilon_{ij}$ (U.L. formulation). |
| ${}^te_{ij}^{IN}$, ${}^te_{ij}^{C}$, ${}^te_{ij}^{P}$, ${}^te_{ij}^{TH}$, ${}^te_{ij}^{VP}$ | Various types of inelastic strains evaluated at time *t* (inelastic analysis): IN inelastic; C creep; P plastic; TH thermal; VP viscoplastic |
| $\underline{e}_r$, $\underline{e}_s$, $\underline{e}_t$ | Unit vectors in the *r*, *s*, and *t* directions (shell analysis). |
| $\bar{\underline{e}}_r$ , $\bar{\underline{e}}_s$ | Unit vectors constructed so that $\bar{\underline{e}}_r$, $\bar{\underline{e}}_s$, $\underline{e}_t$ are mutually orthogonal (shell analysis). |
| E | Young's modulus. |
| $E_a, E_b$ | Young's moduli in the *a* and *b* directions (orthotropic analysis). |
| $E_T$ | Strain hardening modulus (elasto-plastic analysis). |
| ETOL | Convergence tolerance used to measure convergence in energy (solution of nonlinear equations). |
| f(x) | A function that depends on x (solution of nonlinear equations). |
| $\underline{f}(\underline{U})$ | A vector function that depends on the column vector $\underline{U}$ (solution of nonlinear equations). |
| ${}^tf_i^B$ , ${}^tf_i^S$ | Components of externally applied forces per unit current volume and unit current surface area. |
| ${}^tF$ | Yield function (elasto-plastic analysis). |
| ${}^t\underline{F}$ | Vector of nodal point forces equivalent to the internal element stresses. |
| ${}_0^t\underline{F}$ | Vector of nodal point forces equivalent to the internal element stresses (T.L. formulation). |
| ${}_t^t\underline{F}$ | Vector of nodal point forces equivalent to the internal element stresses (U.L. formulation). |
| $\underline{F}_I(t)$ | Column vector containing the inertia forces for all degrees of freedom (dynamic analysis). |
| $\underline{F}_D(t)$ | Column vector containing the damping forces for all degrees of freedom (dynamic analysis). |
| $\underline{F}_E(t)$ | Column vector containing the elastic forces (nodal point forces equivalent to element stresses) for all degrees of freedom (dynamic analysis). |
| g | Acceleration due to gravity. |
| $G_{ab}$ | Shear modulus measured in the local coordinate system *a-b* (orthotropic analysis). |
| h | Cross-sectional height (beam element). |
| $h_k$ | Interpolation function corresponding to nodal point *k*. |

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| $\underline{H}$ | Displacement interpolation matrix (derivation of element matrices). |
| $\underline{H}^S$ | Displacement interpolation matrix for surfaces with externally applied tractions (derivation of element matrices). |
| $I_1, I_2, I_3$ | The invariants of the Cauchy-Green deformation tensor (analysis of rubberlike materials). |
| $\underline{J}$ | The Jacobian matrix relating the $x_i$ coordinates to the isoparametric coordinates (two- and three-dimensional solid elements). |
| ${}^t\underline{J}$ | The Jacobian matrix relating the ${}^tx_i$ coordinates to the isoparametric coordinates (two- and three-dimensional solid elements in geometrically nonlinear analysis). |
| $k$ | Shear factor (beam and shell analysis). |
| ${}^t\underline{K}$ | The tangent stiffness matrix, including all geometric and material nonlinearities. |
| ${}^t_0\underline{K}$ | The tangent stiffness matrix, including all geometric and material nonlinearities (T.L. formulation). |
| ${}^t_t\underline{K}$ | The tangent stiffness matrix, including all geometric and material nonlinearities (U.L. formulation). |
| ${}^t_0\underline{K}_L$ , ${}^t_t\underline{K}_L$ | The contribution to the total tangent stiffness matrix arising from the linear part of the Green-Lagrange strain tensor.<br>${}^t_0\underline{K}_L$ - T.L. formulation<br>${}^t_t\underline{K}_L$ - U.L. formulation |
| ${}^t_0\underline{K}_{NL}$ , ${}^t_t\underline{K}_{NL}$ | The contribution to the total tangent stiffness matrix arising from the nonlinear part of the Green-Lagrange strain tensor.<br>${}^t_0\underline{K}_{NL}$ - T.L. formulation<br>${}^t_t\underline{K}_{NL}$ - U.L. formulation |
| $\hat{\underline{K}}$ | Effective stiffness matrix, including inertia effects but no nonlinear effects (dynamic substructure analysis). |
| ${}^t\hat{\underline{K}}$ | Effective stiffness matrix, including inertia effects and nonlinear effects (dynamic substructure analysis). |
| $\hat{\underline{K}}_c$ | $\hat{\underline{K}}$ after static condensation (dynamic substructure analysis). |
| ${}^t\hat{\underline{K}}_c$ | ${}^t\hat{\underline{K}}$ after static condensation (dynamic substructure analysis). |
| ${}^t\underline{K}_{nonlinear}$ | Nonlinear stiffness effects due to geometric and material nonlinearities (dynamic substructure analysis). |
| ${}^tL$ | Length, evaluated at time *t*. |
| $L_e$ | Element length, chosen using the relation $L_e = c\,\Delta t$ (dynamic analysis). |
| $L_w$ | Wave length of a stress wave (dynamic analysis). |
| $m_{ii}$ | Lumped mass associated with degree of freedom *i* (dynamic analysis). |
| $\underline{M}$ | The mass matrix (dynamic analysis). |
| ${}^tp_{ij}$ | Quantities used in elasto-plastic analysis, defined as $${}^tp_{ij} = -\left.\frac{\partial\,{}^tF}{\partial\,{}^te^P_{ij}}\right\|_{{}^t\sigma_{ij}\ \text{fixed}}$$ |
| ${}^tq_{ij}$ | Quantities used in elasto-plastic analysis defined as $${}^tq_{ij} = \left.\frac{\partial\,{}^tF}{\partial\,{}^t\sigma_{ij}}\right\|_{{}^te^P_{ij}\ \text{fixed}}$$ |
| $r, s, t$ | Isoparametric coordinates (two- and three-dimensional solid elements, shell elements). |
| ${}^t_0\underline{R}$ | Rotation matrix (polar decomposition of ${}^t_0\underline{C}$ ). |
| $\underline{R}$ | Reference load vector (automatic load step incrementation). |
| ${}^t\underline{R}$ | Applied loads vector, corresponding to time *t*. |

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| ${}^{t}\mathfrak{R}$ | Virtual work associated with the applied loads, evaluated at time $t$. |
| RNORM, | Reference load used with force tolerance RTOL (solution of nonlinear equations). |
| RMNORM | Reference moment used when rotational degrees of freedom are present. |
| RTOL | Convergence tolerance used to measure convergence of the out-of-balance loads (solution of nonlinear equations). |
| ${}^{t}s_{ij}$ | Deviatoric stress evaluated at time $t$ (elasto-plastic analysis). |
| ${}^{t}S$ | Surface area, evaluated at time $t$. |
| ${}^{t}_{0}S_{ij}$ | Components of 2nd Piola-Kirchhoff stress tensor, evaluated at time $t$ and referred to the original configuration (basic Lagrangian continuum mechanics). |
| ${}_{0}S_{ij}$ , ${}_{t}S_{ij}$ | Components of increments in the 2nd Piola-Kirchhoff stress tensors: ${}_{0}S_{ij} = {}^{t+\Delta t}_{0}S_{ij} - {}^{t}_{0}S_{ij}$ ; ${}_{t}S_{ij} = {}^{t+\Delta t}_{t}S_{ij} - {}^{t}\tau_{ij}$ |
| ${}^{t}_{0}\underline{S}$ | Matrix containing the components of the 2nd Piola-Kirchhoff stress tensor (T.L. formulation). |
| ${}^{t}_{0}\underline{\hat{S}}$ | Vector containing the components of the 2nd Piola-Kirchhoff stress tensor (T.L. formulation). |
| $t$ , $t+\Delta t$ | Times for which a solution is to be obtained in incremental or dynamic analysis. The solution is presumed known at time $t$ and is to be determined for time $t+\Delta t$. |
| $\bar{t}$ | "Effective" time (creep analysis). |
| $\underline{T}$ | Displacement transformation matrix (truss element). |
| $T_{co}$ | Cut-off period (the smallest period to be accurately integrated in dynamic analysis). |
| $T_n$ | Smallest period in finite element assemblage (dynamic analysis). |
| ${}^{t}u_i$ | Total displacement of a point in the $i$th direction. |
| ${}^{t}\ddot{u}_i$ | Total acceleration of a point in the $i$th direction (dynamic analysis). |
| $u_i$ | Incremental displacement of a point in the $i$th direction. |
| $u_i^s$ | Components of displacement of a point upon which a traction is applied. |
| ${}^{t}_{0}u_{i,j}$ | Derivatives of the total displacements with respect to the original coordinates (T.L. formulation). |
| ${}_{0}u_{i,j}$ | Derivatives of the incremental displacements with respect to the original coordinates (T.L. formulation). |
| ${}_{t}u_{i,j}$ | Derivatives of the incremental displacements with respect to the current coordinates (U.L. formulation). |
| $u_i^k$ | Incremental displacement of nodal point $k$ in the $i$th direction. |
| ${}^{t}u_i^k$ | Total displacement of nodal point $k$ in the $i$th direction at time $t$. |
| $\underline{\hat{u}}$ | A vector containing incremental nodal point displacements. |
| ${}^{t}\underline{\hat{u}}$ | A vector containing total nodal point displacements at time $t$. |
| ${}^{t}\underline{\ddot{U}}$ | Vector of nodal point accelerations, evaluated at time $t$. |
| ${}^{t}\underline{\dot{U}}$ | Vector of nodal point velocities, evaluated at time $t$. |
| ${}^{t}\underline{U}$ | Vector of nodal point displacements, evaluated at time $t$. |
| ${}^{t}_{0}\underline{U}$ | Stretch matrix (polar decomposition of ${}^{t}_{0}\underline{C}$ ). |
| $\underline{v}^{(i)}$ | Column vector used in the BFGS method (solution of nonlinear equations). |

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| ${}^{t}V$ | Volume evaluated at time $t$. |
| ${}^{t}\underline{V}_n^k$ , ${}^{t}V_{ni}^k$ | Director vector at node $k$ evaluated at time $t$ (shell analysis). |
| $\underline{V}_n^k$ | Increment in the director vector at node $k$ (shell analysis). |
| ${}^{t}\underline{V}_1^k$ , ${}^{t}\underline{V}_2^k$ | Vectors constructed so that ${}^{t}\underline{V}_1^k$ , ${}^{t}\underline{V}_2^k$ and ${}^{t}\underline{V}_n^k$ are mutually perpendicular (shell analysis). |
| ${}^{t}\underline{V}_s^k$ , ${}^{t}\underline{V}_t^k$ | Director vectors in the $s$ and $t$ directions at node $k$, evaluated at time $t$ (beam analysis). |
| $\underline{V}_s^k$ , $\underline{V}_t^k$ | Increments in the director vectors in the $s$ and $t$ directions at node $k$ (beam analysis). |
| $\underline{w}^{(i)}$ | Vector used in the BFGS method (solution of nonlinear equations). |
| $W$ | Preselected increment in external work (automatic load step incrementation). |
| ${}_{0}^{t}W$ | Strain energy density per unit original volume, evaluated at time $t$ (analysis of rubberlike materials). |
| ${}^{t}W_P$ | Plastic work per unit volume (elastoplastic analysis). |
| ${}^{t}x_i$ | Coordinate of a material particle in the $i$th direction at time $t$. |
| ${}^{t}x_i^k$ | Coordinate of node $k$ in the $i$th direction at time $t$. |
| ${}_{0}^{t}x_{i,j}$ , ${}_{0}^{t}X_{ij}$ | Components of the deformation gradient tensor, evaluated at time $t$ and referred to the configuration at time 0. |
| ${}_{t}^{0}x_{i,j}$ , ${}_{t}^{0}\underline{X}_{ij}$ | Components of the inverse deformation gradient tensor. |

# Glossary of Greek Symbols

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| $\alpha$ | Parameter used in the $\alpha$-method of time integration.<br>$\alpha = 0$ - Euler forward method<br>$\alpha = 1/2$ - Trapezoidal rule<br>$\alpha = 1$ - Euler backward method |
| $\alpha_k$ | Incremental nodal point rotation for node $k$ about the ${}^t\underline{V}_1^k$ vector (shell analysis). |
| ${}^t\alpha$ | Coefficient of thermal expansion (thermo-elasto-plastic and creep analysis). |
| $\beta$ | Line search parameter (used in the solution of nonlinear equations). |
| $\beta$ | Section rotation of a beam element. |
| $\beta_k$ | Incremental nodal point rotation for node $k$ about the ${}^t\underline{V}_2^k$ vector (shell analysis). |
| $\gamma$ | Transverse shear strain in a beam element. |
| $\gamma$ | Fluidity parameter used in visco-plastic analysis. |
| $\gamma$ | Related to the buckling load factor $\lambda$ through the relationship $$\gamma = \frac{\lambda - 1}{\lambda}$$ |
| ${}^t\gamma$ | Proportionality coefficient between the creep strain rates and the total deviatoric stresses (creep analysis). |
| $\underline{\gamma}^{(i)}$ | Force vector in the BFGS method. |
| $\dfrac{\partial \underline{f}}{\partial \underline{U}}$ | A square coefficient matrix with entries $$\left[\frac{\partial \underline{f}}{\partial \underline{U}}\right]_{ij} = \frac{\partial f_i}{\partial U_j}$$ (solution of nonlinear equations). |
| $\delta$ | When used before a symbol, this denotes "variation in." |
| $\delta_{ij}$ | Kronecker delta; $$\delta_{ij} = \begin{cases} 0; & i \neq j \\ 1; & i = j \end{cases}$$ |
| $\underline{\delta}^{(i)}$ | Displacement vector in the BFGS method. |
| $\Delta \ell$ | "Length" used in the constant arc-length constraint equation (automatic load step incrementation). |
| $\Delta t$ | Time step used in incremental or dynamic analysis. |
| $\Delta t_{cr}$ | Critical time step (dynamic analysis). |
| $\Delta \underline{U}^{(i)}$ | Increment in the nodal point displacements during equilibrium iterations $$\Delta \underline{U}^{(i)} = {}^{t+\Delta t}\underline{U}^{(i)} - {}^{t+\Delta t}\underline{U}^{(i-1)}$$ |
| $\Delta \underline{\bar{U}}$ | Vector giving the direction used for line searches (solution of nonlinear equations). |
| $\Delta \underline{\bar{U}}^{(i)}, \Delta \underline{\bar{\bar{U}}}$ | Intermediate displacement vectors used during automatic load step incrementation. |

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| $\Delta \underline{X}^{(k)}$ | Increment in the modal displacements (mode superposition analysis). |
| $\Delta \tau$ | A time step corresponding to a subdivision of the time step $\Delta t$ (plastic analysis). |
| ${}_0^t\varepsilon_{ij}$ | Components of Green-Lagrange strain tensor, evaluated at time $t$ and referred to time 0. |
| ${}_0\varepsilon_{ij}$ | Components of increment in the Green-Lagrange strain tensor: ${}_0\varepsilon_{ij} = {}^{t+\Delta t}_{0}\varepsilon_{ij} - {}_0^t\varepsilon_{ij}$ |
| ${}_t^t\varepsilon_{ij}^a$ | Components of Almansi strain tensor. |
| $\eta, \xi, \zeta$ | Convected coordinate system (used in beam analysis). |
| ${}_0\eta_{ij}$ | The "nonlinear" part of the increment in the Green-Lagrange strain tensor. |
| $\theta_k$ | Nodal point rotation for node $k$ (two-dimensional beam analysis). |
| $\theta_i^k$ | Nodal point rotation for node $k$ about the $x_i$ axis (beam analysis). |
| ${}^t\theta$ | Temperature at time $t$ (thermo-elasto-plastic and creep analysis). |
| ${}^t\kappa$ | Variable in plastic analysis. |
| $\lambda$ | Lamé constant (elastic analysis). $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ |
| $\lambda$ | Scaling factor used to scale the stiffness matrix and load vector in linearized buckling analysis. |
| ${}^t\lambda$ | Load factor used to obtain the current loads from the reference load vector: ${}^t\underline{R} = {}^t\lambda \underline{R}$ (automatic load step incrementation). |
| ${}^t\lambda$ | Proportionality coefficient in calculation of the plastic strain increments (plastic analysis). |
| $\mu$ | Lamé constant (elastic analysis). $\mu = \frac{E}{2(1+\nu)}$ |
| $\nu$ | Poisson's ratio. |
| $\nu_{ab}$ | Poisson's ratio referred to the local coordinate system $a$-$b$ (orthotropic analysis). |
| $\Pi$ | Total potential energy (fracture mechanics analysis). |
| ${}^t\rho$ | Mass density, evaluated at time $t$. |
| ${}^t\sigma_{ij}$ | Components of stress tensor evaluated at time $t$ in M.N.O. analysis. |
| ${}^t\bar{\sigma}$ | Effective stress (used in creep analysis) ${}^t\bar{\sigma} = \sqrt{\frac{3}{2}\,{}^ts_{ij}\,{}^ts_{ij}}$ |
| ${}^t\sigma_y$ | Yield stress at time $t$ (plastic analysis). |
| $\sigma_y$ | Initial yield stress (plastic analysis). |
| $\sum_m$ | Denotes "sum over all elements." |
| ${}^t\hat{\underline{\Sigma}}$ | Vector containing the components of the stress tensor in M.N.O. analysis. |
| $\tau$ | (as a left superscript)—Denotes a time. Examples: ${}^\tau\underline{K}, {}^\tau\underline{R}$ - linearized buckling analysis; ${}^\tau\underline{K}$ - solution of nonlinear equations |
| ${}^t\tau_{ij}$ | Components of Cauchy stress tensor, evaluated at time $t$. |
| ${}^t\underline{\tau}$ | Matrix containing the components of the Cauchy stress tensor (U.L. formulation). |

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| ${}^{t}\hat{\underline{T}}$ | Vector containing the components of the Cauchy stress tensor (U.L. formulation). |
| $\underline{\phi}$ | A vector containing the nodal point displacements corresponding to a buckling mode shape. |
| $\underline{\phi}_i$ | A vector containing the nodal point displacements corresponding to the *i*th mode shape. |
| $\omega_i$ | Natural frequency of the *i*th mode shape. |
| $\omega_n^{(m)}$ | Largest natural frequency of element *m*. |
| $(\omega_n^{(m)})_{max}$ | Largest natural frequency of all individual elements. |