IMPLEMENTATION OF METHODS IN COMPUTER PROGRAMS; EXAMPLES SAP, ADINA

LECTURE 5

56 MINUTES

LECTURE 5

Implementation of the finite element method

The computer programs SAP and ADINA

Details of allocation of nodal point degrees of freedom, calculation of matrices, the assemblage process

Example analysis of a cantilever plate

Out-of-core solution

Effective nodal-point numbering

Flow chart of total solution process

Introduction to different effective finite elements used in one, two, three-dimensional, beam, plate and shell analyses

TEXTBOOK: Appendix A, Sections: 1.3, 8.2.3

Examples: A.1, A.2, A.3, A.4, Example Program STAP

IMPLEMENTATION OF THE FINITE ELEMENT METHOD

We derived the equilibrium equations

$$\underline{K}\,\underline{U} = \underline{R} \; ; \; \underline{R} = \underline{R}_B + \ldots$$

where

$$\underline{K} = \sum_m \underline{K}^{(m)} \; ; \; \underline{R}_B = \sum_m \underline{R}_B^{(m)}$$

$$\underline{K}^{(m)} = \int_{V^{(m)}} \underline{B}^{(m)^T} \underline{C}^{(m)} \underline{B}^{(m)} \, dV^{(m)}$$

$$\underline{R}_B^{(m)} = \int_{V^{(m)}} \underline{H}^{(m)^T} \underline{f}^{B^{(m)}} \, dV^{(m)}$$

$\underline{H}^{(m)}$ ($k \times N$), $\underline{B}^{(m)}$ ($\ell \times N$), N = no. of d.o.f. of total structure

In practice, we calculate compacted element matrices.

$\underline{K}$ ($n \times n$), $\underline{R}_B$ ($n \times 1$), . . . n = no. of element d.o.f.

$\underline{H}$ ($k \times n$), $\underline{B}$ ($\ell \times n$)

The stress analysis process can be understood to consist of essentially three phases:

1. Calculation of structure matrices K , M , C , and R , whichever are applicable.

2. Solution of equilibrium equations.

3. Evaluation of element stresses.

The calculation of the structure matrices is performed as follows:

1. The nodal point and element information are read and/or generated.

2. The element stiffness matrices, mass and damping matrices, and equivalent nodal loads are calculated.

3. The structure matrices K , M , C , and R , whichever are applicable, are assembled.

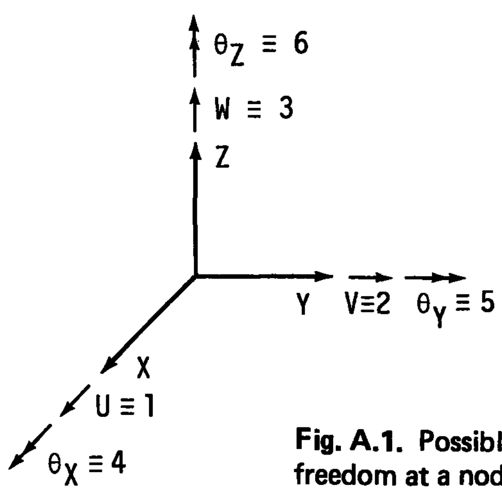
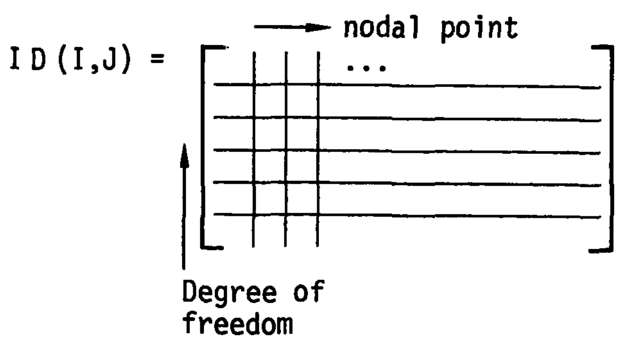


Fig. A.1. Possible degrees of freedom at a nodal point.



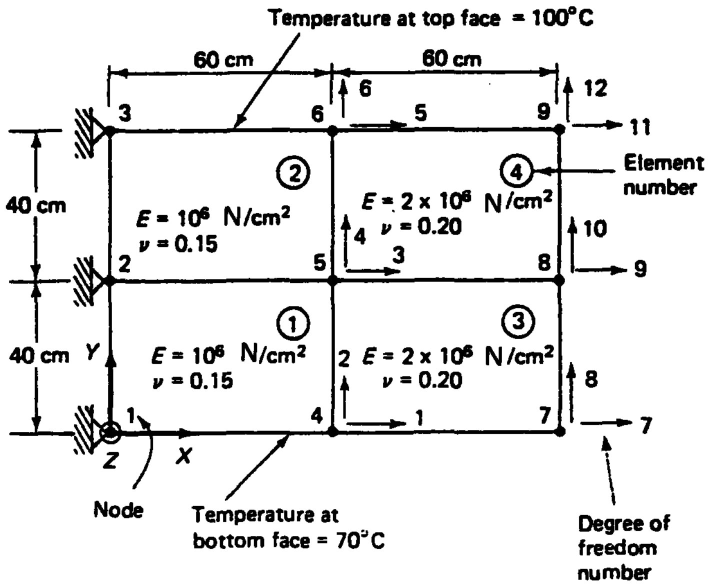


Fig. A.2. Finite element cantilever idealization.

In this case the ID array is given by

$$ID = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

and then

$$ID = \begin{bmatrix} 0 & 0 & 0 & 1 & 3 & 5 & 7 & 9 & 11 \\ 0 & 0 & 0 & 2 & 4 & 6 & 8 & 10 & 12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Also

$$X^T = [\ 0.0 \quad 0.0 \quad 0.0 \quad 60.0 \quad 60.0 \quad 60.0 \quad 120.0 \quad 120.0 \quad 120.0]$$

$$Y^T = [\ 0.0 \quad 40.0 \quad 80.0 \quad 0.0 \quad 40.0 \quad 80.0 \quad 0.0 \quad 40.0 \quad 80.0]$$

$$Z^T = [\ 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0]$$

$$T^T = [70.0 \quad 85.0 \quad 100.0 \quad 70.0 \quad 85.0 \quad 100.0 \quad 70.0 \quad 85.0 \quad 100.0]$$

For the elements we have

Element 1: node numbers: 5,2,1,4;
material property set: 1

Element 2: node numbers: 6,3,2,5;
material property set: 1

Element 3: node numbers: 8,5,4,7;
material property set: 2

Element 4: node numbers: 9,6,5,8;
material property set: 2

CORRESPONDING COLUMN AND ROW NUMBERS

For compacted matrix	1	2	3	4	5	6	7	8
For $\underline{K}_1$	3	4	0	0	0	0	1	2

$$LM^T = [3 \quad 4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 2]$$

Similarly, we can obtain the LM arrays that correspond to the elements 2,3, and 4. We have for element 2,

$$LM^T = [5 \quad 6 \quad 0 \quad 0 \quad 0 \quad 0 \quad 3 \quad 4]$$

for element 3,

$$LM^T = [9 \quad 10 \quad 3 \quad 4 \quad 1 \quad 2 \quad 7 \quad 8]$$

and for element 4,

$$LM^T = [11 \quad 12 \quad 5 \quad 6 \quad 3 \quad 4 \quad 9 \quad 10]$$

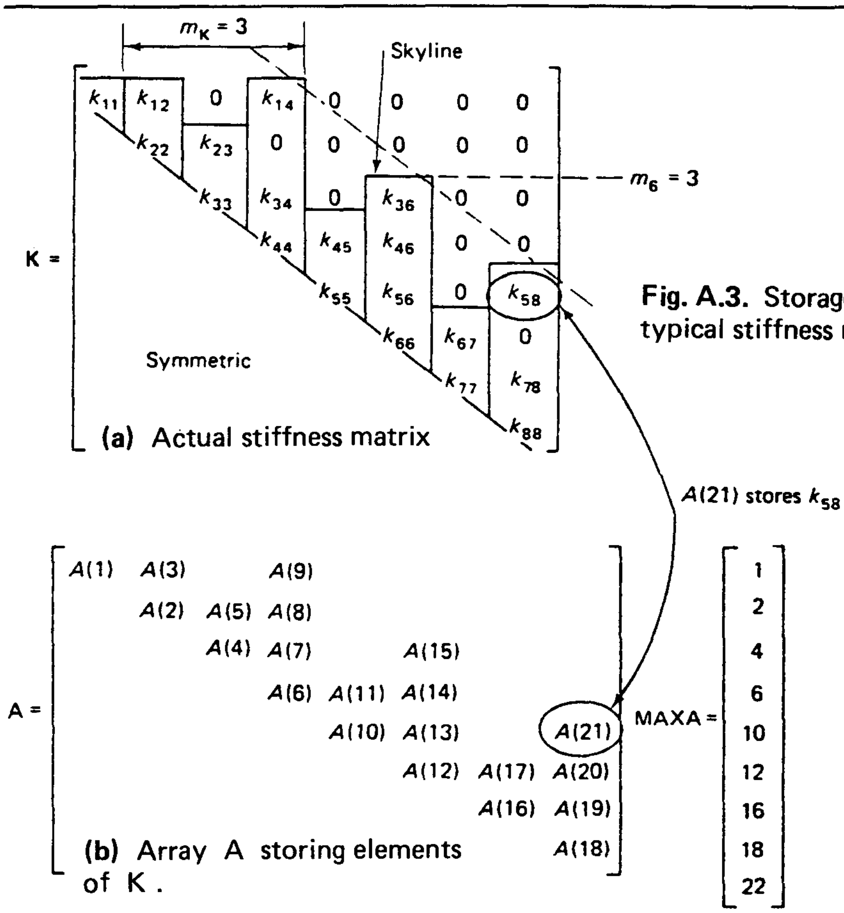


Fig. A.3. Storage scheme used for a typical stiffness matrix.

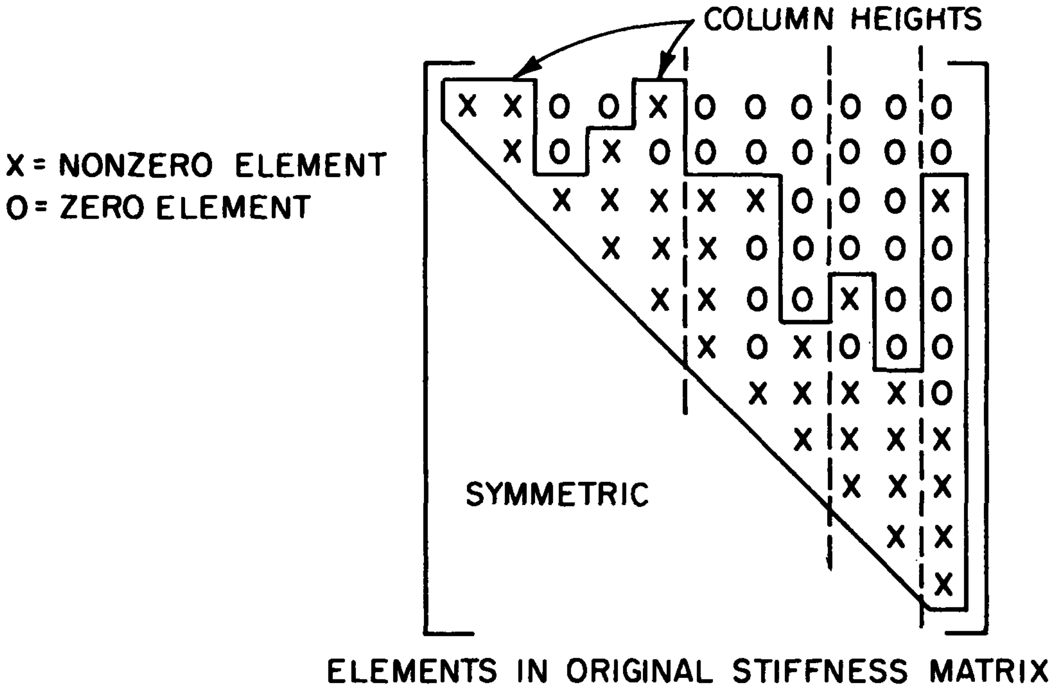


Fig. 10. Typical element pattern in a stiffness matrix using block storage.

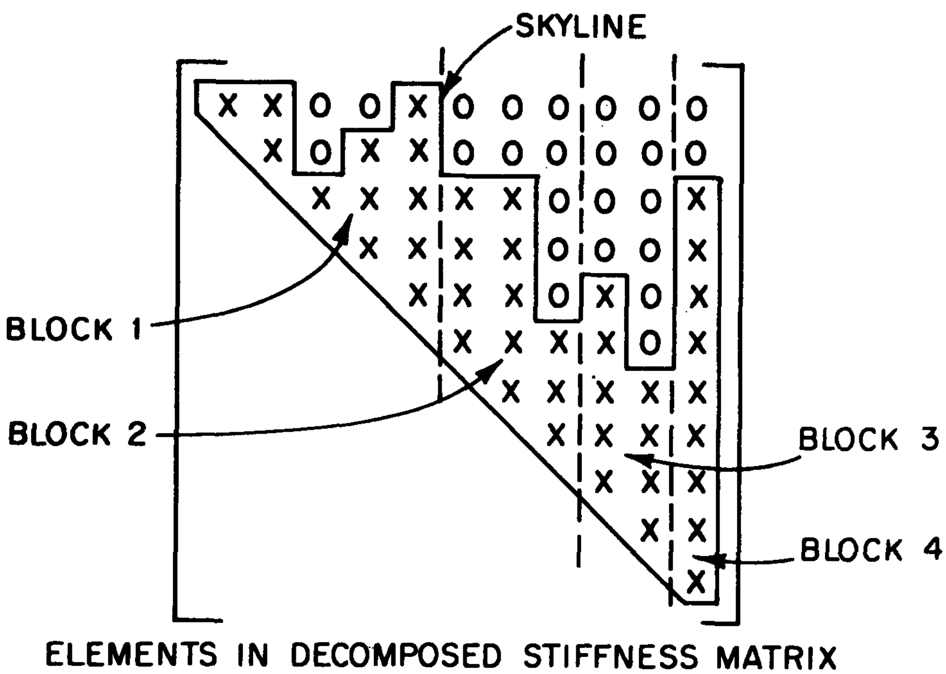
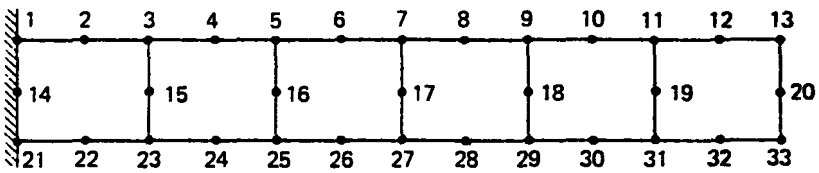
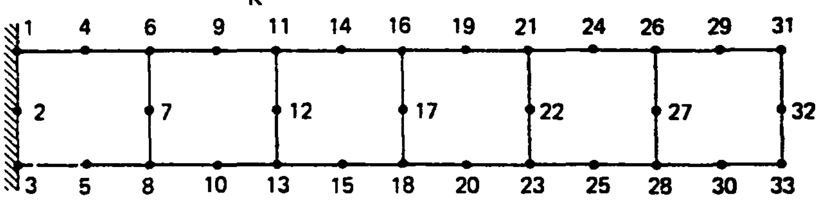


Fig. 10. Typical element pattern in a stiffness matrix using block storage.



(a) Bad nodal point numbering, $m_k + 1 = 46$.



(b) Good nodal point numbering, $m_k + 1 = 16$.

Fig. A.4. Bad and good nodal point numbering for finite element assemblage.

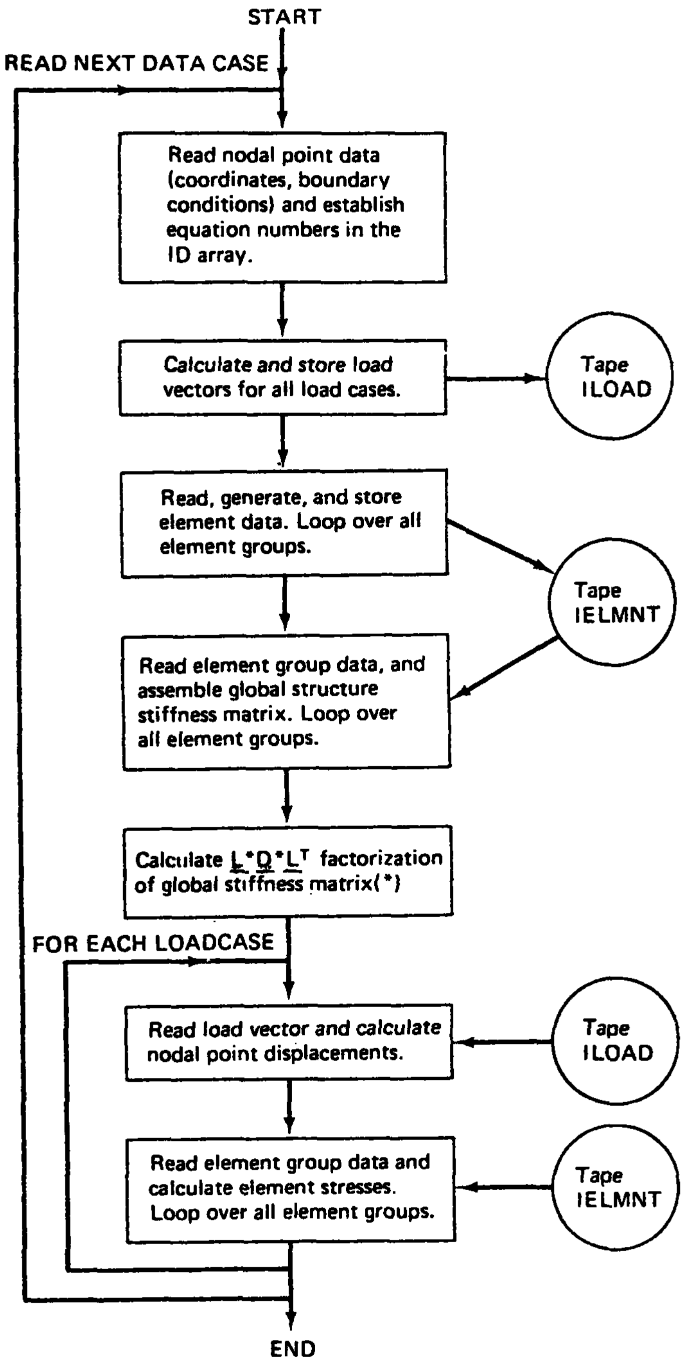


Fig. A.5. Flow chart of program STAP. *See Section 8.2.2.

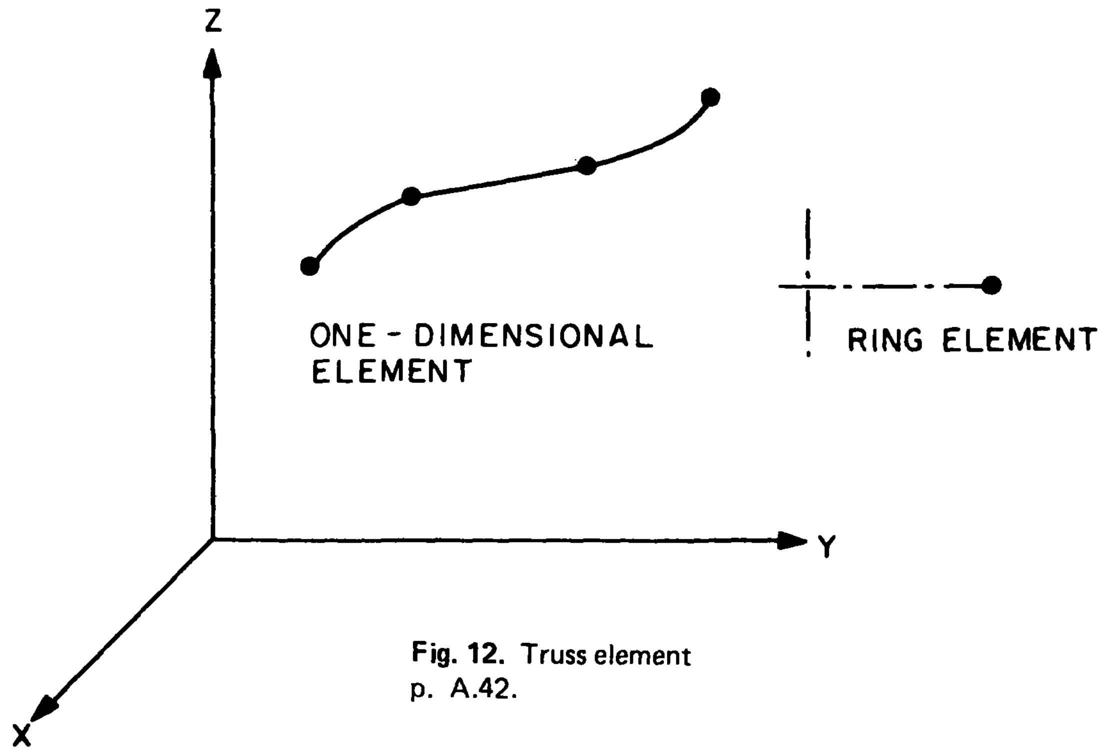


Fig. 12. Truss element
p. A.42.

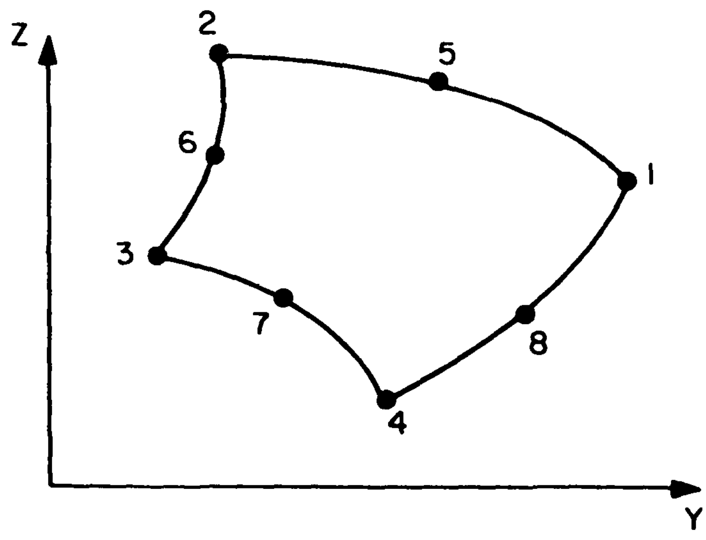


Fig. 13. Two-dimensional plane stress, plane strain and axisymmetric elements.
p..A.43.

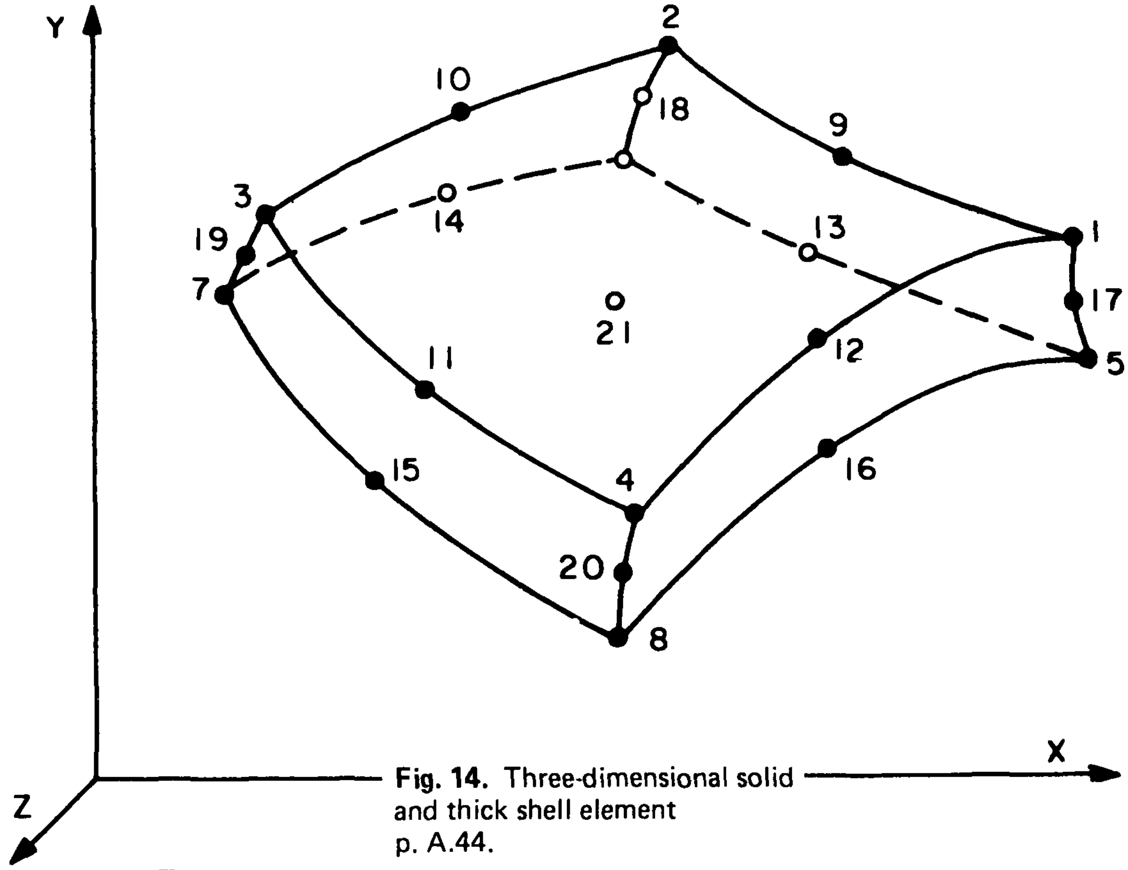


Fig. 14. Three-dimensional solid and thick shell element
p. A.44.

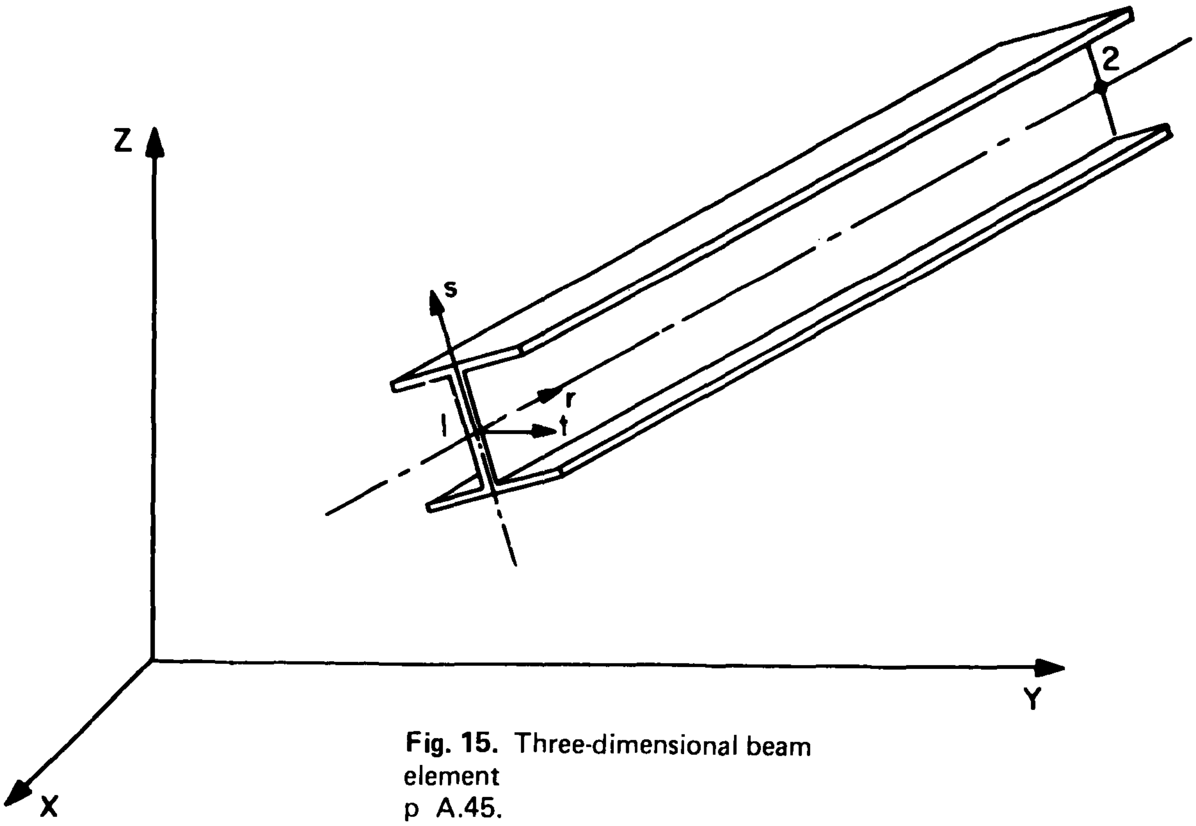


Fig. 15. Three-dimensional beam element
p A.45.

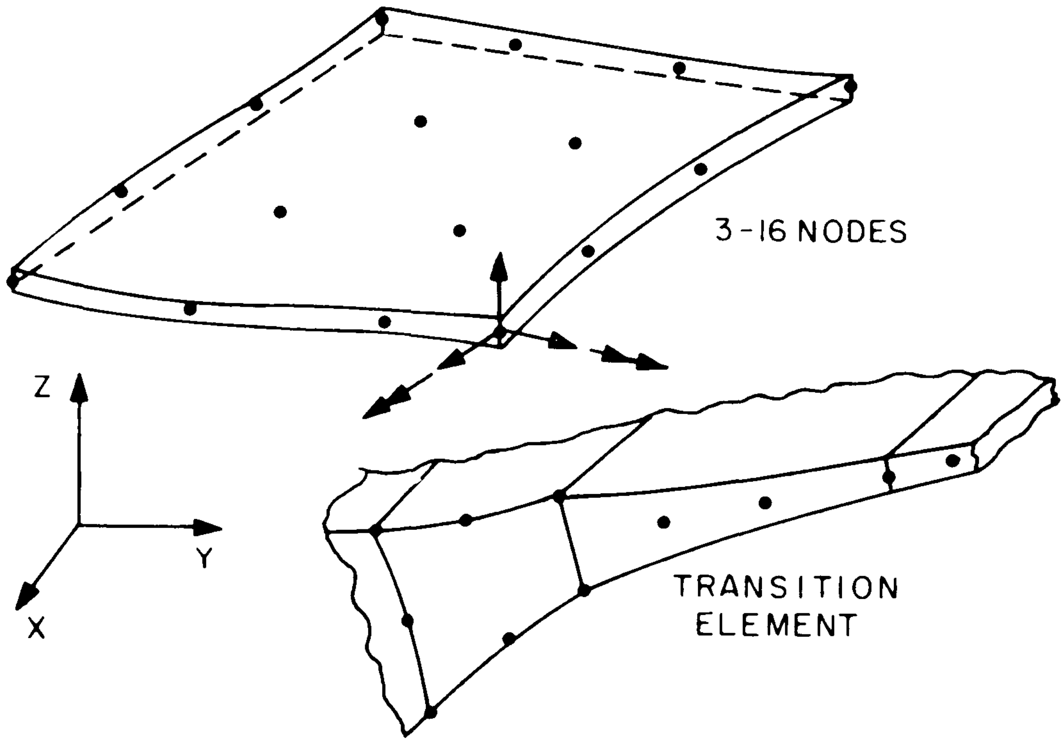


Fig. 16. Thin shell element (variable-number-nodes) p. A.46.