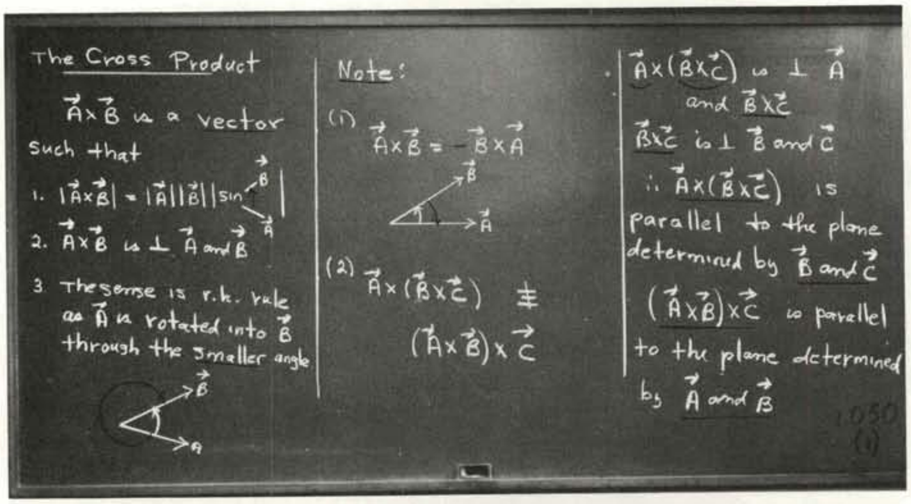
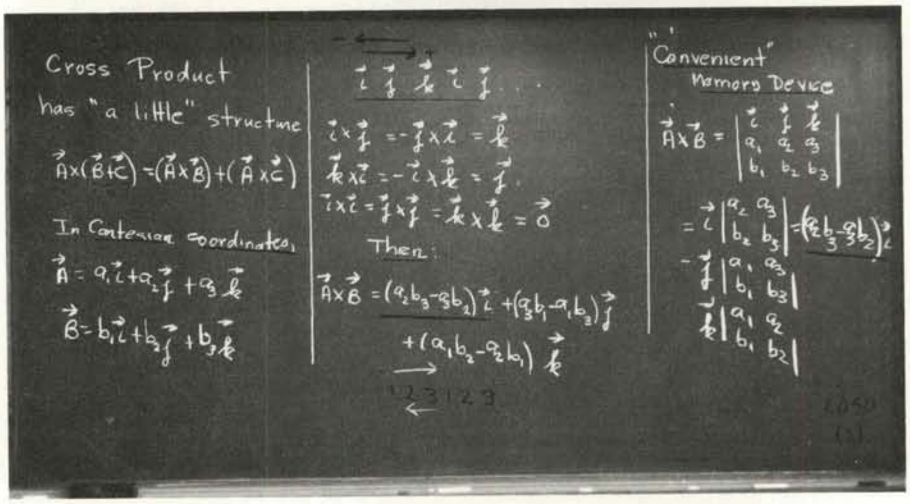
Unit 5: The Cross Product

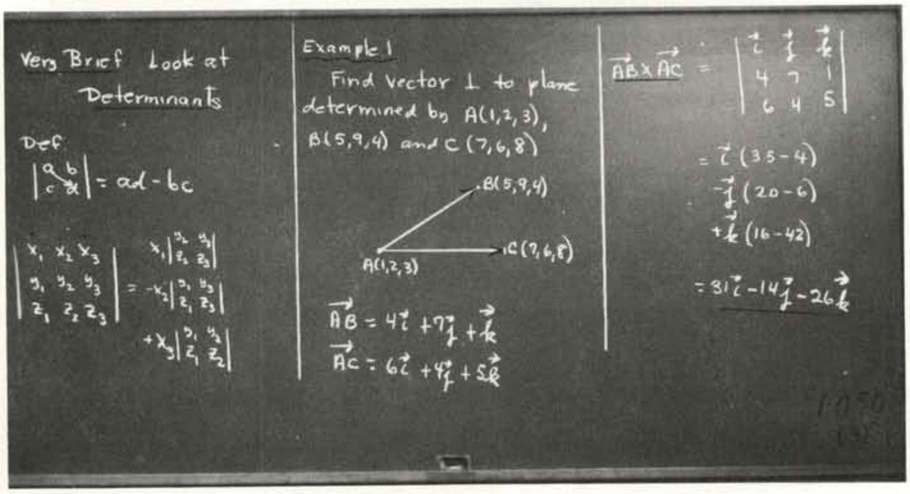
1. Lecture 1.050



a.

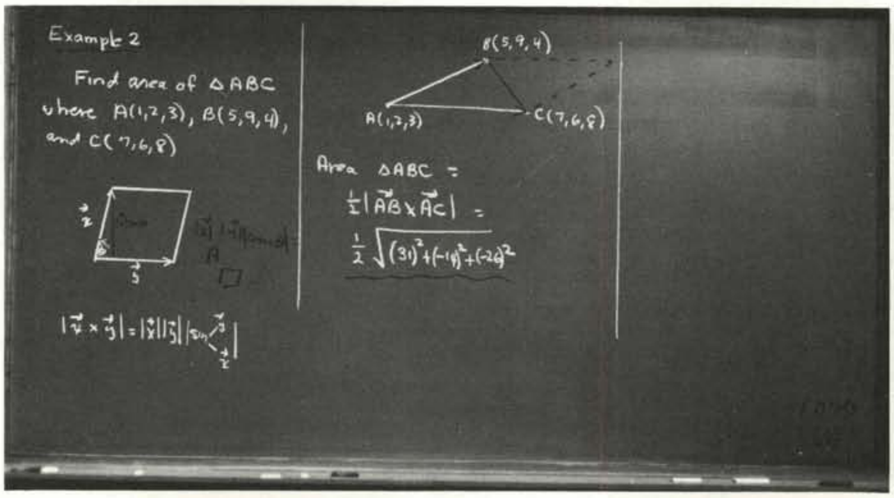


b.



c.

Lecture 1.050 continued

d. 

2. Read Thomas, Sections 12.7 and 12.9.

3. Exercises:

1.5.1(L)

Find a vector which is perpendicular to both $\vec{A} = 3\vec{i} + 4\vec{j} + 5\vec{k}$ and $\vec{B} = 2\vec{i} + 6\vec{j} + 7\vec{k}$.

1.5.2(L)

Let A(1,2,3), B(3,3,5) and C(4,8,1) be points in space.

a. Find a vector perpendicular to the plane determined by A, B, and C.

b. Explain geometrically why $\vec{AB} \times \vec{AC}$ and $\vec{AB} \times \vec{BC}$ can differ, at most, only in their sense.

c. With A, B, and C as above, find the area of ΔABC.

1.5.3(L)

a. Outline a method for finding the distance between two skew lines. (For the definition of a skew line, see the introduction to the solution of this exercise.)

b. Translate the method in (a) into a form which utilizes concepts of vector arithmetic.

c. Find the distance between the two skew lines one of which passes through the points A(1,2,3) and B(4,5,1) and the other of which passes through C(2,3,5) and D(3,6,8).

1.5.4

Let A(2,3,4), B(5,6,8), C(4,5,9), and D(6,11,14) be given points in space. Find the distance between the skew lines $\vec{AB}$ and $\vec{CD}$.

1.5.5(L)

a. Describe the direction of $(\vec{A} \times \vec{B}) \times \vec{C}$.

b. Use the answer in (a) to show why $(\vec{A} \times \vec{B}) \times \vec{C}$ and $\vec{A} \times (\vec{B} \times \vec{C})$ need not be equal vectors.

1.5.6

Let $\vec{A} = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{B} = 3\vec{i} + 5\vec{j} + 4\vec{k}$, and $\vec{C} = 6\vec{i} + 8\vec{j} + 9\vec{k}$. Use the techniques of Exercise 1.5.5 to find a vector in the plane determined by $\vec{A}$ and $\vec{B}$, and which is perpendicular to $\vec{C}$.

1.5.7

Use the result that $(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{B}\cdot\vec{C})\vec{A}$, together with the appropriate arithmetic properties of the cross product to express $\vec{A} \times (\vec{B} \times \vec{C})$ as a linear combination of $\vec{B}$ and $\vec{C}$ (i.e., in the form $p\vec{B} + q\vec{C}$, where p and q are scalars).

1.5.8

a. Express $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$ as a linear combination of $\vec{A}$ and $\vec{B}$.

b. Geometrically, what does the vector $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$ represent?

1.5.9

a. Vectors are drawn from the origin to the points A, B, and C in space. In terms of the vectors $\vec{OA}$, $\vec{OB}$, and $\vec{OC}$, express the condition that these three vectors (or the four points O, A, B, and C) lie in the same plane.

b. Use the result of (a) to conclude that the vectors $\vec{A}(1,1,1)$, $\vec{B}(2,3,4)$, and $\vec{C}(3,4,5)$ originating at a common point lie in the same plane.

c. A parallelepiped has one vertex at 0(0,0,0) and three other vertices at A(1,1,1), B(2,4,3), and C(3,4,5). Find the volume of this parallelepiped.