Inverse Functions and Logarithms

A function assigns an **output** $\boldsymbol{y = f(x)}$ to each **input** $\boldsymbol{x}$

A one-to-one function has different outputs y for different inputs x

For the **inverse function** the input is y and the output is $\boldsymbol{x = f^{-1}(y)}$

Example If $y = f(x) = x^5$ then $x = f^{-1}(y) = y^{\frac{1}{5}}$

KEY If $y = ax + b$ then solve for $x = \dfrac{y-b}{a}$ = inverse function

Notice that $\boldsymbol{x = f^{-1}(f(x))}$ and $\boldsymbol{y = f(f^{-1}(y))}$

The **chain rule** will connect the derivatives of f^{-1} and f

The great function of calculus is $\boldsymbol{y = e^x}$

Its inverse function is the **"natural logarithm"** $\boldsymbol{x = \ln y}$

Remember that x is the exponent in $y = e^x$

The rule $e^x e^X = e^{x+X}$ tells us that $\boldsymbol{\ln(yY) = \ln y + \ln Y}$

Add logarithms because you add exponents: $\boldsymbol{\ln(e^2 e^3) = 5}$

$(e^x)^n = e^{nx}$ (multiply exponent) tells us that $\boldsymbol{\ln(y^n) = n \ln y}$

We can change from base e to base 10: New function $\boldsymbol{y = 10^x}$

The inverse function is the logarithm to base 10 Call it log: $\boldsymbol{x = \log y}$

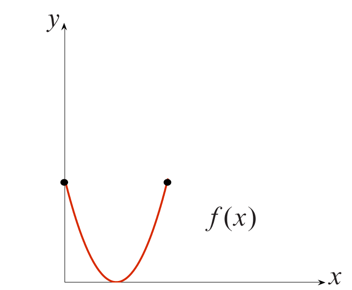
Then $\log 100 = 2$ and $\log \dfrac{1}{100} = -2$ and $\log 1 = 0$

We will soon find the beautiful derivative of $\ln y$ $\boldsymbol{\dfrac{d}{dy}(\ln y) = \dfrac{1}{y}}$

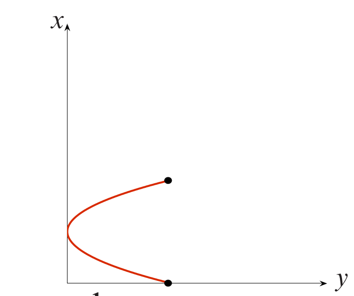
You can change letters to write that as $\boldsymbol{\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}}$

Practice Questions

1. What is $x = f^{-1}(y)$ if $y = 50x$?
2. What is $x = f^{-1}(y)$ if $y = x^4$? Why do we keep $x \geq 0$?
3. Draw a graph of an increasing function $y = f(x)$. This has different outputs y for different x. **Flip the graph (switch the axes) to see $\boldsymbol{x = f^{-1}(y)}$**
4. This graph has the same y **from two $\boldsymbol{x}$'s. There is no $\boldsymbol{f^{-1}(y)}$**



$\boldsymbol{f(x)}$ **is NOT one-to-one**



$\boldsymbol{f^{-1}(y)}$ **is NOT a function**

5. The natural logarithm of $y = 1/e$ is $\ln(e^{-1}) = ?$ What is $\ln(\sqrt{e})$?
6. The natural logarithm of $y = 1$ is $\ln 1 = ?$ and also base 10 has $\log 1 = ?$
7. The natural logarithm of $(e^2)^{50}$ is ? The base 10 logarithm of $(10^2)^{50}$ is ?
8. I believe that $\boldsymbol{\ln y = (\ln 10)(\log y)}$ because we can write y in two ways $y = e^{\ln y}$ and also $y = 10^{\log y} = e^{(\ln 10)(\log y)}$. Explain those last steps.
9. Change from base e and base 10 to **base 2**. Now $y = 2^x$ means $\boldsymbol{x = \log_2 y}$. What are $\log_2 32$ and $\log_2 2$? Why is $\log_2(e) > 1$?