Topics in Fluid Dynamics: Dimensional Analysis, the Coriolis Force, and Lagrangian and Eulerian Representations

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Author's Preface: This collection of three essays grew out of my experience teaching the introduction to fluid mechanics as part of the MIT/WHOI Joint Program in Oceanography (Fluid Dynamics of the Atmosphere and Ocean, 12.800). Students enter the Joint Program with widely varying experience in mathematics and physics; the goal of this introductory course is to help each student learn the concepts and mathematical tools that will be the essential foundation for their later study of geophysical fluid dynamics and for their research in oceanic and atmospheric sciences.

There are a number of modern comprehensive textbooks that can serve the purpose of this course very well. However, it seemed to me that there were three specific topics that could benefit from a greater depth of treatment than a comprehensive text can afford; these were (1) dimensional analysis, (2) the Coriolis force, and (3) Lagrangian and Eulerian representations of kinematics. A quick treatment of either of these first two topics leaves most students in a state of complete mystery, and the third topic is either omitted or barely noted in introductory texts. But no matter how forcefully stated, this is obviously a highly personal judgment — what is clear and sufficient for one student (or instructor) may be inadequate for another student (or instructor) having a different background or level of interest. For example, the origin of the Coriolis force is described rigorously in many classical mechanics texts. However, to most physics or engineering students the Coriolis force amounts to little more than Foucault pendulums and deflected rockets — curious, but not crucial. To a student of the atmosphere or ocean, the Coriolis force is something that will be encountered nearly every working day and so a few additional hours devoted to a thorough understanding of its origin and consequences will, for them, be time well spent.

With that as the motive and backdrop, I set out to write three essays dealing with each topic in turn and with the hope of making a clear and accessible written source for introductory-level graduate students. Some topics are accompanied by software and numerical problems (available from the Matlab File Central archive). Compared with the necessarily brief treatment of most introductory texts, these essays go into greater depth, and occasionally make explicit mention of the relevant contrast class, i.e., a useful explanation must show what *is*, and it can be very useful and illuminating to consider what alternatives may exist and to understand at least a part of what *is not*.

These essays, and one could say fluid mechanics generally, are highly mathematical. Readers are assumed to have a background in applied mathematics including some exposure to partial differential equations. It is one thing to solve a certain class of equation in the context of a mathematics class where the equation will be more or less expected, and quite another to encounter the same equation in a fluid mechanics problem where it may arise without warning as a consequence of an approximation or simplification. Hence, the necessary mathematics is introduced slowly and deliberately, but with the assumption that it will not be completely new to the reader.

The first two of these essays, *Dimensional analysis of models and data sets; scaling analysis and similarity solutions* and *A Coriolis tutorial*, have a modest and clearly defined scope. Examples treated in these essays are often geophysical or fluid mechanical, but dimensional analysis in particular has an extremely wide range of application. The scope and the goal of the third essay, *Lagrangian and Eulerian representations of fluid flow: kinematics and the equations of motion*, is in some respects more ambitious. At one level the goal is to introduce the kinematics of fluid flow and specifically the notion of Lagrangian and Eulerian representations of fluid flow. That is challenging in itself. But on a deeper level the goal is to define a theme for fluid mechanics by addressing the kind of question that lurks in the minds of most students — what is it that makes fluid mechanics different from the rest of classical (solid particle) mechanics, and while we are at it, why is fluid mechanics so difficult? I suspect that these big questions are answered only in part.

These essays are pedagogical in style and in purpose. They are in the public domain for all personal, educational purposes and they may be freely copied for classroom use. I would be pleased to hear your questions or comments, and especially grateful for suggestions that might make them more accessible or more useful for your purpose.

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