MITOCW | MITRES_10_S95F20_0502_300k

PROFESSOR: So another important source of convection in an air filled room is buoyancy due to differences in the density of the air as the temperature varies.

Even relatively small variations in temperature can lead to significant flows.

There's another dimensionless number, which controls the appearance and strength of such flows, which is the Rayleigh number, written Ra.

And this is also a physical property of the-- it's a combination of physical properties of the fluid plus the geometry.

So in this case, the relevant geometrical scale is the height, because this is a gravitational instability.

So in the Rayleigh number, we have gravity.

I'll just define all these-- a gravitational acceleration, which is 9.8 meters per second squared.

We have-- well, if we define the change in air density relative to the initial air density that's caused by changes in temperature, if the temperature changes aren't too big, there is a linear response, which is defined by the thermal expansion coefficient beta, so beta delta T.

So beta is the thermal expansion coefficient.

And this, for air, is something like 3.1 times 10 to the minus 3 inverse Kelvins.

OK, so basically we have gravity times the change in density.

That's the buoyancy force per volume.

And so we can write that as beta delta T. Or I should say, delta T is T maximum minus T minimum.

So there's some temperature change.

So the example we're going to consider is a cold plate above a hot plate with a fluid in between.

And then we have also some other parameters.

So we have-- the length comes in cubed now.

So it's very sensitive to the size of the system.

And then we also have the kinetic viscosity of air, which I'll write again here.

This also appeared in the Reynolds number.

And for air, this is 1.5 times 10 to the minus 5 meters squared per second, roughly.

And then finally, we have alpha T, which is the thermal diffusivity.

So this parameter gives a sense of how quickly heat energy is transmitted by conduction and diffusion through the fluid.

And thermal diffusivity, turns out, is pretty close to the kinematic viscosity for a gas, like air.

And the reason is that the ratio of thermal-- well, in other words, the ratio of the kinematic viscosity to the thermal diffusivity for air is around 0.7.

And this ratio, by the way, is called Pr-- the Prandtl number-- which is also very important in these sorts of flows.

And for gases, basically kinematic viscosity refers to the diffusion and momentum in the air, whereas the thermal diffusivity alpha is the diffusion of heat energy.

And in a gas, both those processes occur by collisions of molecules.

And since it's the same mechanism, you have roughly the same order of magnitude of those quantities.

So basically, all these quantities enter the Rayleigh number.

And the way to think about, qualitatively, what the Rayleigh number is telling us is the ratio of buoyancy force to viscous stress, which is trying to fight that motion as we talked about before, but also heat diffusion or thermal diffusion, which is also kind of fighting it, because it spreads out the temperature gradient because this is a motion that is naturally driven whenever a temperature gradient exists which is unstable.

So because this beta is typically positive-- so when you heat the fluid, it expands.

That's certainly the case for most gases and even for many liquids.

Then what I've sketched here is an unstable density gradient, where if the cold is above the hot, there's a heavy fluid above a light fluid.

And at conditions of low Rayleigh number, this is stable.

And in particular, if the Rayleigh number is less-- for this particular case of two fixed plates and an infinite layer of fluid, if the Rayleigh number is less than 1708, then we have a stable situation.

Or it can be at least meta-stable.

It won't go spontaneously unstable, so at least local-- stable to small perturbations.

But then at this critical Rayleigh number of 1708, we start to get some spontaneous flows, because what's happening is that the heavy fluid above, which I've sketched in blue-- the cold fluid-- it wants to sink to the bottom, whereas the red, warmer fluid is lighter and wants to rise to the top.

And so it has to find a way to do that.

And eventually, it breaks symmetry and just starts forming convection.

And that is so-called natural convection.

So you have plumes of hot fluid rising and cold fluid sinking, driven at first by fairly regular arrays.

But as you increase the Rayleigh number even further, and if you increase it a lot, then you eventually get to a complicated turbulent flow.

So if you increase the Rayleigh number on the order of 10 to the 4, you may have some unsteady situations as we saw with the vortex shedding.

But here, if you go to a very high Reynolds number-- Rayleigh number, I should say-- greater than about 10 to the 9, then you again get turbulence.

So simply these temperature variations are strong enough-- those buoyancy forces-- to completely destabilize the fluid and generate a turbulent mixture where the hot and cold are very quickly mixing.

And I've sketched here the hot and cold as still being separate.

But in fact, due to diffusion, they will kind of also be re-equilibrating all the time as well, although the temperature gradient is needed to kind of maintain that flow.

So let's see how big the Rayleigh number is in different situations of interest for indoor air now, as well.

So if we look, let's say, in a room-- so let's have-- this is another room.

And let's imagine again, we have our heating and ventilation air conditioning.

Let's say, it's an air conditioning unit on the top, which is dumping in some cold air.

And it's giving it some velocity.

So the velocity is associated with the Reynolds number.

That inertia will lead to destabilization and vortices and mixing by itself.

But let's see what the effect is of the temperature difference, OK?

So we're injecting cold air.

And we usually put it on top, because we actually want good mixing in the room.

That's how these systems are designed.

We also inject heat normally from below, for example, from the lower sections of the wall or from the floor.

And so if we were-- and in fact, I could just mention, if we were to heat, we would do that.

And if we're doing air conditioning or cooling, we would do it from above.

And so basically, there is an unstable gradient like this.

And let's actually put some numbers in here.

So what if we say that the temperature difference between the fluid we're injecting, whether it's heating or cooling, relative to the sort of background air in the room, which is, let's say, closer to the target temperature-- is approaching target temperature-- let's say, it's only 10 degrees C. So that seems like not a very big difference.

But then we go back to our height, which is our length scale.

And we say it's 2.7 meters, just as sort of a standard ceiling height.

If you plug in the properties of air with these numbers, the Rayleigh number is actually 10 to the 10.

It's enormous.

So that tells you that if you-- and that's partly because of the huge scale here, right?

So H comes in cubed.

So if you have 2 or 3 meters of height, that's a lot of height.

And so if you are maintaining that kind of temperature difference across such a height, you're going to be generating very serious convection in that system.

So what's happening is that besides the fact that you're blowing and generating flows by inertia, you also have these thermal flows going on, which can be very, very strong in a system when there's even just a few degrees of temperature difference.

You've probably seen dust in the air near a sunny window, which allows you to visualize the flow.

And even if nobody's moving-- the air is fairly still-- you might see plumes of rising air in one location or sinking in another air.

And if you look closely, those plumes may actually have very complex convective instabilities and turbulence even, even when the temperature differences are not so great.

And in fact, we can see such things.

For example, if we have, let's say, a window-- and let's say that it's cold outside.

And it's warm inside.

Then just simply that temperature gradient means that there's a colder air near the surface that wants to sink.

And so the flow rate is going to look-- or the flow is going to look something like a boundary layer flow of fluid that is sort of falling near the surface.

And if the Reynolds number gets high enough, this flow can actually become itself unstable as it kind of goes down the surface.

So you can see that these rising or falling plumes of natural convection near vertical services that are heated or cooled relative to the environment can also lead to complex flows.

And actually, a good example of that is the flow that occurs around a person just simply due to the temperature.

So if you look very closely at a person-- I'm not going to draw it very well here.

But let's just say, we have a person.

And that's supposed to be a head, OK?

If you look very closely, the body has a temperature which is usually higher than the ambient by at least 10 degrees if not more.

And if you now plug in a little bit smaller size-- let's say, we plug in 30 or 40 centimeters.

And we-- so let's do that actually.

So if we say that here, maybe, H would be on the order of-- well, since this was 3 meters, we'll go down to 0.3 meters.

So we'll drop the size roughly by a factor of 10.

But it comes in cubed .

So that drops the Rayleigh number by a factor of 1,000.

So if we still keep our delta T at 10 degrees-- and it might actually be much more than that-- the Rayleigh number around this person's head, just simply by virtue of the heat generated by the body, can be of order 10 to the 7.

So it may not be quite into the turbulent regime.

But it's certainly in the regime where there'll be some unsteady complicated flows due only to natural convection.

And we're not even talking about the person moving, which gives you even more flow.

And so what it actually looks like if you look closely-- the air around a person is actually rising almost like a chimney, driven by these thermal flows.

And those flows even can go turbulent or at least generate some vortical structures.

And of course, these kinds of flows are just due to temperature.

In all these cases due to the HVAC and also due to the person, there are these convective flows that we've talked about from inertia, which also contribute to mixing.

So as we'll talk about shortly, we also know that this person is breathing.

Let's say, they're just breathing through the nose, even.

Then there's some puffs that are generated.

And you've got the thermal stuff going on.

Also, the air that you're breathing out is warmer than the ambient.

So it tends to want to rise as well.

So I hope I've convinced you here that-- I'll write this as, these could be buoyant respiratory jets and puffs.

So I guess the first part of this section is just to convince you that the conditions in a room are such that we have good reason to believe that there is significant mixing of the air, either due to inertial effects from movement, from ventilation air flows, or from thermal effects, as we have sketched here.

And so at least, that gives us a beginning of a justification for our assumption of a well-mixed room.

So in this video from Linden's group at the University of Cambridge, we can see a visualization of the airflow around a person who is speaking or just breathing.

And the videos are taken by a differential synthetic schlieren imaging method, which allows you to see basically the changes in density in the flow.

And what we see are these thermal plumes of warm air rising past the body due to the difference in the body temperature and the ambient air temperature.

And we also see, on top of that, repeated puffs coming from the breathing, which interact with those plumes and also themselves have buoyant and turbulent flows therein.

In the next video, we see how different the flows are when masks are worn.

So we can still see the thermal body plume rising vertically past the person's face.

But now the mask is preventing the transfer of momentum to the fluid to push forward these puffs.

And instead, we see the leaking of some of the breathed, exhaled air rising, almost entrained in the turbulent thermal plume rising upwards rather than being ejected forward.

And so this helps to eliminate short-range transmission due to those puffs and really brings us closer to an airborne model of a well-mixed room.