PROFESSOR: So as an aside, for those of you who have an understanding of probability theory and stochastic processes, let me just explain why it is valid to bound the main transmission rate in defining the indoor reproductive number when we might be more concerned about bounding the probability of a transmission.

So let's let a capital T be a random variable which describes the random number of transmissions that occur in the room.

And this is specifically for the case for one infector or infected person and n minus 1 susceptible.

So again, the situation of the reproductive number where for every person that comes in we want to know is there going to be transmission, and typically, only one infected person would be seen.

So this is a random variable, and let's let ft of n be the probability density function that gives the probability of little n transmissions.

And then we can define the risk of a transmission, the risk of at least one transmission, as the probability that this random variable takes on a value, which is greater than or equal to 1.

OK.

Well, in terms of the probability density function then, that would be a sum from 1 to infinity of the ft event, basically.

So we're just adding the probabilities that we're not seeing a transmission to those-- or possible transition to those different numbers of people.

Now I'd like to do a little calculation to get an upper bound on this quantity.

So we can say that this is less than or equal to the sum from n equals 1 to infinity of n ft of n.

Now, this is just a mathematical trick here.

So n refers to the natural numbers 1, 2, 3, 4, et cetera.

Those are all positive numbers and they're all greater than or equal to 1.

So if I take 1 in this expression over here and replace it with little n, I'm only increasing the value of that sum, because also, this ft is a probability density that has to be positive.

And then now I can also say that this is actually equal to throwing in n equals 0, because that is a term that is actually identically 0.

So I can change the summation.

And then by definition here, this thing is the expected value of the number of transmissions, because I'm summing the number of transmissions little n times the probability of that event occurring.

So that is the definition of the average.

So what we're seeing here is that the risk of a transmission is rigorously bounded above by the expected number of transmissions.

And so therefore, if we require that this is less than epsilon, our risk tolerance that we've just introduced, this is a conservative bound on the true risk, which let's say here is defined by rt.

So if your goal is to control the probability of having at least one transmission, so basically to ensure that no transmissions occur, then you would do well to bound the expected number of transmissions, because that's an upper bound.

It can also be shown that as epsilon goes to 0, so we're talking about very low probabilities of transmission, and oftentimes that is the case, then rt is asymptotically the same as the expected number of transmissions.

So this overall risk of a transmission and the expected number are the same.

And in fact, that's one way to understand sort of even the definition of a probability in terms of an expected number of events.

And we are typically thinking of cases where epsilon is much less than 1.

So for those of you that have some background in probability, you may recognize that what I've just done here is an example of a much more general result, which is called Markov's Inequality.

So now we can safely proceed by continuing to work with average values of all the quantities of interest.