

MITOCW | Investigation 3, Part 2

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MARK HARTMAN: Are we ready?

AUDIENCE: Yes.

MARK HARTMAN: So I think in general, we saw that the black-body model fits our observations pretty well. And I think what we'd like to do now is to just-- let's think about how can we tell a good fit. Because we're saying we're taking-- again, write down today's date, the time, [INAUDIBLE] for a few minutes. What is our observation in this case? Chris, what is our observation?

AUDIENCE: That the black body fits better than the power law.

MARK HARTMAN: Well, don't go even that far yet. Let's back up. What is the data that we collected? What is our observation about this object?

AUDIENCE: The plot.

MARK HARTMAN: The plot. Which plot?

AUDIENCE: The black body and the power [? log. ?]

MARK HARTMAN: Are the black body and power law an observation?

AUDIENCE: No, no.

MARK HARTMAN: No, they're not.

AUDIENCE: [INAUDIBLE]

MARK HARTMAN: What did we actually collect, Chris?

AUDIENCE: We collected the [? counts ?] and the energy.

MARK HARTMAN: OK, we looked at the number of counts as a function of energy, or the intensity as a function of energy. What do we call that?

AUDIENCE: Power?

MARK HARTMAN: No. Chris?

AUDIENCE: The flux?

MARK HARTMAN: OK, we're collecting the flux at each energy. If I had a graph that had intensity as a function of energy, what do I call this-- let's say there's a staircase thing. What do we call this?

AUDIENCE: Spectrum.

MARK HARTMAN: OK. Nice and loud, Steve.

AUDIENCE: Spectrum.

MARK HARTMAN: We call this a spectrum. We observed the spectrum, spectrum of a neutron star. Was the spectrum different when we did the two different fits?

AUDIENCE: No.

MARK HARTMAN: No. We're fitting to the same data. We always look at that same spectrum. So our observation is the data that we collect from the object, that spectrum. Now, what, in this case, is our model? What model or models did we use?

AUDIENCE: Black body and power law.

MARK HARTMAN: OK.

AUDIENCE: Histogram.

MARK HARTMAN: Nice and loud.

AUDIENCE: Black-body and power-law models.

MARK HARTMAN: OK. We do have, Juan, a histogram, but a histogram is both-- it's the observation. It's the spectrum here.

But Steve just said we looked at two different models. We looked at a black-body model, which is a mathematical way to describe that curve. We also looked at a power-law model. So our model, we had two. Let me just say we had a black-body model, and we had a power-law model.

So each one of those, we tried to get the model-- and it usually showed up as green. We tried to get the model to go as close to this as we could. This model allows us to make some

predictions. What are the predictions that we were able to make?

Let's just take for the black-body [? model. ?] What predictions did it allow us to make? What values did we get because we fit the black body?

AUDIENCE: The green was more precise for the black-body [? scale. ?]

MARK HARTMAN: OK. So maybe one model fit better, but what was in the output file for each one of these models? Remember? Think back to what we did this morning. We changed a property of the black body to get it to fit the [? sun. ?] What property of the model did we have to change?

AUDIENCE: The [INTERPOSING VOICES]

MARK HARTMAN: Hang on. Let's go back to raising our hands, OK? So what property did we predict? OK, I saw Chris, and then Steve. Go ahead, Chris.

AUDIENCE: [? We ?] changed the temperature [INAUDIBLE].

MARK HARTMAN: OK, we had to change the temperature. So a black-body model allows us to make a prediction of temperature. Anything else? Steve, was that what you were going to say? OK. What other information did we get from fitting the black-body model? What else was in that text file? Bianca? Nice and loud.

AUDIENCE: Flux.

MARK HARTMAN: OK. We predicted the temperature, and we also predicted the flux. That's useful. That's interesting. We can do something with that. Go ahead, Juan.

AUDIENCE: Energy.

MARK HARTMAN: Energy. Where did it predict the energy?

AUDIENCE: Oh. Wrong one. Sorry.

MARK HARTMAN: Unfortunately, in this case, we measure temperature in units of keV. We're going to show you how to switch that back. But for the power law, what predictions were we allowed to make? You should be writing this down. Were we able to predict a temperature? What were we able to predict, Nikki?

AUDIENCE: For the power law?

MARK HARTMAN: For the power-law model. Hang on [? one sec. ?]

AUDIENCE: Yeah. [? We got ?] magnetic field of the [? sun. ?]

MARK HARTMAN: OK. Did we get a number that said magnetic field equals? What did we get? Did we get a temperature? No? What did we get in that text file? Juan.

AUDIENCE: Power-law index.

MARK HARTMAN: OK, we got the index. Remember, when we fit the power law this morning, we changed the power-law index, and that changed the shape of the model. So we got a power-law index. And what else did we get? Lauren?

AUDIENCE: Oh, I was--

MARK HARTMAN: Oh, you going to say--

AUDIENCE: Power-law index, yeah.

MARK HARTMAN: OK. Was there anything else in that text file for the power law?

AUDIENCE: Power-law index.

MARK HARTMAN: Yeah, power-law index. Whenever you get a piece of text that is tossed out at you, you always need to read it very, very carefully. You also got a prediction of the flux from the power-law model. Every time you fit a model to a spectrum, it's going to give you the flux, and then some other parameters that describe that model.

For anybody who is taking calculus or may soon be taking calculus, the flux is simply the area underneath this curve. So what we do for each case is if we have a model, what the computer does is it figures out the area underneath the curve of the model. And that's what it gives you back for the flux.

But there's actually a couple of different predictions of flux, and we're going to get to that in just a minute. But here's the way that we've been thinking about things. We have an observation. We have a model that we now want to fit to that observation, and it allows us to make some predictions.

Now, just because we fit a power law-- I think everybody agreed that the black-body model fit

better. So let's take a look at what do we mean by if something fits better. And I think most people have this already.

When you're looking at data like this, what we would like to do is we want to look at a couple of different ways for us to figure out which fit is better. There's three different ways that we can figure it out, three ways to tell a good fit. The first is to check the fit by eye. What does it mean to check the fit by eye? What do you think?

AUDIENCE: See if the lines match up/

MARK HARTMAN: See if the lines match up.

AUDIENCE: Reading--

MARK HARTMAN: Reading.

AUDIENCE: --the data.

MARK HARTMAN: Oh, reading the data? OK. What did you say?

AUDIENCE: Comparing.

MARK HARTMAN: Comparing. You want to compare. Now, we don't necessarily always have two models that we want to pick. But we want to compare the model to the data.

So if we check the fit by eye, we see if something is a good fit. Good equals the model passes near most data points, or rather-- I don't like to use data points-- most observation points.

AUDIENCE: What's wrong with data?

MARK HARTMAN: It's just not specific enough, because data, we could say, is numbers that we figured out from something. So these measurements of temperature flux, those can be data. But we want to say that the model passes near most observation points. Again, the observation is the spectrum itself, and the model is the line or the mathematical shape that we try to get close to those observations.

So that's the first way to do it, which I think everybody said. Everybody said, if you look at the model and the green line passes pretty close, then that's a good fit. And we saw, in our case, the black-body model line went pretty close.

The second way that you can do it is you can check what we call the reduced chi-squared statistic. Now, I'm going to put this in brackets because when you get your data out, it's just going to say the reduced statistic. What this is, this chi-squared-- chi is a Greek letter. That's chi-squared. It is a measurement that tells you how close is your model to the observation.

And a very, very simple way to think about it, it is one number. So we'll say that this is a number which is the average number of error bars away. Well, let's just say the number of error bars between observation and model.

Let's just do a quick example. So it's the average number of error bars between the observation and the model, average meaning averaged over all points in your spectrum. So say I had intensity versus energy, and I had this data point, that data point, and this data point. And I had error bars, which are indications of how well do I know really where that value is.

Say I had error bars, which we normally show like that. That means this value is probably right about here at this intensity, but it could be a little higher. It could be a little bit lower. If I had a model that went like this, that's not a very good fit, right?

So let's say at this point of energy, my observation is down here, but my prediction is actually up there. At this energy, I'm predicting that I'm going to get that much flux or that much intensity. At this energy, I'm predicting I'm going to get that much. And at this energy, I'm predicting I'm going to get that much.

In this case, my prediction is different from my model by-- here's the width of one error bar. So I'm going to say 1, 2, 3. So here's the width of my error bar-- 1, 2. That's three error bars away. In this case, I've got-- there's the size of my error bar-- 1, 2, maybe 2.8. And in this case, I've got-- here's the size of my error bars-- 1, 2.

So what I would do is I'd take the average of how far away am I in both cases. So in this case, my chi-squared is going to be 3 plus 2.8 plus 2. If I wanted to take the average of how far away that is-- that's reduced chi-squared, and so they put this little χ or this ν down here. That's 3 plus 2.8 plus 2 divided by 3.

AUDIENCE: Why 3?

MARK HARTMAN: Because that's the number of data points that I have. So on average, this is maybe 2.8, something like that. So my reduced chi-squared statistic here is going to be about 2.8. So it's not a very good fit.

What I want is that instead of that, I want to be able to say-- I want my model to kind to do this, right? Maybe it doesn't quite fit all of them, but the reduced chi-squared-- here, that's only maybe half an error bar away. Here, that's right on. And then here, maybe that's one error bar away.

So a good chi-squared, which means you've got a good fit-- so we're going to say good equals close to 1. If that number is close to 1, that means, on average, you're only about one error bar away when you predict using your model to what you actually see with the data. Yeah, Bianca?

AUDIENCE: So in the blue region of the nebula that we saw before, if it matched it completely on every single point, is that 1 or 0 because there's no error bar?

MARK HARTMAN: In this case, we didn't show the error bars when we looked at the Crab Nebula spectrum. So it's kind of hard to tell unless you know how big the error bar is. Typically, you're not going to get something that fits that perfectly. And I think in that case, the data was actually fudged a little bit so that would be a perfect power law.

But on average, if it's less than 1, what that means is probably your error bars are a little bit too big. [? Because ?] then, if you're so close to the actual value-- so it should be around 1. And I will say that this is the last time that we're going to get models that fit great. Because you will have [? seen, ?] for the black body, what was the reduced statistic that we got? Anybody?

AUDIENCE: What's [? that? ?]

MARK HARTMAN: What was the reduced statistic that we got when we fit the black-body model here?

AUDIENCE: 12.58.

AUDIENCE: I got 4.07.

MARK HARTMAN: OK, 4.07. It should be around 4. You got 12?

AUDIENCE: [INAUDIBLE]

MARK HARTMAN: OK.

AUDIENCE: 9.915.

AUDIENCE: Yeah, around [? 4. ?]

MARK HARTMAN: OK. So in our case, we had the black body was around 4. The power law was around what?

AUDIENCE: 12.

MARK HARTMAN: Around 12. So this is another way to check. OK, yeah, it looks good by eye, but this is another way. You don't want to count on just one of these ways to figure it out. Because eventually, we're going to have you fit different spectra models to the same source that we don't know what it is. In this case, we knew that it was supposed to be a black body. In other cases, we don't necessarily know. So you'll have to figure out which one of these fits the best.

Now, the last thing that we can look at-- and this is probably the most important one-- is we want to look at a-- so three is the physical interpretation of-- what did I say-- model parameters. We're going to get some predictions about this. We're going to get a prediction of the temperature. We're going to get a prediction of the power-law index. We're going to get a prediction of the flux.

We want to look at those values and see, do those actually make sense? Because if we get a measurement of the temperature of our neutron star, if we come out with a temperature of 100 kelvin, that's really cold. Would a really cold object be producing X-rays?

AUDIENCE: No.

MARK HARTMAN: Probably not, right? So we want to look physically at our model parameters, see if they make sense. So what I would like to do is I want to ask you another, just a quick question. Because what was the-- so we know that our temperature should come out to be about right. It's about 0.1 keV.

Each keV of energy-- so this [? is ?] we're going to look at temperature of black body-- and I'll just shorten that to BB, Black Body-- fit to neutron star. What did we get? What was your value for the temperature? Go ahead, Steve.

AUDIENCE: [? 0.1 ?]

AUDIENCE: 0.0971122 keV.

MARK HARTMAN: That's what you said?

AUDIENCE: Yeah.

MARK HARTMAN: OK. So this is actually an interesting point for us to bring out. We're going to start measuring a lot more numbers. Whenever we have numbers that have so many decimals after them, typically, a good rule of thumb is to keep maybe two decimal places or two significant figures. Any number that's not 0 is significant.

So when we get a number like this, what we actually want to say is, well, it's not that we know these numbers for sure-- because remember, we're adjusting our model. And if we adjust it by just a little tiny bit, it's not really going to change how it looks that much. So we're going round off, and we're going to say that the temperature is about 0.097 keV.

AUDIENCE: Can you do it in scientific notation?

MARK HARTMAN: Yep, we could write that as scientific notation. That's 9.7 times 10 to the minus second keV. Or I think a couple of other people got about-- and that's about 0.1 keV.

A way to convert this between units, a temperature of about 1 keV is going to be equal to about 1 times 10 to the sixth kelvin, so 1 million kelvin. So let's think about this. KeV is just a nice way to measure this. It's just a unit. So this is for temperature.

AUDIENCE: How much is 1 keV in Celsius?

MARK HARTMAN: So 1 keV is about 1 times 10 to the sixth kelvin, so 1 million kelvin. The kelvin and Celsius scales are only offset by a couple of hundred degrees each. So that's about 1 million degrees Celsius as well. So if we have 0.1 keV, how hot are we?

AUDIENCE: What's that?

MARK HARTMAN: So if we have 0.1 keV as our temperature for our neutron star, and we know that 1 keV is a million--

AUDIENCE: [? 100,000. ?]

MARK HARTMAN: Yep. We can say temperature equals 0.1 keV in temperature times 1 keV per 1 times 10 to the sixth kelvin. That gives us 1 times 10 to the fifth kelvin, or 100,000.

Now, anybody who's doing the X-ray binaries project or the active stars project, do you remember an estimate for how hot the surface of a neutron star might be?

AUDIENCE: [INAUDIBLE] [? give me ?] a second.

MARK HARTMAN: No, not off the top of your head. But what do you think, Juan?

AUDIENCE: [INAUDIBLE]

MARK HARTMAN: Go ahead and keep looking, but 100,000 kelvin is pretty reasonable. It seems really high. But for objects that emit X-rays, most objects are around a million kelvin-- so maybe like 100,000 kelvin, all the way up to 10 or 20 million kelvin. What do you think, Steve? You found something?

AUDIENCE: Yeah, something like that.

AUDIENCE: [INAUDIBLE] to temperatures of 10 million to 100 million kelvins.

MARK HARTMAN: OK, so somewhere between 10 million and 100 million kelvins for other stars. So yeah, this one is actually fairly cool. That's why this particular star was observed, I think, because it is fairly cool. And we can see that because the black-body shape was shifted to the left. So that makes sense. So then black body makes sense.