

## MITOCW | MIT\_IIT-JEE\_Video1\_OCW\_720p

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Hi everyone, welcome to this series of videos for the topic of vectors.

This chapter is important for mathematics and it also forms the basis for the next chapter, that is 3-Dimensional Geometry.

There will be series of videos for this topic.

In this first video, I will just like to talk about very basic stuff.

I will try to define vectors.

I believe that there is a lot of confusion among students about even basic things like position vector, line vector, unit vector and magnitude.

I think if you can really focus on this video, and try to understand the different between position vector, line vector and unit vector that will clarify a lot of things for you.

Let me start by defining the position vector.

First, before I go into position vector, let me just briefly summarize vectors for you in case you are not familiar.

I believe you have already learned about vectors in chapter of physics like kinematics, forces.

You should know that vectors are quantities that have both magnitude and direction.

And they are very important for scientific studies in general.

Let me start by defining position vector.

So position vector, is a vector connecting origin to a point.

And as the names suggest, basically this position vector is about position of one point.

I think this is where a lot of students get confused.

A good thing about vectors is that they will be always be in 3 dimensions and they will be really helpful in the next chapter of 3 dimensional geometry.

It is important for you to understand position vectors and like line vector, unit vector.

So let us just initially focus on position vector.

So what is the definition: a position vector is a vector connecting origin to a point.

Let us say we have a point P with coordinates  $P_x P_y P_z$ .

Let us call this x axis, y axis and z axis.

Position vector is a vector connecting origin, which is O to the point P, and pointing in direction on P.

This direction is very important because if it is in the opposite direction, then the vector will be opposite.

When we write OP vector (this is called OP vector), the first letter is from where the arrow line connects to second point (or the second alphabet, which is P in this case).

OP vector also signifies the direction of the vector.

What I am trying to say is that if you have OP vector then the vector will start from O and end at P.

In this case, OP vector means  $P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$ . (I hope you are familiar with  $\hat{i}$ :  $\hat{i}$  means the direction of x-axis) .. +  $P_y \hat{j} + P_z \hat{k}$ .

So x axis, y axis and z axis.

Another point you might want to note is that magnitude of OP vector is  $(P_x^2 + P_y^2 + P_z^2)^{1/2}$ .

Let us do a quick example.

If we have a point A with coordinates (3,2,1).

What is position vector and its magnitude?

And the answer is: OA vector =  $3 \hat{i} + 2 \hat{j} + \hat{k}$  and OA vector magnitude =  $(3^2 + 2^2 + 1^2)^{1/2} = 14^{1/2}$ .

Position vector is always for a point.

Please do not forget this basic definition of position vector.

Trust me that it will happen sometimes during the chapter of vector that you will get confused if you forget this definition.

So please try to remember that position vector is always and always for a point.

Now let us try to move to new type of vectors called the line vector.

Position vector is for a point.

Line vector is for a line.

As simple as that.

If you have a line, let us say connecting two points A and B.

If I write a vector here and b vector here, what does this mean?

a vector is for point A and b vector is for point B.

Whenever you see something like this in brackets, it means it is for a point, or it is a position vector.

In other words, a vector is OA vector and b vector is OB vector.

And OA vectors and OB vector are position vectors of point A and B.

Even this basic definition gets people really confused.

So whenever you see a point and something written in the bracket, that always means a position vector.

So what does line vector mean?

Line vector means connecting two points.

If we have AB vector: starting from A and ending at B.

Arrow goes from A to B.

This vector is defined as b vector - a vector.

Or in other words,  $AB = OB - OA$ .

Let us try to take one quick example to help us understand this concept.

What is the vector joining  $\sin(\theta) \hat{i} + \cos(\theta) \hat{j}$  and  $(0,0,1)$ .

What is its magnitude?

Let us try to find the answer.

It might be little confusing because of the way it is written here.

$\sin(\theta) \hat{i} + \cos(\theta) \hat{j}$  is basically a point for which you have been given a position vector.

$\theta$  can be any parameter basically  $\theta$  is an angle and a parameter here.

We have not been given a value but we can still calculate the answer.

b vector is  $\hat{k}$ .

What will be  $b - a$ ? It will be,  $\hat{k} - \sin(\theta) \hat{i} - \cos(\theta) \hat{j}$ .

And thus magnitude of AB vector would be similar to the way we defined vector magnitude: squaring all the components i.e.  $(-\sin(\theta))^2 + (-\cos(\theta))^2 + 1^2$  (1.2). And  $\sin^2(\theta) + \cos^2(\theta) = 1$ . So AB vector magnitude is  $2^{1/2}$ .

In this part I just wanted to emphasize that line vector is for a line and position vector is for a point.

Now there are also vectors which people get confused about.

Here, both the vectors were fixed i.e. the vectors started from A and went to B.

This started from O and went to P.

However, you should realize that sometimes vectors are not fixed. They are only fixed in the magnitude and the direction but aren't fixed in the position.

For instance, if I have been given a vector a and I have been told that it is  $\hat{i} + \hat{j} + \hat{k}$  and I have not been given any other information about this.

What would this vector represent? This vector could represent a lot of things.

For instance, it can represent a point such that OA vector for a point (1,1,1) would be  $\hat{i} + \hat{j} + \hat{k}$ .

Similarly, this can also represent a line vector which it can be  $(b_{\text{vector}} - a_{\text{vector}})$ .

It can join two points and the difference can come as  $\hat{i} + \hat{j} + \hat{k}$ .

By free vector I just know the direction and magnitude. It is free to translate in 3-D space.

Let us say this is the vector. If this vector freely translates  $\theta$  it is not changing its direction and it is not changing its magnitude.

If you have not been specified whether a vector starts from origin or a position, or it joins two points, it can also be a free vector.

I want to emphasize this you will see later in this chapter that many times people get confused about free vectors as they are not able to understand it. They also get confused about the vector connecting two points, and when they see notation  $\hat{a}$  a\_vector in the brackets and b\_vector in the brackets.

Just to summarize: whenever you see a point, it always means a position vector.

Whenever you see a vector joining two points, it means a line vector which can be calculated like this.

Whenever you see something just written like this and not specified whether it is a point or whether it is a line vector, that means it is a free vector which only has a fix magnitude and a direction and can move freely around the space.

Till now I have only talked about the magnitude of the vector here: you can just take  $x\_component^2 + y\_component^2 + z\_component^2$  and take a square root of that.

However, there is something that is also important for a vector and that is its direction.

So let me start by defining unit vector.

Whenever I write a vector\_a, I can also write vector\_a as magnitude of vector\_a into unit vector.

What does this mean?

This means that a vector has some magnitude which you can calculate by squaring x\_component, y\_component, z\_component, and adding them and taking a square root. And this a\_cap is nothing but a unit vector.

Something which I really prefer to use is to call it "direction".

Unit vector and direction are synonymous.

So whenever you see a vector: it is multiplication of its magnitude and its direction (or its unit vectors).

For instance, if we have to calculate unit vector of a\_vector. Then you can write a\_cap as vector\_a / | a | Vector\_a in this case is  $\hat{i} + \hat{j} + \hat{k}$ .

And  $| a | = (1^2 + 1^2 + 1^2)^{1/2}$ , and this means  $(\hat{i} + \hat{j} + \hat{k})/3^{1/2}$ .

In other words I can also write that  $\mathbf{a\_vector} = 3^{1/2} (\hat{i} + \hat{j} + \hat{k}) / 3^{1/2}$ .

This is the direction, this is the unit vector and this is the magnitude.

Let us write another question.

What is the direction of x-axis?

Direction means a unit vector. So in that case  $\mathbf{a\_cap}$  would be direction or something with the magnitude of 1. We know the direction of x-axis is  $\hat{i}$  and it also has a magnitude of 1. So it is just  $\hat{i}$ .  $\hat{i}$  is nothing but a direction.

So whenever I write  $P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$ , it means I have moved  $P_x$  in the x direction, and then I have moved  $P_y$  in the y direction, and then I have moved  $P_z$  in the k direction.

That is how you define direction by unit vectors.

In this video we covered very basic stuff.

Position vector refers to a point.

Line vector refers to vector joining two points.

Free vectors refers to a vector which is neither a point nor a line, and something that can move freely around the space though it has a fixed magnitude and fixed direction.

Unit vector means the direction where you divide the vector by its magnitude. We did a quick calculation to see that.

I hope this was clear. Just take some time and digest these things. And in the next video we will talk about addition of vectors and section formula. Thank you.