Hi!

I'm Pritish Today, we will be talking about Trigonometry.

This is the first video on Trigonometry.

I would like to start with "Why do we want to study Trigonometry?" and "What is Trigonometry?" You would certainly know that Euclidean geometry is very fundamental to how we think about nature.

The way we visualize things in Physics, for example, we use Euclidean geometry in the way we think.

And often, Euclidean geometry requires some amount of innovative thinking to reason about problems.

And often when you want to talk about Physics, you don't want geometry to be your bottleneck in thinking about Physics.

So Trigonometry and Coordinate Geometry are two subjects which try to make a more systematic study of Geometry.

And what is "Trigonometry"?

Trigonometry is about a study of angles in an "algebraic" way.

You would feel more comfortable doing Algebra Algebra is something you can do in a more mechanical way.

Whereas, Geometry often requires an innovative leap.

Using trigonometry, you'll be able to reason about geometric problems in a more algebraic way.

Let's start with the very basics of what is an angle.

You know from basic Euclidean geometry, that if you have a circle of radius 'r', then the perimeter of the circle is equal to 2.\pi.r.

We know this from basic geometry.

A basic reasoning will tell you that length of the arc of half a circle is \pi.r.

Similarly, if I take a quarter circle, the length of the arc is (\pi/2).r.

In general if I pick out an angle of, say, \alpha degrees (like this could be 60 degrees), then you can easily reason that the length of an arc with angle \alpha degrees is (\alpha/360).(2.\pi.r).

This is just a basic ratio and proportion argument.

Let me write this as ((2.\pi.\alpha)/360).r.

We are more generally taught since school to think in terms of degrees.

The full circle has 360 degree.

But now when we have to write the length of an arc, we have to write an annoying expression ((2.\pi.\alpha)/360).r.

It would be much nicer to call this expression ((2.\pi.\alpha)/360) as the "angle" itself.

Instead of \alpha degrees if we could call ((2.\pi.\alpha)/360) as the "angle", then it would be much nicer.

As there are different ways to measure objects, for example, length, you can measure it in meters or inches.

similarly this is a different unit for measuring angles.

This is angle in "radians".

Thinking in terms of radians is very important.

My math teacher used to tell me that when you are learning a new language, for example, if I am trying to speak in English, if I want to learn it well, I should not think in a different language.

For example, I should not think in Hindi and translate to English every time I am speaking.

You should really start thinking in English, if you want to learn English well.

Similarly here, thinking in terms of radians is often the better thing to do because now the length of arc with angle \theta radians is simply (r.\theta).

So that's why thinking in terms of radians is often the more natural way to think about angles.

Just to be more familiar with this conversion, lets see that 360 degrees is '2 \pi' radians.

The perimeter of the entire circle is '2.\pi.r' and '2\pi' is the angle of the entire circle.

Similarly, 180 degrees is \pi. 90 degrees is \pi/2 and so on.

So we just saw how to measure angles in degrees and radians and we said that radians is the more natural unit to think about angles.

So that now the length of the arc with angle of \theta radians is just r.\theta.

In the rest of the video, we will talk about trigonometric ratios, we'll define what they are and we'll talk about some trigonometric identities.

These are the algebraic tools which will help us reason about angles in a more algebraic way.

Let's look at an angle \theta.

Whenever we write \theta as an angle, it is assumed that it is in radians, as we know that radians is the more natural unit to think about angles.

Let's say the radius is 'r', and we just saw that the length of the arc is r.\theta.

Now there are other interesting parameters that you would want to know about the angle.

For example, if we drop a perpendicular from the top vertex, what is the height of this line segment?

Let's call the vertices as A, B and C.

This side BC is "opposite" to the angle.

The side AC is the side that is "adjacent" to the angle.

And the side AB is the "hypotenuse".

So we would like to know the height of the line segment BC is.

One thing that you know from similarity of triangles is that suppose I have two triangles like this.

You know from similarity of triangles that BC/AB = DE/AD.

(Note that angles BCA and DEA are right angles.) These are two triangles which have all three angles the same, and therefore they are "similar" and hence the ratio of sides are equal.

Because of similarity of triangles, this ratio is purely a property of the angle, and is not related to any of the sides.

This ratio is defined as the "sine" of the angle.

To remember it more easily, let's call sin(\theta) as opposite/hypotenuse.

Now we can see that since the hypotenuse is 'r', the length of the opposite side is now 'r.(sin \theta)'.

Similarly, now we can define the ratio of the adjacent/hypotenuse.

So that is defined as cos \theta.

Again, since the hypotenuse is 'r', the adjacent side is of length r.(cos \theta).

In terms of this ratio, this will be AC/AB = AE/AD.

And by similarity of triangles, this is just a property of the angle, and does not depend on the sides of the triangle.

Now, we'll see our first trigonometric identity.

From Pythagoras' theorem, we know that for any right angled triangle, the sum of squares of the two sides is equal to the square of the hypotenuse.

So let's write down here : (opp)^2 + (adj)^2 = (hyp)^2.

This is what Pythagoras' theorem tells us.

If we divide by (hyp)^2 throughout, we get that (opp/hyp)^2 + (adj/hyp)^2 = 1.

Now, if you look at our definitions of sin \theta and cos \theta, this tells us that, (sin \theta)^2 + (cos \theta)^2 = 1.

A usual convention to write (sin \theta)^2 is that you don't write the brackets all the time.

So what we do is, write it as sin^2(\theta) and cos^2(\theta).

So as to not confuse with, for example, sin(\theta^2).

This would mean sin(\theta^2), whereas this is (sin \theta)^2