Chapter 5

Chapter I: The MU–Puzzle

Has the dog Buddha-nature? MU! – Zen Koan

5.1 Abstract

A simple formal system (the MIU-system) is presented, and the rader is urged to work out a puzzle to gain familiarity with formal systems in general. A number of fundamental notions are introduced: string, theorem, axiom, rule of inference, derivation, formal system, decision procedure, working inside/outside the system. *GEB pp. viii*

5.2 Formal Systems

See chapter on *Tools for Thinking*

In regards to this *particular* formal system, we find that we have the following pieces of the puzzle.In order to distinguish which levels we are working on, we will employ the `typewriter font.` Remember the rules only go one way!

LETTERS: `M, I, U`
OUR SOUL POSSESSION: `MI`
RULE I: If you possess a string who last letter s `I`, you can add on a `U` at the end.
RULE II: Suppose you have `Mx`. then you may add `Mxx` to your collection.
RULE III: If `III` occurs in one of the strings in your collection, you may make a new string with `U` in place of `III`.
RULE IV: If `UU` occurs inside one of your strings, you can drop it.

5.3 Theorems, Axioms, Rules

Let's keep track of definitions for easy reference.

- A **string** is simply an ordered sequence of `M`'s, `I`'s, and `U`'s.
- A **theorem** is a string produced (proved) by the rules of the formal system.
- An **axiom** is a starting point assumed to be true in a formal system.

- The rules detailed above are **rules of production** or **rules of inference**.
- A **derivation** is a demonstration of how to produce one theorem from another theorem.

5.4 Inside and Outside the System

What are some more difference between people and machines? Hofstadter talks a lot about *observing* patterns, but who is doing the observing and from where?

5.5 Jumping out of the System

Do you really think that being able to jump out of a task and look for patterns is an inherent property of intelligence?

What do you think of the following?

> Of course, there are cases where only a rare individual will have the vision to perceive a system which governs many people' lives, a system which had never before even been recognized as a system; then such people often devote their lives to convincing other people that the system really is there, and that it ought to be exited from! (pp. 37)

What or who does this make you think of?

- Karl Marx and Communism
- Anarchism
- Socialism today and working peoples
- The Media
- The Government
- The Church
- The School
- Culture

It all has to with cycles and loops! Is Hofstadter advising us to avoid repetition in thought and action?

5.6 M-Mode, I-Mode, U-Mode

- Mechanical Mode (M-Mode)
- Intelligent Mode (I-Mode)
- Un-Mode (U-Mode)

Hofstadter calls the U-Mode a"Zen way of approaching things." (pp. 39) What does this mean?

5.7 Decision Procedures

OUR THEOREMHOOD TEST: Wait until the string in question is produced; when that happens, you know it is a theorem – and if it never happens, you know that it is not a theorem.
Is this a good test? NO! We want something that can terminate in finite time!

5.8 Study Questions

1. In principle, any three letters could have been chosen for the puzzle. Why were "**M**","**I**", and "**U**" employed?
2. On the first page, four strings of letters are given to be strings of the MIU-system. What generality can you draw as to what constitutes such a string?
3. What does it mean to be "in possession of a string"?
4. What is the product of *RULE I* acting on the string **MUUUIUI**? What about the string **MIIIUIU**?
5. What is the product of *RULE II* acting on the string **MUUUIUI**?
6. The text states that from **MU** you can get **MUU**. Doesnt that imply that you can also get **MU**? If so, then the puzzle is solved!
7. What is the product of *RULE III* acting on the string **MUUUIUI**? On **MIIIUIU**?
8. What is the product of *RULE IV* acting on the string **MUUUIUI**? On **MIIIUIU**?
9. How does a theorem differ from an axiom (both in mathematics and the sense used in GEB)?
10. Is the notion of "truth" different for a theorem than an axiom?
11. How sure are you that if **MI** is the sole axiom, the *MIU-system* can produce no theorem that does not begin with **M**?
12. Is it possible for humans to act unobservantly?
13. Hofstadter suggests that the numbers 3 and 2 play important roles in the *MIU-system*. What roles?
14. Provide an example of how you used the *M-mode* in considering the puzzle.
15. Provide an example of how you used the *I-mode* in considering the puzzle.
16. Provide an example of how you used the *U-mode* in considering the puzzle.
17. Will the decision tree shown in Figure 11 produce every theorem of the *MIU-system* from the sole axiom **MI**?

18. Suppose we leave *MIU* and take on the observable universe. If there existed a decision tree that could generate every true statement (modeled after Figure 11), could I say that all truth is knowable? Would it help if I told you that the decision tree reached a decision after no more than 1000 steps? By the way, if each decision is yes-no, then the tree could conceivably deal with 2^{1000} questions, a number close to 1 with 300 zeros after it, far more than the number of electrons in the universe.

19. Does there exist a litmus test for theoremhood in the MIU-system?

Gödel, Escher, Bach: A Mental Space Odyssey
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