Chapter 3

Introduction: A Musico-Logical Offering

"Normal is a Distribution" – Unknown

3.1 Introduction to the Introduction

As we have finally reached the beginning of the book proper, these notes should mirror the book as best as possible. Let us not forget that these notes are an ode to GEB, not to propositional calculus! As such there will be a chapter for each chapter and dialogue, and sections for each section of the chapters found in GEB.Here on out, this section will be occupied normally by Hofstadter's own *abstract* for each chapter found in the overview of GEB.

3.2 Abstract

The book opens with the story of Bach's *Musical Offering.* Bach made an impromptu visit to King Frederick the Great of Prussia, and was requested to improvise upon a theme presented by the King. His improvisations formed the basis of that great work. The *Musical Offering* and its story form a theme upon which I "improvise" throughout the book, thus making a sort of "Metamusical Offering". Self-reference and its interplay between different levels in Bach are discussed; this leads to a discussion of parallel ideas in Escher's drawings and then Gödel's Theorem. A brief presentation of the history of logic and paradoxes is given as background for Gödel's Theorem. This leads to mechanical reasoning and computers, and the debate about whether Artificial Intelligence is possible. I close with an explanation of the origins of the book – particularly the why and wherefore of the Dialogues. *GEB pp. viii*

3.3 Bach

Of all the angles GEB uses to "illuminate" the same concept, music is perhaps my weakest side. As such, I will down play the role of music, but still try to give its proper place. I think the important thing to gather from this section is the type of intellectual "playing" that Hofstadter

likes to engage in. In many ways all of GEB is one big puzzle; much like a fugue. The kind of "high-brow cleverness" that Hoftstadter speaks of, will inspire the rest of the book (pp 7).

3.4 Canons and Fugues

Let's go ahead and record some definitions and facts about canons and fugues (pp. 8-9). A canon is a piece of music where a single theme is repeated and "played against itself." Hofstadter gives us several ways in which a **canon** can have complexity:

1. A copy of the theme is played a fixed time later.
2. The theme is staggered in time and pitch.
3. The theme is played at different speeds.
4. The theme is inverted.
5. The theme is played backwards $\Rightarrow$ **Crab Canon!**

A **fugue** is a canon with more flexibility and opportunity for creative expression.

The two quotes of the section seems to be:

> Such an information-preserving transformation is often called an *isomorphism,* and we will have much traffic with isomorphisms in this book. (p. 9)

> It is itself one large intellectual fugue, in which many ideas and forms have been woven together, and in which playful double meanings and subtle allusions are commonplace. (pp 9-10)

The second quote is a classic example of Hofstadter talking about GEB within GEB! Even if you can't pick up all of GEB's clever layers, at least it has the decency to tell you that multiple layers do exist!

3.5 An Endlessly Rising Canon

Here we meet out first definitions of **Strange Loops** and **Tangled Hierarchies**.

> In this canon [*Musical Offering*], Bach has given us our first example of the notion of *Strange Loops.* The "Strange Loop" phenomenon occurs whenever by moving upwards (or downwards) through the levels of some hierarchial system, we unexpectedly find ourselves right back where we started. (pp 10)

> Sometimes I use the term *Tangled Hierarchy* to describe a system in which a Strange Loop occurs. (pp 10)

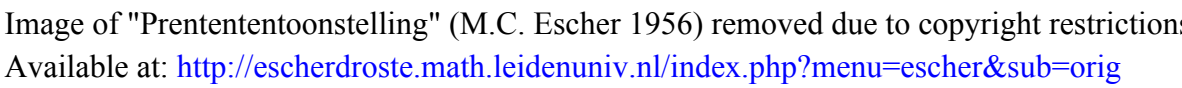
Image of "Prentententoonstelling" (M.C. Escher 1956) removed due to copyright restrictions.
Available at: http://escherdroste.math.leidenuniv.nl/index.php?menu=escher&sub=orig

3.6 Escher

As far as the introduction goes, I believe this section is the most significant. By using images of Escher we begin to muse on what the actual structure of thought and existence is.

> Implicit in the concept of Strange Loops is the concept of infinity, since what else is a loop but a way of representing an endless process in a finite way?...In some of his drawings, one single theme can appear on different levels of reality...But the mere presence of these two levels invites the viewer to look upon himself as part of yet another level, and by taking that step, the viewer cannot help getting caught up in Escher's implied chain of levels, in which, for any one level, there is always another level above it of greater "reality" and likewise, there is always a level below, "more imaginary" than it is. This can be mind-boggling in itself. However, what happens if the chain of levels is not linear, but forms a loop? What is real,then, and what is fantasy? The genius of Escher was that he could not only concoct, but actually portray, dozens of half-real, half-mythical worlds, worlds filled with Strange Loops, which he seems to be inviting his viewers to enter.

When reflecting on this quote, Escher reflects in Hofstadter's mind's eye, but Mandelbrot flashes in mine. *Fractals* and *Fractal Geometry* is such an apparent part of nature, Hofstadter's quote gains even more legitimacy than previously thought. Please see the appendix for a tour of the Mandelbrot set. Perhaps it will convince you of Hofstadter's thinking.

3.7 Gödel

The essence of what Gödel did, was to use number theory to encode statements about provability of statements within a specific formal system. He then, in a very rigorous and brilliant way, encoded the statement:

This statement is not provable.

What if this statement is provable? If it is provable, then it is true since provable statements are necessarily true. If it is true, then it is not provable! $\Rightarrow$ Contradiction! What if this statement is not provable? That is exactly what the statement asserts, so it is **true AND not-provable**.

3.8 Mathematical Logic: A Synopsis

Here we meet the founders of logic. The important example to take away is **Russell's Paradox**. The paradox is phrased in multiple ways. One version considers the set Ω of all sets not members of themselves. Now ask yourself, "Is Ω a member of itself? If it is, then it isn't and if it isn't then it is. Some examples of self-containing sets might be the set of all sets, the set of all things not George Bush, etc.

Perhaps that still seems convoluted. Consider a popular variant known as the **Barber's Paradox**. There is a town in Tumbolia which has a barber who shaves all and only people who don't shave themselves. Does our barber shave himself or not?

3.9 Banishing Strange Loops

The idea of a hierarchy was thought to prevent paradoxes from creeping into mathematics. The idea is that we keep normal non-self referential language and restrict it to an **object language** call it L1. Then if we want to talk about statements in T1, we need to talk in an expanded **metalanguge** call it L2. But if we want to talk about L2 we need to operate in an expanded metalanguage L3, and so on...

This way sentences like *This sentence is not true.* could never be expressed in L1, because the sentence it self isn't in the domain of L1. However a sentence like *Snow is white.* is perfectly acceptable for L1. We could then say something in L2 like *'Snow is white.' is true.*

3.10 Consistency, Completeness, Hilbert's Program

Just remember the definitions.

A system is **consistent** if it cannot prove both a statement P true and its negation *not-P* true.

A system is **complete** if every true statement if provable.

3.11 Babbage, Computers, Artificial Intelligence...

Convergence of three disciplines:

1. Theory of Axiomatic Reasoning
2. Study of Mechanical Computation
3. Psychology of Intelligence

What is intelligence? How can it be programmed? Hofstadter comes up with a list of essential abilities for intelligence to have (p. 26):

- to respond to situations very flexibly;
- to take advantage of fortuitous circumstances;
- to recognize the relative importance of different elements of a situation;
- to find similarities between situations despite differences which may separate them;
- to draw distinctions between situations despite similarities which may link them;
- to synthesize new concepts by taking old concepts and putting them together in new ways;
- to come up with ideas which are novel.

3.12 ... and Bach

Do you think it is possible for a computer to compose music?

3.13 "Gödel, Escher, Bach"

Each dialogue is patterned on a different piece of Bach. The dialogues are meant to provide an intuitive understanding of the concepts before the technical understanding in the following chapter. Similarly in this course we will have dialogues and meta-dialogues preceding each reading assignment.

3.14 Study Questions

1. What property does "*Row, Row, Row Your Boat*" and "*Frere Jacques*" have in common? Would any tune work just as well? Suggest one as a test case.

2. Hofstadter claims that *Good King Wenceslas* works as an inversion. You can decide for yourself. (ask for music if you really want)

3. What's the connection between Bach's endlessly rising canon and Escher's *Waterfall*?

4. A quote from GEB (p15): "*Implicit in the concept of Strange Loops is the concept of infinity, since what else is a loop but a way of representing an endless process in a finite way?*" There's a lot to think about here. Try to elaborate on your conception of infinity, either musically, artistically, or through language. Consider using Escher's *Metamorphosis, Mobius Strip I*, or *Mobius Strip II* (p276) to assist you. For additional help, you might recall the *Autumn Floods* section of Chuang-Tzu: "*The Lord of the River said, 'Men who debate such matters these days all claim that the minutest thing has no form and the largest thing cannot be encompassed. Is this a true statement?'*" Are there things about infinity that you find paradoxical? What are they?

5. Are the following statements true or false? What's the problem? "*All Cretans are liars.*" – Uttered by Epimenides the Cretan *I always lie. This statement is false.*

6. Translate the following sentence into language the guy in the street could readily understand: *All consistent axiomatic formulations of number theory include undecidable propositions.*

7. Accept for the moment Hofstadter's statement that "This statement of number theory does not have any proof in the system of *Principia Mathematica*" can indeed be translated into a statement of number theory. We are interested in two questions: 1) Is it a true statement? and 2) Can it be proved in the system of *Principia Mathematica*? Describe the possibilities.

8. Consider **R**, the set of all run-of-the-mill sets (as defined in the text). Is **R** itself a run-of-the-mill set or is it a self-swallowing set?

9. Which Box Contains the Gold? Two boxes labeled **A** and **B**. A sign on box **A** says "The sign on box **B** is true and the gold is in box **A**". A sign on box **B** says "The sign on box **A** is false and the gold is is in box **A**". Assuming there is gold in one of the boxes, which box contains the gold?

10. What is Hofstadter's main objection to the Hierarchical Theory of Types?

11. Identify the hierarchy of object language and metalanguage in the following statement: *The definition of ambiguous is not ambiguous.* Parse the statement in both sensible and paradoxical ways. Would punctuation help to clarify the intended meaning?

12. heterological: Is "*heterological*" (as defined in the text) heterological?

13. meta...: Compose a metasentence.

14. strange loop: Think of an example of a strange loop, besides what you've read. How tight is it?

15. consistent system: What would be necessary to make a system inconsistent?

16. complete system: What would be necessary to make a system incomplete?

17. infinity: What is its essential property?

Gödel, Escher, Bach: A Mental Space Odyssey
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