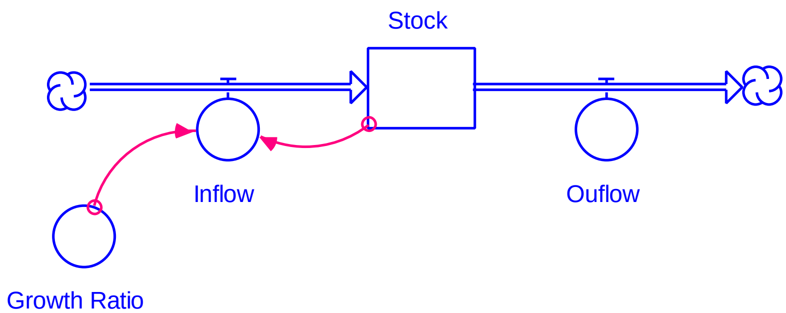
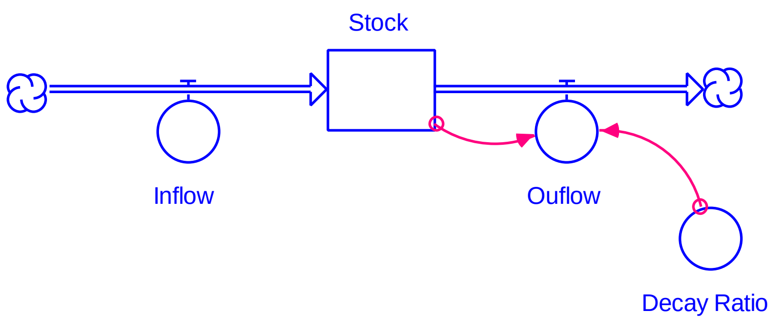
Beginner Modeling Exercises

Section 4

Mental Simulation: Adding Constant Flows





Prepared for
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Alan E. Coronado, 1996
Vensim Examples added October 2001

Table of Contents

Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems causes nearly all dynamic behavior. To use system dynamics successfully as a learning tool, one must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with computer simulation software.[1] Computer simulation is a very useful tool for exploring systems. However, one should be able to use the other simulation tool of system dynamics: *mental simulation.* A strong set of mental simulation skills will enhance ability to validate, debug and understand dynamic systems and models.

This paper provides guidelines to mentally simulate first-order (single-stock) feedback systems containing constant flows into or out of the stock. These guidelines are presented as the following three steps of mental simulation: first, *calculate equilibrium*; then, *determine the behavior mode* that the system will exhibit; and finally, *sketch the expected behavior*. Application of these three steps will first be demonstrated on a positive feedback system containing an outflow and then on a negative feedback system with an inflow. Practice exercises will follow the examples. In order to obtain a better understanding, the reader is encouraged to make a serious attempt at solving the exercises before looking up the answers in the back. It is assumed that the reader has experience mentally simulating simple positive and negative feedback systems.[2]

1. Positive Feedback with Constant Outflow

The purpose of this section is to demonstrate application of the following insights to the mental simulation of positive feedback systems containing constant outflows:

? Adding constant flows to a positive feedback system shifts equilibrium away from zero.

---

[1]There are several commercial system dynamics simulation packages available for both Windows and Macintosh. Road Maps is geared towards the use of STELLA II which is available from High Performance Systems (603) 643-9636. Road Maps can be accessed through the internet at http:/sysdyn.mit.edu/.

[2] For practice exercises consult the "Beginner Modeling Exercises: Mental Simulation" papers on "Positive Feedback" (D-4487) by Jospeh Whelan and "Negative Feedback" (D-4536) by Helen Zhu.

? Constant flows do not change the characteristics of exponential growth generated by positive feedback. Thus even in the presence of steady flows, the doubling time can be used to estimate system behavior.

The model to be simulated is built by a scientist interested in breeding a population of fruit flies in order to assure a steady supply for use in experiments. The model contains a stock, representing the fruit fly population, which is subject to two flows. The inflow corresponds to reproductive growth. Since fruit flies reproduce rapidly, adding about half a population every day, the scientist estimates a reproduction ratio of 50% per day. The outflow corresponds to a constant rate of removal of specimen from the stock. In this example the scientist desires a steady supply of about 50 fruit flies per day. In order to determine the right amount of fruit flies needed to begin breeding them, the scientist builds the stock-and-flow model in Figure 1.

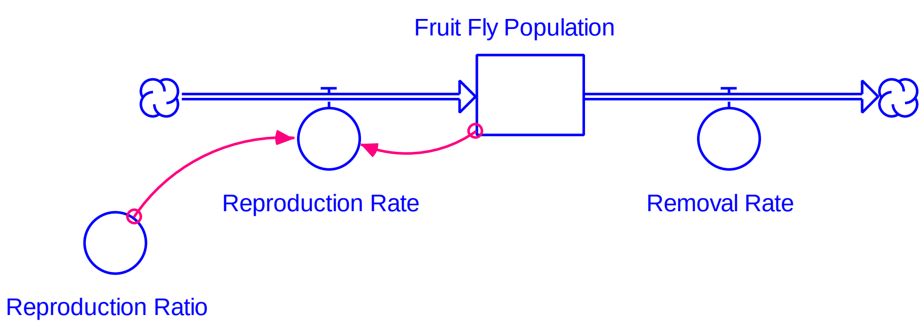


Figure 1: Fruit fly population model

The quantity of fruit flies the scientist needs should allow for the population to remain stable, with a balance struck between the removal and reproduction of fruit flies. Thus the scientist will simulate the system behavior in equilibrium. This will be performed in three steps:

1. Calculate equilibrium.

The constant outflow shifts the equilibrium of the positive feedback system away from zero. The equilibrium stock for a first-order system can be obtained by equating the sum of flows into the stock to the sum of flows out of the stock. Thus for the fruit fly system, equilibrium is found by solving the following equation:

Reproduction Rate = Removal Rate,

or:

Fruit Fly Population * Reproduction Ratio = Removal Rate.

Solving this equation we obtain the equilibrium fruit fly population:

Fruit Fly Population = Removal Rate / Reproduction Ratio

= 50 / 0.5 = 100 fruit flies.

2. *Determine the behavior mode.*

First-order positive feedback systems tend to exhibit either exponential growth away from equilibrium, negative exponential change towards equilibrium, or equilibrium. Since we are simulating the behavior of the stock at stability, 100 fruit flies, this last behavior mode is the one we are looking for.

3. Sketch the expected behavior.

Since the system is in equilibrium, the graph will be a horizontal line at 100 fruit flies as shown in Figure 2. Figure 2 also presents the equilibrium graph of the population for the case where there is no constant outflow. From comparing the two graphs it can be concluded that adding the constant outflow shifts the equilibrium of this positive feedback system away from zero.

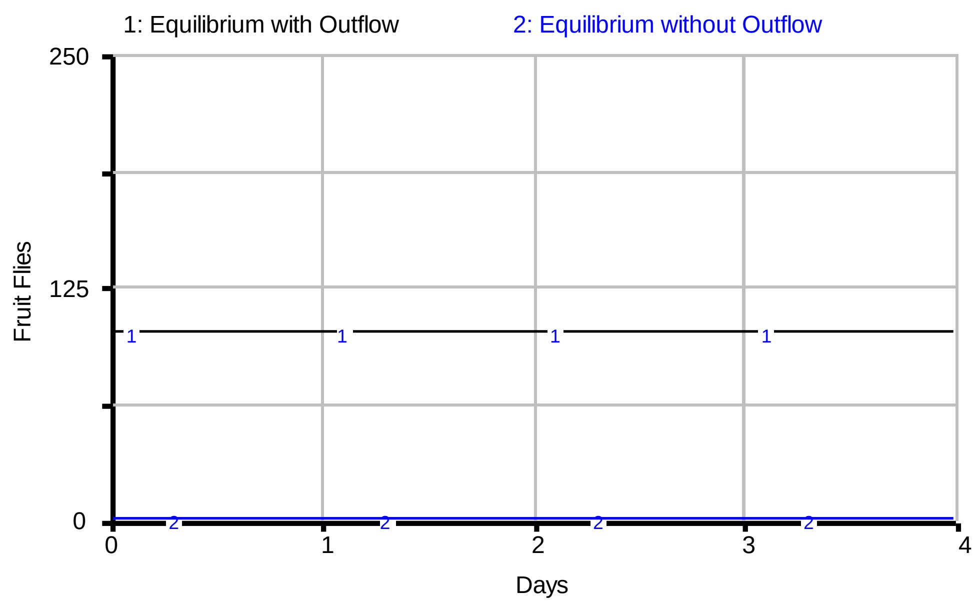


Figure 2: Change in equilibrium as a result of outflow

Since the mental simulation indicates that the population will be stable at 100 fruit flies, the scientist decides to order that amount. However, the lab supplies company mistakenly sends 120 fruit flies instead. The scientist quickly predicts the population behavior in three steps:

1. Calculate equilibrium.

The scientist remembers that the population is at equilibrium when there are 100 fruit flies.

2. Determine the behavior mode.

When there are 120 fruit flies the population clearly is not in equilibrium. Instead, there are *more* fruit flies than at equilibrium. Thus the behavior mode is *exponential growth* away from equilibrium.

3. *Sketch the behavior.*

Since the constant outflow does not change the exponential behavior generated by the positive feedback loop, doubling time can be used to estimate behavior. The doubling time is approximated by:

Doubling Time = 0.7 / Reproduction Ratio = 0.7 / 0.5 = 1.4 days.

Does this mean that the stock of 120 fruit flies doubles to 240 in just 1.4 days? If this assertion is true, then the system will behave exactly as if there were no outflow! Thus, it is obvious that the 120 fruit flies will not grow to 240 in 1.4 days.

From this last observation it might seem as if the doubling time does not describe the rate at which 120 fruit flies reproduce. This observation is misleading because the doubling time is being applied to the wrong stock. Clearly, the exponential growth generated by positive feedback does not describe the behavior of the 100 fruit flies that are being removed at the same rate that they reproduce (remember that the population is at equilibrium when there are 100 specimens).

Instead, doubling time refers only to exponential growth. Only the additional 20 fruit flies that are not subject to removal grow exponentially, unhindered by the constant outflow. Thus, the behavior of the 120 fruit flies can be predicted by dividing the population up into two groups: the 100 fruit flies at equilibrium and the 20 fruit flies subject to positive feedback.

The key to sketching behavior is graphing the two behaviors separately and then adding them up to produce the behavior of the population as a whole.[3] First, the graph of the 20 fruit flies that are subject to pure positive feedback is graphed as shown in Figure 3. The doubling time of 1.4 days allows for a quick sketch.

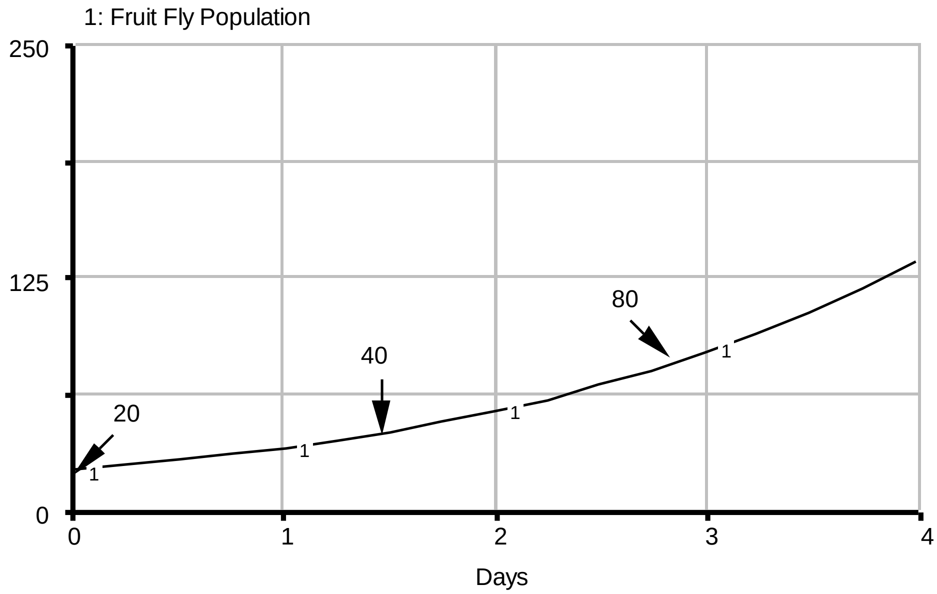


Figure 3: Exponential growth of twenty additional fruit flies

The predicted behavior for the system as a whole is obtained by adding the equilibrium graph, obtained in Figure 2, to the graph in Figure 3. Since the new equilibrium is represented by a horizontal line at 100 fruit flies, adding these behavior modes is tantamount to shifting the exponential curve up by the amount of the new equilibrium. Figure 4 shows the final behavior estimate for the system. The exponential growth generated by the system with the outflow is compared to that without the outflow. The previous and new equilibriums are also compared.

[3] Mathematically, this procedure of adding behavior modes to produce the total system behavior is called "superposition." Superposition is only possible for linear systems, such as those being used in this paper.

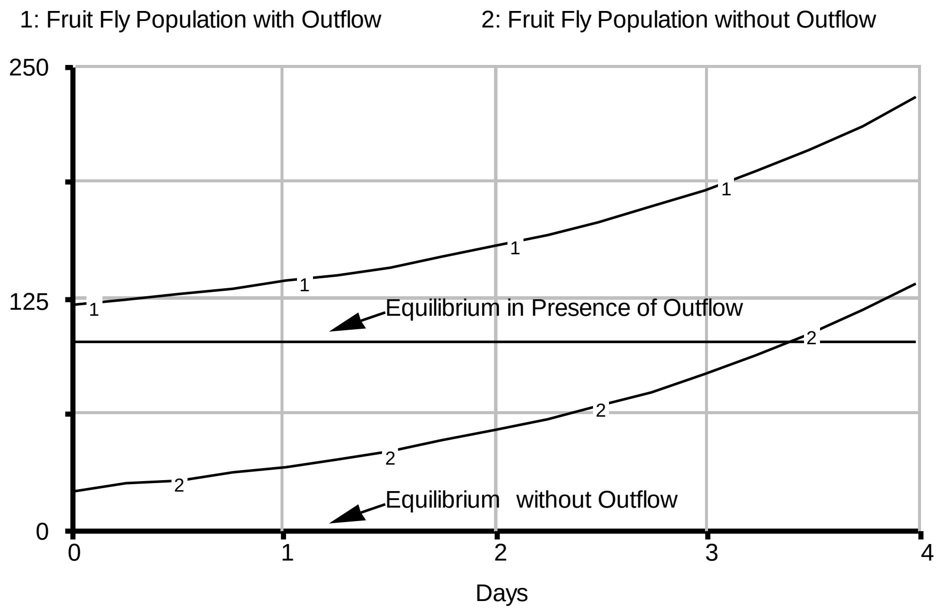


Figure 4: Mental simulation graph of fruit fly population behavior

In Figure 4 we notice that addition of the constant flow did not change the exponential behavior generated by the positive feedback. As a result, sketching positive feedback system with a constant outflow is simple. Just add the two behavior modes: exponential growth generated by positive feedback, and the new equilibrium resulting from addition of the constant outflow. The first is estimated using the doubling time and the second is calculated from the equilibrium relation. The behavior of the system as a whole is found by adding up these two behaviors. This operation amounts to shifting the exponential growth upwards, so that it starts from the new equilibrium.

2. Exercise 1: Nobel Prize Fund

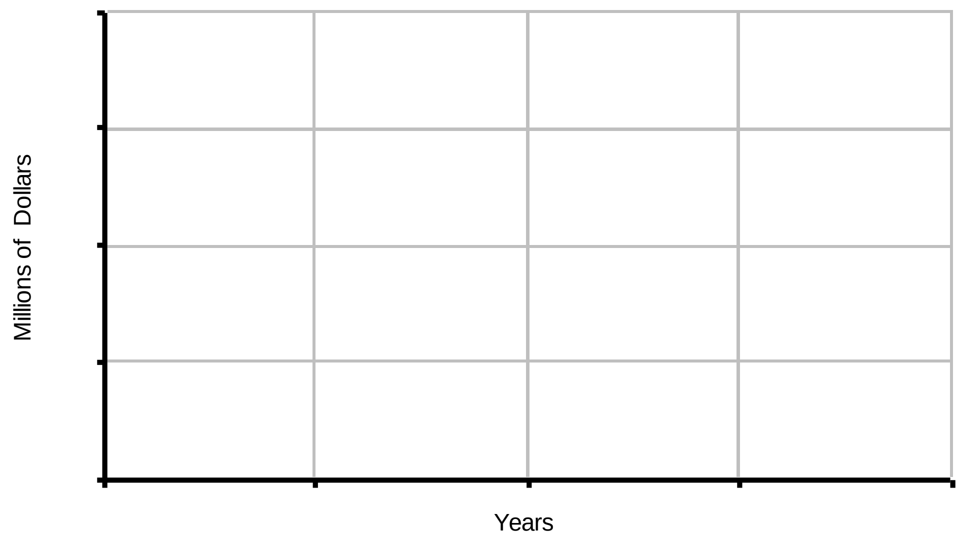
Every year the Nobel Prize Foundation distributes approximately a total of $6,000,000 in cash prizes to those who, during the preceding year, have conferred the greatest benefit on mankind in one of the following areas: Chemistry, Literature, Medicine, Physics, Economics, and Peace. These prizes are financed through interest accumulated on a bank account.

A. Draw a stock-and-flow model that describes the behavior of the Nobel Prize Fund. Treat the prizes in different categories as separate outflows from the bank account.

B. Draw a model that describes the behavior of the Nobel Prize Fund, this time treating the prizes as one big prize, i.e. as a single aggregated flow.

C. The Nobel Prize Fund earns enough interest to offset the cash lost as a result of the awards given. Assuming the interest rate is 10%, what is the minimum balance of the Nobel Prize Fund?

D. Sketch the account behavior assuming the Fund contains $30,000,000 at a time zero. Accuracy is not necessary; a drawing describing the basic behavior of the account is sufficient. For simplicity, treat the accumulation of interest and the withdrawal of cash prizes, as smooth, continuous functions, i.e. that they occur evenly throughout the year.



E. Suppose the Nobel Prize Fund is actually greater than the minimum needed for it to remain steady. This assumption is reasonable as it is unrealistic to expect the account to be exactly, to the last cent, equal to the minimum amount needed to not deplete. Now, suppose the Nobel Prize Foundation members have decided that they have enough money to fund a Nobel Prize "for those who have conferred the greatest benefit on mankind" in the field of System Dynamics. Supposing the Fund contains $60,500,000, how much can the System Dynamics Nobel Prize distribute in cash, assuming money is not taken from the other prizes to fund this new prize?

3. Negative Feedback with Constant Inflow

This section will guide the reader through the mental simulation of a negative feedback system containing a constant inflow. The following insights will prove useful to the mental simulation process:

? Adding constant flows to a negative feedback system shifts equilibrium.

? Constant flows do not change the characteristics of exponential decay produced by negative feedback. As a result, halving time remains a useful mental simulation tool.

The negative feedback system to be simulated is a draining sink that contains an added inflow produced by a leaking faucet. The rate of draining is proportional to the volume of water in the sink. For this specific sink the proportionality constant, or draining fraction, is about 0.1/s. Water flows in at a rate of 30 cm^3/s. The model for the system is depicted in Figure 5.

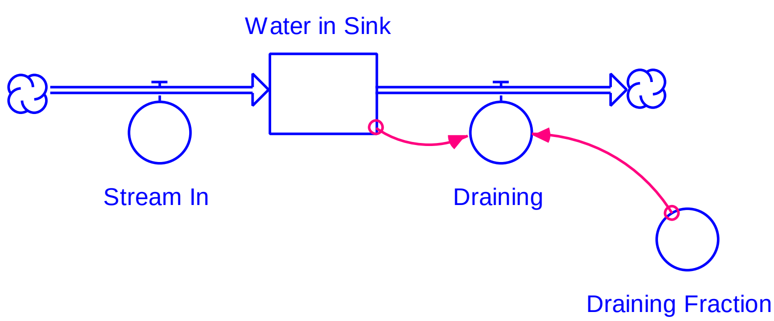


Figure 5: Model for draining sink with constant inflow

Now let us mentally simulate the behavior of the system when it is in equilibrium:

1. *Calculate equilibrium.*

In the absence of an inflow, the system is in equilibrium when the sink is empty. Adding a steady, exogenous flow shifts the equilibrium volume. To find out by how much, the equilibrium condition for first-order systems is used. In other words, the sum of inflows into the stock is equated to the sum of outflows. The inflow is simply a constant stream in. The outflow is given by the product of the volume by the draining fraction. Equating these terms we obtain:

Stream In = Volume * Draining Fraction.

Solving this equation we obtain the equilibrium volume of water:

Volume = Stream In / Draining Fraction = 30 / 0.1 = 300 cm^3.

2. *Determine the behavior mode.*

In first-order negative feedback systems the stock tends to approach equilibrium asymptotically, either from above or from below. Besides asymptotic behavior, the stock can exhibit equilibrium. For this simulation we are attempting to estimate the behavior of the system when the stock is at 300 cm^3, which represents equilibrium.

3. *Sketch the expected behavior mode.*

Since the system is in equilibrium, the graph will be a horizontal line with volume equal to 300 cm^3, as shown in Figure 6. The result of adding the inflow to the negative feedback system has been to shift equilibrium from 0 cm^3 to 300 cm^3.

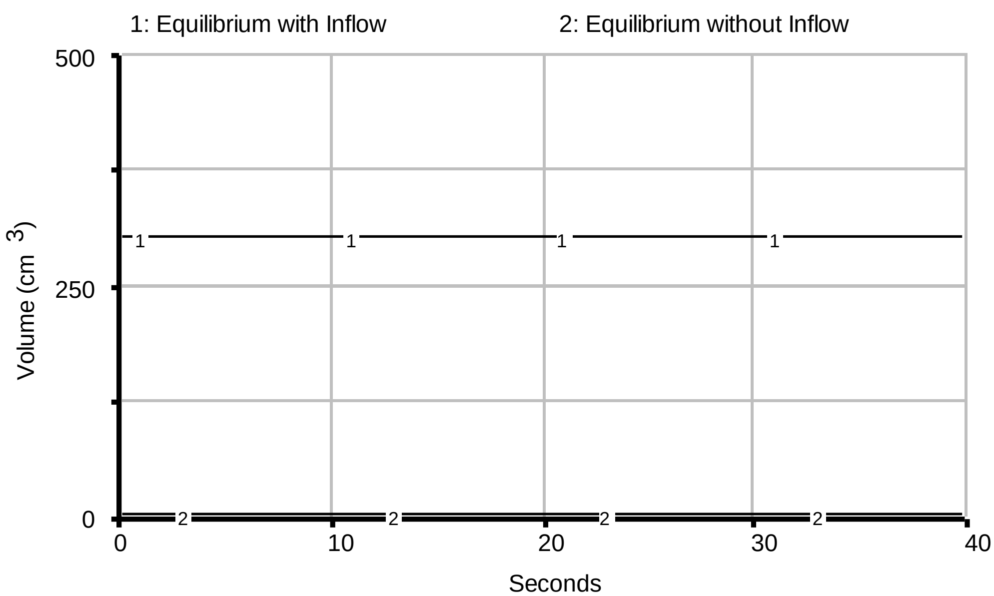


Figure 6: Equilibrium resulting from addition of inflow

Now let us simulate the sink system for the case when the sink contains 500 cm^3 of water at the beginning of the simulation.

1. *Calculate equilibrium.*

From the previous simulation the equilibrium volume is known to be 300 cm^3.

2. *Determine the behavior mode.*

For this simulation, the initial volume of water, 500 cm^3, is *greater* than the equilibrium value. Thus the system approaches equilibrium from *above*.

3. *Sketch the behavior.*

The behavior of the system as a whole can be decomposed into two separate parts that can be graphed separately. From the 500 cm^3 of water present at the beginning of the simulation, 300 cm^3 of it are in equilibrium. The remaining 200 cm^3 are subject to draining. The graph of the equilibrium component was obtained in the previous simulation. Now we shall proceed to sketch the behavior of the volume subject to draining. Subsequently, the behavior modes will be added to obtain the behavior for the system as a whole.

Draining of the 200 cm^3 of water can be sketched quickly using the half-life, which is approximated by:

Half-Life = 0.7 / Draining Fraction = 0.7 / 0.1 = 7 seconds.

Having obtained the half-life, a quick sketch, resembling Figure 7, can be obtained for the 200 cm^3 of water subject to draining.

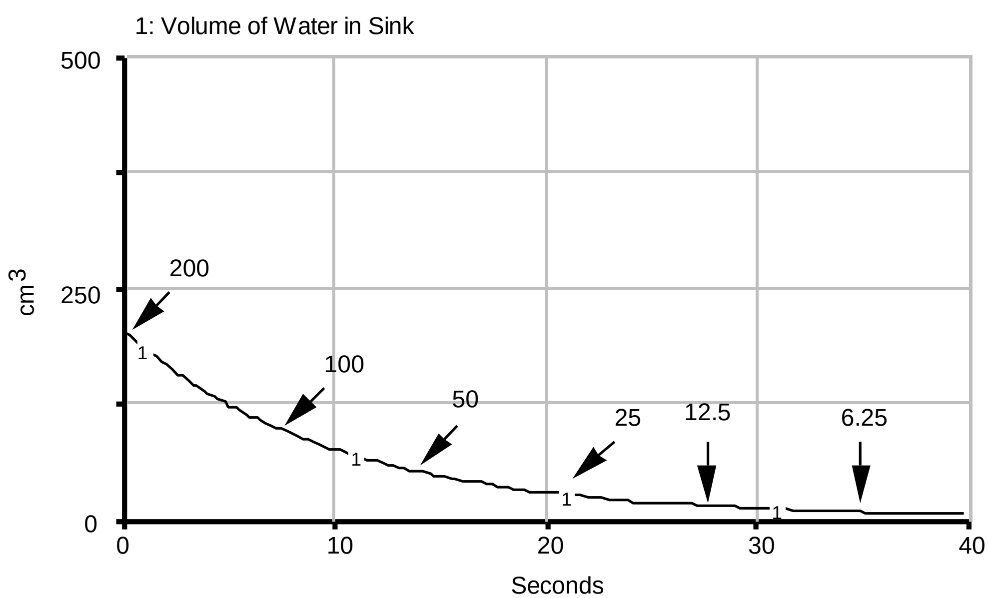


Figure 7: Exponential decay of water subject to draining

To obtain the sketch for the behavior of the system as a whole, the sketch for the 300 cm^3 of water in equilibrium, obtained in the previous example, is added to the sketch that was just obtained, representing exponential decay of 200 cm^3 of water. The result of adding the behavior modes is shown in Figure 8. The conclusion to draw from the graph is that addition of the constant flow has shifted the equilibrium, or goal, that the system wants to reach. However, it has not changed the time constant of the feedback.

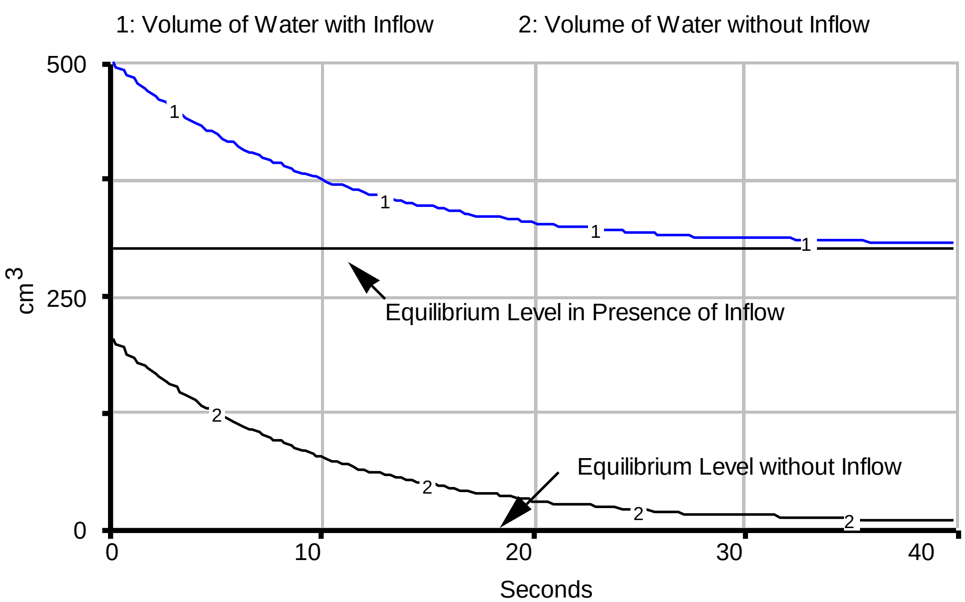


Figure 8: Mental simulation graph of water volume behavior

4. Exercise 2: Memorizing Song Lyrics

Victor loves listening to Italian opera. While he loves singing, he cannot remember the lyrics of these songs unless he listens attentively. Thus, he has decided that he will listen carefully to his favorite aria, and try to memorize each word. At first, as the song starts playing, he memorizes most words. However, as the song progresses, and Victor has already memorized many words, he starts forgetting some of the earlier words.

A. Sketch a model which shows how the stock of words that Victor remembers—while the song is being played—changes. Assume that the stream of words played is constant enough to allow modeling it as a constant inflow into Victor's consciousness. Moreover, assume that Victor forgets words at a rate proportional to the total number of words he remembers, at any given moment, and inversely proportional to some constant time-to-forget.

B. Victor comes up with a model which contains a stock of remembered words that is augmented by a constant stream of words, and decreased by a negative feedback loop which represents the words being forgotten. Victor does a variety of tests, listening to many arias, and comes to the following conclusions: for most arias a word is sung about every two seconds (0.5 words/second); for arias three minutes or longer, he remembers usually around forty-five words. Assuming his model is fairly accurate, what would the "time to forget" be? (Hint: Use the equilibrium relation for first-order systems.)

C. Using this model, how many words will Victor recall after listening carefully to a 10 minute long aria?

D. Victor eventually gets bored of listening to so much Italian opera and wants to listen to faster music. He goes to the record store and buys a Bob Dylan CD. These songs however are played at a rate of about two words per second (2 words/second), rather than one word every two seconds (0.5 words/second). Assuming that the time constant for forgetting the lyrics while the songs are playing is the same as that for the Italian arias, how would the behavior of the system, i.e. how does the stock of words he remembers while the song is being played, change? (A qualitative description is sufficient.)

E. If it takes longer to forget Bob Dylan lyrics—maybe because English is easier to remember than Italian—would the rate at which Victor forgets words be greater or less than before? Would he remember more or fewer words than for an aria of comparable length?

___

___

___

5. Review

The three steps to mentally simulating a first-order feedback system containing constant flows are as follows:

I. Calculate equilibrium:

- Sum of inflows = sum of outflows

II. Determine behavior mode:

- Equilibrium
- Diverge exponentially from equilibrium (positive feedback)
- Converge exponentially towards equilibrium (negative feedback)

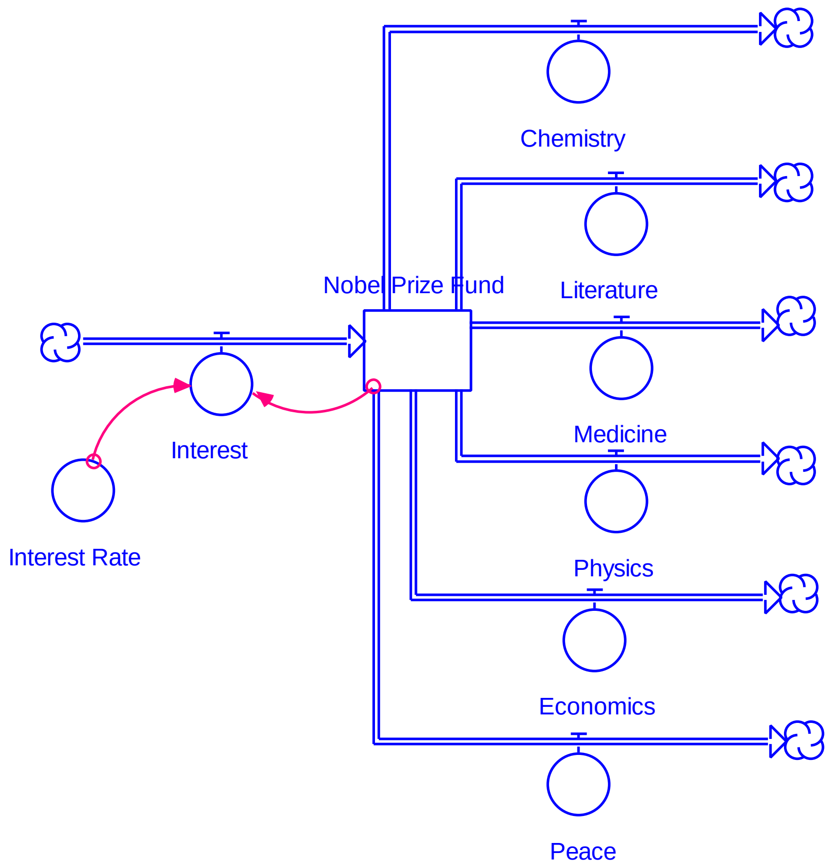
III. Sketch behavior:

1. Sketch equilibrium.
2. Sketch exponential behavior using time constant.
3. Add the behavior modes.

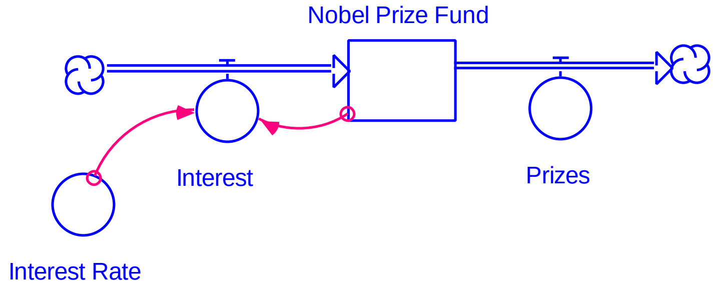
6. Solutions to Exercises

6.1 Solutions to Exercise One

A. The model contains a positive feedback loop, which represents interest payments, and six constant outflows, one for each prize.



B. This model predicts the same behavior for the bank account as the previous one. However, it is much simpler:



This model illustrates a virtue of aggregating variables in a model when possible. Doing so can simplify the model, and hence calculations, without changing the behavior of the variables being observed (such as the Nobel Prize Fund). Furthermore, this example demonstrates that the lessons we have learned for systems with one constant flow can be generalized to any first-order system containing more than one constant flow.

C. The bank account is at minimum. The removal of cash is balanced by the accrual of interest. Thus the equilibrium condition applies:

Outflow = Inflow

Removal of Cash = Accrual of Interest

Prizes = Fund * Interest Rate.

Solving this equation in terms of the Fund gives:

Fund = Prizes / Interest Rate = $6,000,000 / 0.10 = $60,000,000.

D. In order to mentally simulate the behavior of the Nobel Prize Fund when it begins at $30 million, let us follow the three steps for mentally simulating first-order systems.

1. *Calculate equilibrium.*

From the solution to Part C we know the account is at equilibrium when it contains $60,000,000.

2. *Determine the behavior mode.*

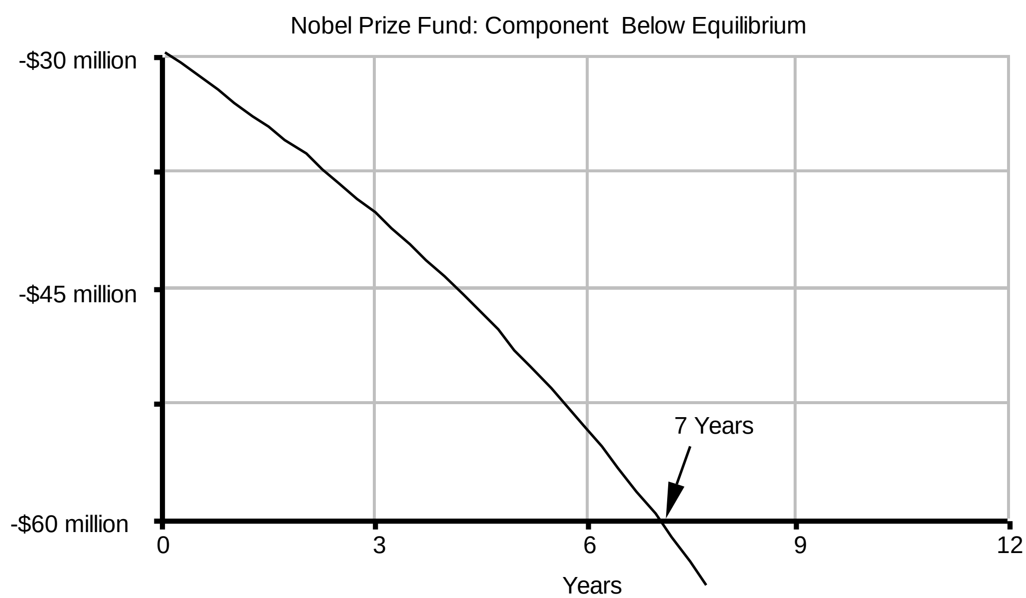
When there are only $30,000,000, the account is clearly not in equilibrium. There are *fewer* dollars than at equilibrium. Thus the behavior mode is *negative exponential growth* away from equilibrium.

3. *Sketch the behavior.*

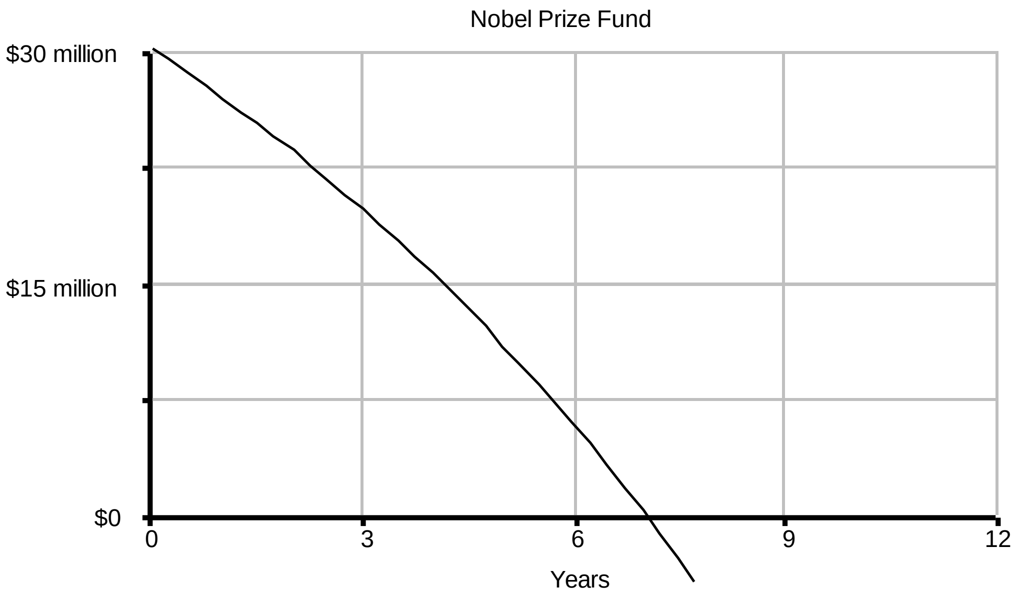
We must calculate the doubling time in order to determine by how much the fund deficit grows. By fund deficit is meant the amount by which the fund is below equilibrium. In this case the Fund’s value is initially equal to the sum of the equilibrium value, $60,000,000, and the amount below equilibrium, —$30,000,000. The graph of the component of the value that is at equilibrium is a horizontal line at $60,000,000. The graph of the account component below equilibrium is negative exponential growth with the following doubling time:

Doubling Time = 0.7 / Interest Rate = 0.7 / 0.10 = 7 years.

Using the doubling time, the sketch for the component of the Fund below equilibrium is as follows:



Notice that the amount doubles in 7 years from —$30,000,000 to —$60,000,000. Now, this behavior mode, that is the behavior of the component of the Fund below equilibrium, must be added to the equilibrium graph to obtain the graph of the behavior of the Fund as a whole. Sketching the final graph amounts to shifting the graph we just obtained by $60,000,000, which is the equilibrium value resulting from addition of the constant outflow.



Because of the model's simplicity, the Fund's value is allowed to become negative! A more comprehensive model might take into account the efforts by the Nobel Prize Committee to improve the balance, either by adding funds or by reducing the value of cash prizes. Also, in reality no bank would be willing to hold an account with a negative balance.

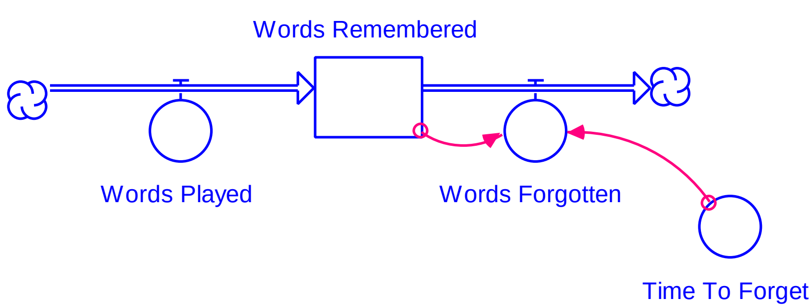
E. We use the equilibrium equation again to obtain the answer:

System Dynamics Prize = Funds available for prize * Interest rate

=$500,000 * 0.10 = $50,000.

6.2 Solutions to Exercise Two

A. Assuming that the flow of words is constant, we obtain the following model:



B. When the words are played at a rate of about one word every two seconds, he seems to reach an equilibrium of about 45 words remembered. Using the equilibrium equation below, we derive the time constant:

Words played = Words Forgotten,

= Words Remembered / Time to Forget,

Solving in terms of the time constant we obtain:

Time to Forget = Words Remembered / Words Played = 45 / 0.5 = 90 seconds.

C. Once Victor has reached equilibrium, he will remember about forty-five words, assuming that the song is still playing and the rate of words being played remains constant.

D. A faster stream of words played amounts to increasing the inflow of words entering Victor's memory. By changing the inflow, the equilibrium stock of words will shift. Solving in terms of Words Remembered, the equilibrium equation derived in part B becomes:

Words Remembered = Words Played * Time to Forget.

From the relation, increasing Words Played increases the stock of words remembered at equilibrium. Thus, Victor will remember more words when he listens to the Bob Dylan CD. Increasing the inflow of words shifts the stock of words remembered upward.

E. From the equilibrium equation in part D, increasing Time to Forget will increase the number of words Victor remembers at equilibrium. The reason is that increasing the time to forget decreases the rate at which words are forgotten. Given the same constant inflow, equilibrium shifts upward.

7. Appendix: Model Documentation

<u>Fruit Fly Population Model</u>

Fruit_Fly_Population(t) = Fruit_Fly_Population(t-dt) + (Reproduction_Rate - Removal_Rate) * dt

INIT Fruit_Fly_Population = 100

DOCUMENT: The Fruit Fly Population was initialized at 100 fruit flies for the equilibrium example, and 120 fruit flies for the second example, which involved exponential growth (*fruit flies*).

INFLOWS:

Reproduction_Rate = Fruit_Fly_Population * Reproduction _Ratio

DOCUMENT: Rate at which fruit flies reproduce (*fruit flies / day*).

OUTFLOWS:

Removal_Rate = 50

DOCUMENT: Fruit flies removed daily (*fruit flies / day*).

Reproduction_Ratio = 0.5

DOCUMENT: Ratio of fruit flies added to the population per day (*1 / day*).

<u>Nobel Prize Fund Model</u>

Nobel_Prize_Fund(t) = Nobel_Prize_Fund(t-dt) + (Interest - Prizes) * dt

INIT Nobel_Prize_Fund = 60000000

DOCUMENT: The Nobel Prize Fund contains about $60,000,000 at equilibrium (*dollars*).

INFLOWS:

Interest = Nobel_Prize_Fund * Interest_Rate

DOCUMENT: Rate of interest payments to the fund (*dollars / year*).

OUTFLOWS:

Prizes = 6000000

DOCUMENT: Amount of cash prizes awarded yearly (*dollars / year*).

Interest Rate = 0.10

DOCUMENT: Interest rate paid on bank accounts (*1 / year*).

<u>Model for Draining Sink</u>

Water_in_Sink(t) = Water_in_Sink(t-dt) + (Stream_In - Draining) * dt

INIT Water_in_Sink = 100

DOCUMENT: Water in the sink is at equilibrium when there are 300 cubic centimeters. For the example involving exponential decay, the water in the sink began at 500 cubic centimeters (cm^3).

INFLOWS:

Stream_In = 30

DOCUMENT: Rate at which water flows into the sink (cm^3 / s). Assumed to be constant.

OUTFLOWS:

Draining = Water_in_Sink * Draining_Fraction

DOCUMENT: Rate at which water drains from the sink (cm^3 / s).

Draining Fraction = 0.1

DOCUMENT: Fraction of volume of water which flows out the drain per second (*1 / s*).

<u>Memorizing Song Lyrics Model</u>

Words_Remembered(t) = Words_Remembered(t-dt) + (Words_Played -
Words_Forgotten) * dt

INIT Words_Remembered = 0

DOCUMENT: At the beginning of the song it is assumed that Victor does not know any of the lyrics (*words*).

INFLOWS:

Words_Played = 0.5

DOCUMENT: Rate at which words are played (*words / s*). Assumed to be constant.

OUTFLOWS:

Words_Forgotten = Words_Remembered / Time_To_Forget;

DOCUMENT: Rate at which Victor forgets words (*words / s*).

Time_To_Forget = 90

DOCUMENT: Time it takes for Victor to forget a word on average (*words / s*).

8. Bibliography

Forrester, Jay W. (1961). Industrial Dynamics. Portland, OR: Productivity Press.

Forrester, Jay W. (1968). Principle of Systems. Portland, OR: Productivity Press.

Goodman, Michael R. (1974). Study Notes in System Dynamics. Portland, OR: Productivity Press.

Roberts, Nancy et al. (1983). Introduction to Computer Simulation. Portland, OR: Productivity Press.

Vensim Examples:

Beginner Modeling Exercises

By Aaron Diamond

October 2001

1. Positive Feedback with Constant Outflow

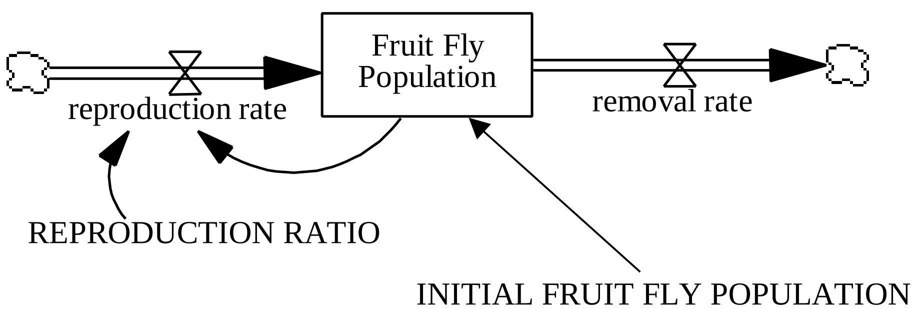


Figure 9: Vensim Equivalent of Figure 1: Fruit fly population model.

Documentation for Fruit Fly population model

(1) FINAL TIME = 4

Units: day

The final time for the simulation.

(2) Fruit Fly Population= INTEG (+reproduction rate-removal rate, INITIAL FRUIT FLY POPULATION)

Units: fruit flies

The Fruit Fly Population was initialized at 100 fruit flies for the equilibriun example, and 120 fruit flies for the second example, which involved exponential growth.

(3) INITIAL FRUIT FLY POPULATION=100

Units: fruit flies

(4) INITIAL TIME = 0

Units: day

The initial time for the simulation.

(5) removal rate=50

Units: fruit flies/day

Fruit flies removed daily.

(6) reproduction rate=Fruit Fly Population*REPRODUCTION RATIO

Units: fruit flies/day

Rate at which fruit flies reproduce.

(7) REPRODUCTION RATIO=0.5

Units: 1/day

Ratio of fruit flies added to the population per day.

(8) SAVEPER = 1

Units: day

The frequency with which output is stored.

(9) TIME STEP = 0.0625

Units: day

The time step for the simulation.

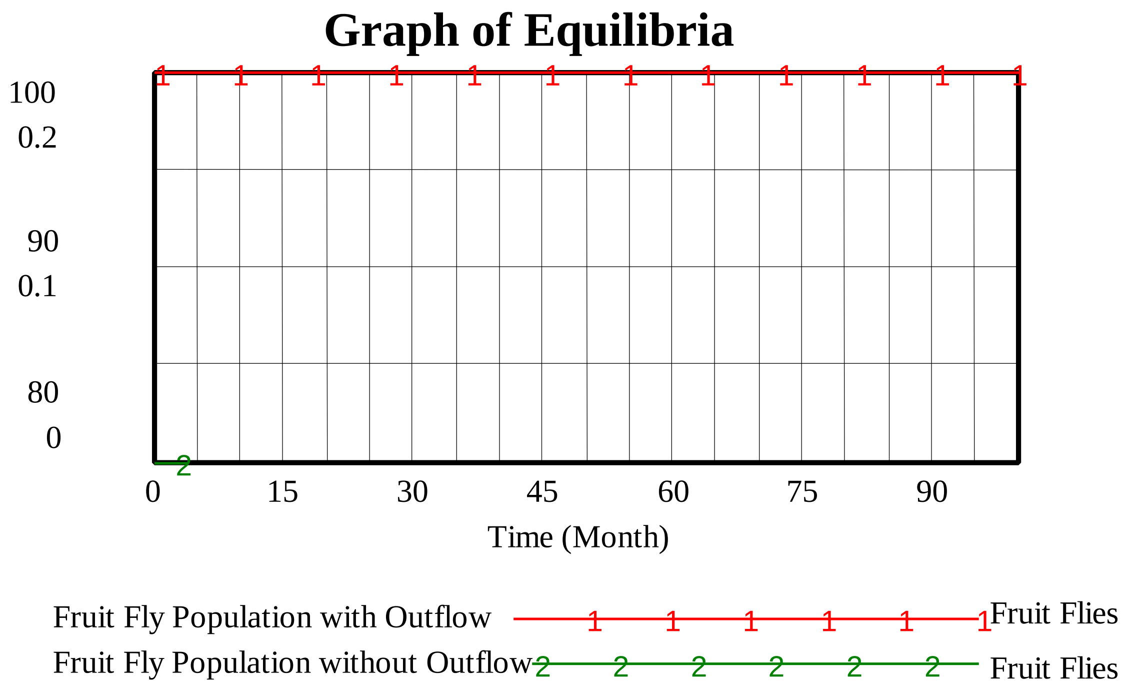


Figure 10: Vensim equivalent of Figure 2: Change in equilibrium as a result of outflow

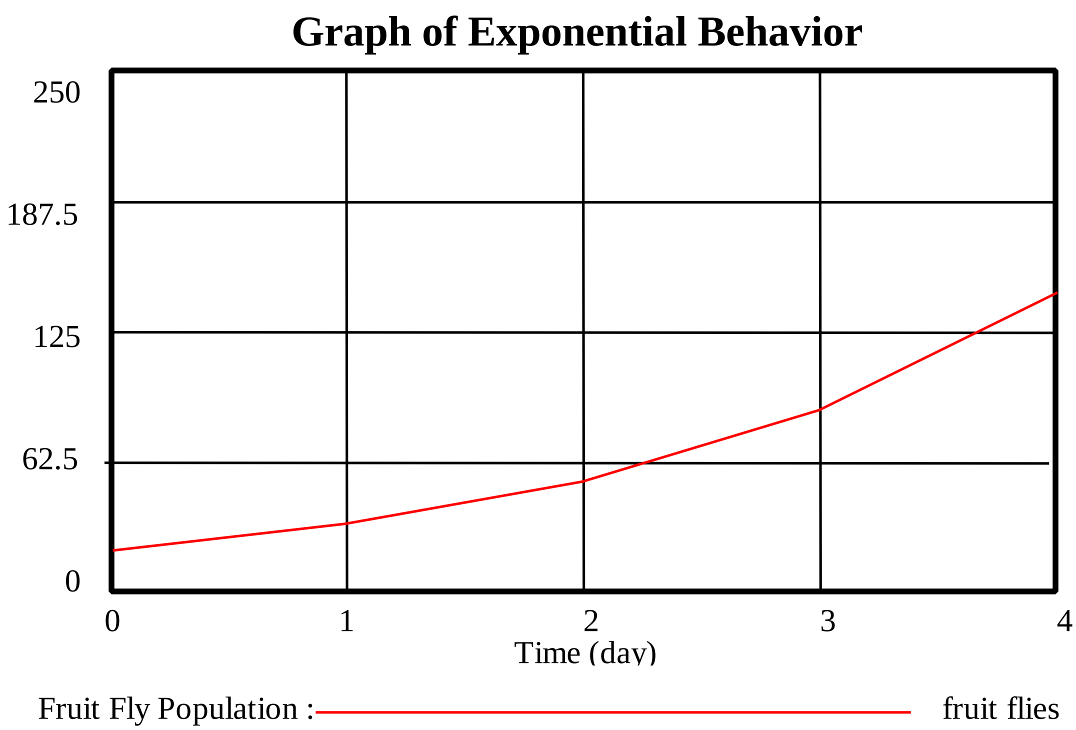


Figure 11: Vensim Equivalent of Figure 3: Exponential growth of twenty additional fruit flies

2. Exercise 1: Nobel Prize Fund

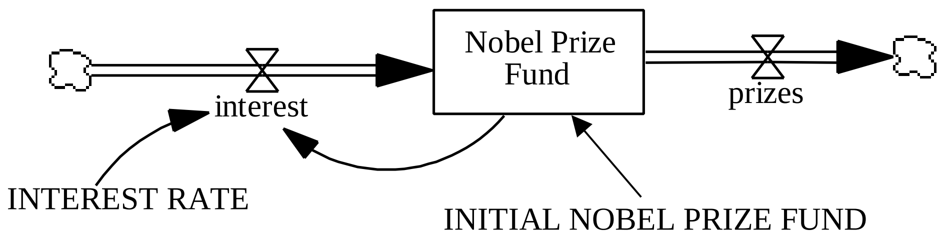


Figure 12: Vensim Equivalent of Figure 7.1, B

Documentation for Nobel Prize Fund model

(1) FINAL TIME = 12

Units: year

The final time for the simulation.

(2) INITIAL NOBEL PRIZE FUND= 6e+007

Units: dollars

(3) INITIAL TIME = 0

Units: year

The initial time for the simulation.

(4) interest=Nobel Prize Fund*INTEREST RATE

Units: dollars/year

Rate of interest payments to the fund.

(5) INTEREST RATE=0.1

Units: 1/year

Interest rate paid on bank accounts.

(6) Nobel Prize Fund= INTEG (interest-prizes, INITIAL NOBEL PRIZE FUND)

Units: dollars

The Nobel Prize Fund contains about $60,000,000 at equilibrium.

(6) prizes=6e+006

Units: dollars/year

Amount of cash prizes awarded yearly.

(7) SAVEPER =TIME STEP

Units: year

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: year

The time step for the simulation.

3. Negative Feedback with Constant Inflow

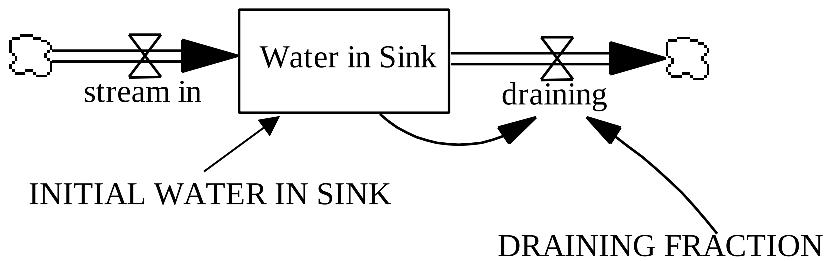


Figure 13: Vensim Equivalent of Figure 5: Model for draining sink with constant inflow

Documentation for draining sink model

(1) draining=Water in Sink*DRAINING FRACTION

Units: cm^3/s

Rate at which water drains from the sink.

(2) DRAINING FRACTION=0.1

Units: 1/s

Fraction of volume of water which flows out the drain per second.

(3) FINAL TIME = 40

Units: s

The final time for the simulation.

(4) INITIAL TIME = 0

Units: s

The initial time for the simulation.

(5) INITIAL WATER IN SINK=100

Units: cm^3

(6) SAVEPER = TIME STEP

Units: s

The frequency with which output is stored.

(7) stream in=30

Units: cm^3/s

Rate at which water flows into the sink. Assumed to be constant.

(8) TIME STEP = 0.0625

Units: s

The time step for the simulation.

(9) Water in Sink= INTEG (+stream in-draining, INITIAL WATER IN SINK)

Units: cm^3

Water in the sink is at equilibrium when there are 300 cubic centimeters. For the example involving exponential decay, the water in the sink began at 500 cubic centimeters.

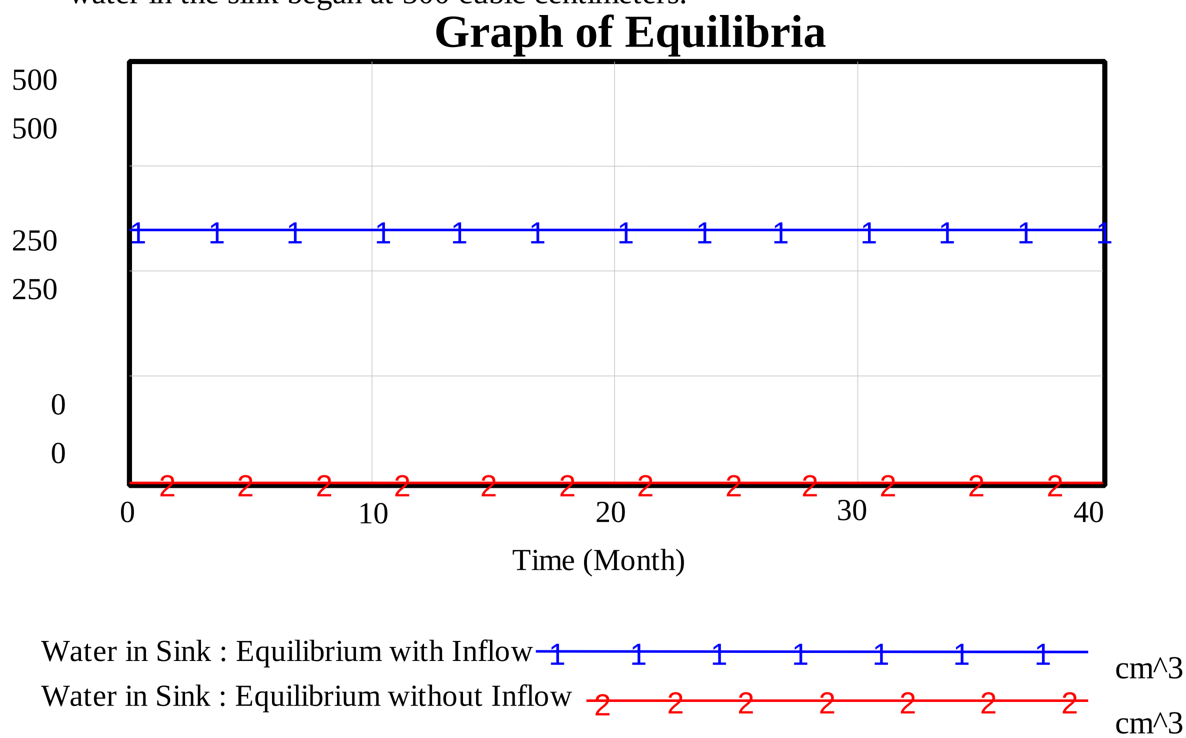


Figure 14: Vensim Equivalent of Figure 6: Equilibrium resulting from addition of inflow.

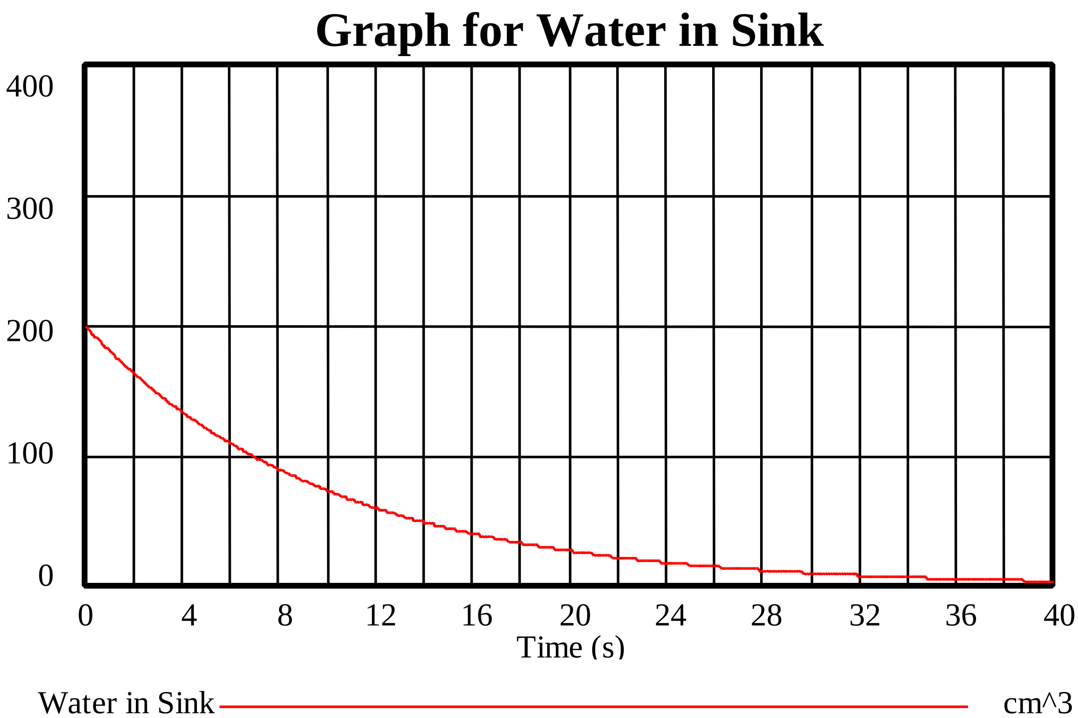


Figure 15: Vensim Equivalent of Figure 7: Exponential decay of water subject to draining.

4. Exercise 2: Memorizing Song Lyrics

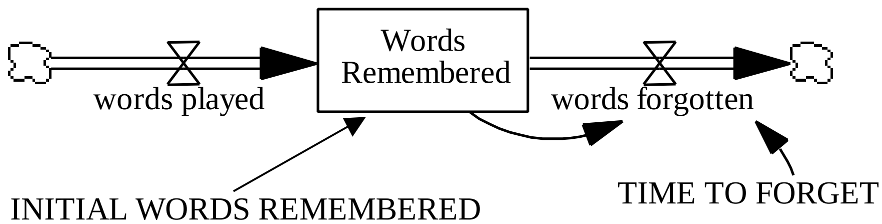


Figure 16: Vensim Equivalent of 7.2, A.

Documentation for Memorizing Song Lyrics model

(1) FINAL TIME = 100

Units: s

The final time for the simulation.

(2) INITIAL TIME = 0

Units: s

The initial time for the simulation.

(3) INITIAL WORDS REMEMBERED= 0

Units: words

(4) SAVEPER = TIME STEP

Units: s

The frequency with which output is stored.

(5) TIME STEP = 0.0625

Units: s

The time step for the simulation.

(6) TIME TO FORGET=90

Units: s

Time it takes for Victor to forget a word on average.

(7) words forgotten=Words Remembered/TIME TO FORGET

Units: words/s

Rate at which Victor forgets words.

(8) words played=0.5

Units: words/s

Rate at which words are played. Assumed to be constant.

(9) Words Remembered= INTEG (+words played-words forgotten, INITIAL WORDS REMEMBERED)

Units: words

At the begining of the song it is assumed that Victor does not
know any of the lyrics.