Optimal Control of High-Volume Assemble-to-Order Systems

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December 15, 2003

presentation for
15.764 The Theory of Operations Management
March 4, 2004

Presentation by Ping Xu

This summary presentation is based on: Plambeck, Erica, and Amy Ward. "Optimal Control of High-Volume Assemble-to-Order Systems." Stanford University, 2003.

Motivation

- Assembly-to-Order
  - hold component inventories
  - rapid assembly of many products
  - Dell - grown by 40% per year in recent years. PC industry - grown by less than 20% per year.
  - GE, American Standard, BMW, Timbuk2, National Bicycle.

- Challenges of ATO
  - product prices?
  - production capacity for component (supply contract)?
  - dynamically ration scarce components to customer orders?

Overview

- Literature review
- Model formulation
  - Dynamic control problem
  - Static formulation
- Asymptotic analysis
- Delay bound and expediting component option

Literature

- ATO survey by Song and Zipkin (2001)
- not FIFO assembly
  - Agrawal and Cohen (2001), Zhang (1997)
- one component and multi-product assembly sequencing — multi-class, single-server queue
  - Wein (1991) , Duenya (1995)
  - Maglaras and Van Mieghem (2002), Plambeck, Kumar, and Harrison (2001)
- *fill rate* constraints
  - Lu, Song, and Yao (2003), Cheng, Ettl, Lin, and Yao (2002)
  - Glasserman and Wang (1998)

Model Formulation

Sequence of events:
1. set product prices, component production rates – remain fixed throughout time horizon
2. dynamically sequence assembly of outstanding product orders

Objective:
minimize infinite horizon discounted expected profit

Trade-off:
inventory vs. customer service (assembly delay, cash flow)

Operational Assumptions:
– assembly is instantaneous given necessary components
– customer order for each product are filled FIFO

Model Formulation - notations

J	components
K	finished products
a_{kj}	no. of type j components needed by product k
p_k	product price
γ_j	component production rate
O_k	product demand arrival renewal process, rate $\lambda_k(p)$
C_j	component arrival renewal process, rate γ_j
c_j	component unit production cost
$A_k(t)$	cumulative no. of type k orders assembled up to t
$u = (p_u, \gamma_u, A_u)$	admissible policy (prices, production rates, assembly sequence rule)
$Q_{u,k}(t)$	order queue-length, $= O_{u,k}(t) - A_{u,k}(t) \geq 0$
$I_{u,j}(t)$	inventory levels, $= C_{u,j}(t) - \sum_{k=1}^{K} a_{kj} A_{u,k}(t) \geq 0$

Model Formulation - technical assumptions

$\lambda(p)$ is continuous, differentiable, and the Jacobian matrix is invertible. guarantees $p(\lambda)$ is unique, continuous, and differentiable.

Customer demand for product k is strictly decreasing in p_k, but may be increasing in $p_m, m \neq k$. $\frac{\partial \lambda_k(p)}{\partial p_k} < 0$ while $\frac{\partial \lambda_k(p)}{\partial p_m} \geq 0, m \neq k$.

Increase in the price of one product cannot lead to an increase in the total rate of demand for all products. $\frac{-\partial \lambda_k}{\partial p_k} > \sum_{m \neq k} \frac{\partial \lambda_m}{\partial p_k}$.

Revenue rates for each product class, $r_k(\lambda) = \lambda_k p_k(\lambda)$ are concave.

Renewal processes O_k and C_j started in steady state at time zero.

Model Formulation - profit expression

infinite horizon discounted profit:

$$\Pi = \sum_{k=1}^{K} \int_{o}^{\infty} p_k e^{-\delta t} dA_k(t) - \sum_{j=1}^{J} \int_{0}^{\infty} c_j e^{-\delta t} dC_j(t)$$

$$\Downarrow$$

$$\Pi = \sum_{k=1}^{K} \left(\int_{0}^{\infty} p_k e^{-\delta t} dO_k(t) - \int_{0}^{\infty} Q_k(t) \delta e^{-\delta t} dt \right) - \sum_{j=1}^{J} \int_{0}^{\infty} c_j e^{-\delta t} dC_j(t),$$

where $Q_k(t)$ is the order queue-length

$$\int_{0}^{\infty} e^{-\delta t} dO_k(t) - \int_{0}^{\infty} e^{-\delta t} dA_k(t) = \int_{0}^{\infty} \delta e^{-\delta t} Q_k(t) dt$$

Model Formulation - static planning problem

if we assume that demand and production flow at the long run average rates continuously and deterministically,

$$\bar{\pi} \quad = \quad \max_{p \geq 0, \gamma \geq 0} \sum_{k=1}^{K} p_k \lambda_k(p) - \sum_{j=1}^{J} \gamma_j c_j$$

$$\text{s.t.} \quad \sum_{k=1}^{K} a_{kj} \lambda_k(p) \leq \gamma_j, \qquad j = 1, ..., J$$

– optimal solution (p^*, γ^*) assumed to be unique, positive. the first order condition imply that all constraints are tight (p^*, γ^*).

– $\bar{\pi}$ is an upper bound on the expected profit rate.

want to show that under high volume conditions, the optimal prices and production rates are close to (p^*, γ^*).

Asymptotic analysis - high demand volume conditions

any strictly increasing sequence $\{n\}$ in $[0, \infty)$, n tends to infinity. order arrival rate function λ^n, where $\lambda_k^n(p) = n\lambda_k(p), \quad k = 1, ..., K.$

$n\bar{\pi}$ upper bounds the expected profit rate in the n^{th} system,

$$\Pi^n \leq \int_0^\infty n\bar{\pi}\delta e^{-\delta t} dt = \delta^{-1} n\bar{\pi}$$

plug $(p^*, n\gamma^*)$ into the n^{th} system, $n^{-1}\Pi_{(p^*, n\gamma^*, A^n)} \to \delta^{-1}\bar{\pi}$ as $n \to \infty$, given that $n^{-1}Q^n \to 0$ a.s., as $n \to \infty$.

Asymptotic analysis - proposed assembly policy

component shortage process:

$$S_j(t) \quad = \quad \sum_{k=1}^{K} a_{kj} O_k(t) - C_j(t) = \sum_{k=1}^{K} a_{kj} Q_k(t) - I_j(t), \quad j = 1, ..., J$$

min. instantaneous cost arrangement of queue-lengths and inventory levels $(Q^*(S), I^*(S))$,

$$\min_{Q,I \geq 0} \qquad \sum_{k=1}^{K} p_k^* Q_k$$

$$\text{s.t.} \qquad I_j = \sum_{k=1}^{K} a_{kj} Q_k - S_j \geq 0, \quad j = 1, ..., J$$

Asymptotic analysis - proposed assembly policy

for the n^{th} system, the review period $l^n = n^{-\alpha}$, where $\alpha = (4(3+2\epsilon_1))^{-1}(6+5\epsilon_1) > 1/2$

Asymptotic analysis - system behavior

(See Theorem 1 on page 12 of the Plambeck and Ward paper)

Review on Brownian Motion

A standard Brownian Motion (Wiener process) is a stochastic process W having
1. continuous sample paths
2. stationary independent increments
3. $W(t) \sim N(0, t)$

A stochastic process X is a Brownian motion with drift μ and variance σ^2 if

$$X(t) = X(0) + \mu t + \sigma W(t), \quad \forall t$$

then $E[X(t) - X(0)] = \mu t$, $Var[X(t) - X(0)] = \sigma^2 t$.

variance of a Brownian motion increases linearly with the time interval.

Optimality of Nearly Balanced Systems

(See Theorem 2 on page 15 of the Plambeck and Ward paper)

System with delay constraints

propose a near-optimal discrete review control policies, which both sequences customer orders for assembly and expedites component production in an ATO system with delay constraints.