

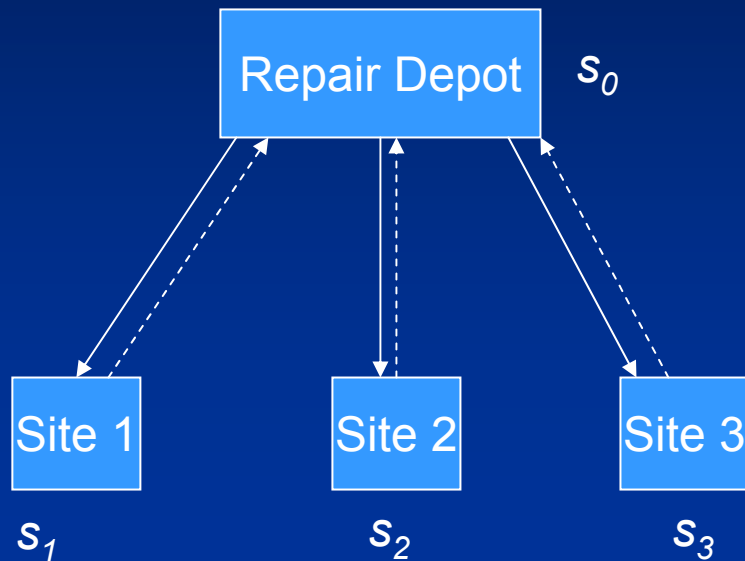
“A Multi-Echelon Inventory Model for a Repairable Item with one-for- one Replenishment”

Steve Graves, 1985

Management Science, 31(10)

Presented by Hongmin Li

Problem



- Failed item is replaced at the site from the site's inventory if available; otherwise the shortage lasts until a replacement arrives from the depot
- Failed item is sent to depot for repair. It enters the repair process upon arrival at the depot and goes into the depot inventory upon completion
- Depot ships a replacement if available; otherwise, the depot backorders the request and fill it when available

Assumptions

- Failure process at each site is a compound Poisson process that depends upon the required no. of working items, not the actual no., thus indep. of the status of the site
- One-for-one replenishment
- Shipment time from depot to site is deterministic (T_1)
- Ample repair capacity at the depot

Notations

- $Q_i(t)$
Outstanding orders at site i at time t
- $Q(t)$
Aggregate outstanding orders at the sites at time t
- $B(t|s_0)$
Backorders at time t at the depot given s_0
- $D_i(t_1, t_2)$
Failures at site i over the time interval $(t_1, t_2]$
- $D(t_1, t_2)$
Aggregate failures at all sites over the time interval $(t_1, t_2]$

“replacement requests that have yet to be filled”

Primary Result

- $Q(t+T_1) = B(t|s_0) + D(t, t+T_1)$
- No depot backorders at t can arrive at the sites by $t+T_1$ and no failure occurring after t can be replenished before $t+T_1$
- $B(t|s_0)$ and $D(t, t+T_1)$ are indep. r.v.s since depot back orders at t depend only on failures that occur prior to t

In order to find $Q_i(t)$...

- Convolve the distribution of $B(t|s_0)$ and $D(t, t+T_1)$ to obtain the distribution of $Q(t)$
- Disaggregate $Q(t)$ into $Q_i(t)$...

$$B(t|s_0) = [Q_0(t) - s_0]^+$$

$Q_0(t)$ the total no. of failed items in the system at t

Assuming Ample Repair Capacity

- $Q_0(t)$ is the occupancy level in a $M|G|\infty$ queue where the service time includes the in-transit time to the depot T_1 and the repair time
- Palm's theorem
 - Steady state distribution of $Q_0(t)$ is Poisson

Disaggregation of $Q(t)$

- Assuming depot backorders are filled FCFS:
The likelihood that any outstanding order is from site i is directly proportional to site i 's failure rate λ_i , thus conditional distribution of $Q_i(t)$ is binomial.

(See equation 3 in the 1985
Graves paper)

Comparison to the METRIC approximation

- METRIC approximates $Q_i(t)$ as the occupancy level of an independent $M|G|_\infty$ queue. Thus Q_i is Poisson.
- METRIC approximation for the case of a deterministic transit time to the sites is equivalent to approximating the depot backorder level by a Poisson r.v.

Exact Models

- Determine the distribution of Q_0
 - Ample capacity
use Palm's theorem (assuming Poisson or compound Poisson failure processes):
 - General shipment time, Poisson failure process, k parallel lines

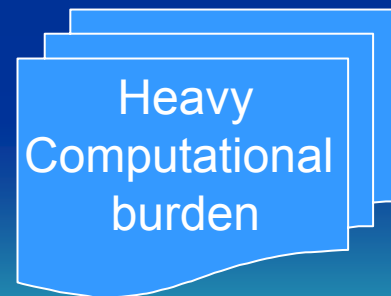
$$Q_0 = \textit{In-transit} + \textit{In-repair} + \textit{In-repair-queue}$$

Occupancy level in
an $M|G|\infty$ queue

an $M|M|k$
system

Exact Models

- For every value of s_0
 - Find distribution of backorders at the depot
 - Disaggregate using conditional distribution $Pr[Q_i=j|Q=k]$, which is determined by the priority scheme for filling the outstanding orders



Approximate Model

- From $Q(t+T_1) = B(t|s_0) + D(t, t+T_1)$ and binomial conditional distribution of the outstanding orders at each site, the mean and variance of each site's outstanding orders can be expressed in terms of the mean and variance of the depot backorder levels:

Graves (1983)

Approximate Model

- From $B(s_0) = [Q_0 - s_0]^+$

(See equations 6 and 7
in the 1985 Graves paper)

- Thus we can compute the mean and variance of $B(s_0)$ for all values of s_0 of interest given the distribution of Q_0

Approximate Model

- Next approximate the distribution of Q_i by a negative binomial distribution

(See equations 8, 9, and 10
in the 1985 Graves paper)

Justification

- Exploratory investigation suggests that the distribution of Q_i will be unimodal and will have variance strictly greater than its mean

Test Problem

- Specify the following
 - the expected repair cycle time
 - the aggregate demand rate
 - the depot stock level
 - the site fill rate
- Test shows that METRIC errs 11.5%, the approximate model errs 0.9%

Summary

- Provides a general framework for determining the distribution of net inventory levels in a multi-echelon system
- Presents an approximate model assuming ample repair capacity, Poisson failure process and FCFS rule

Questions