Principles of valuation

Class 2
Financial Management, 15.414

Today

Principles of valuation

- Present value
- Opportunity cost of capital

Reading

- Brealey and Myers, Chapters 2 and 3

Valuation

Applications

- Real assets (capital budgeting)
- Bonds (financing decisions)
- Stocks and firms (financing decisions, M&A, …)

Common feature
Invest cash today in exchange for expected, but generally risky, cashflows in the future.

Time	0	1	2	3	4	⋯
Cost	CF_0					⋯
Payoff		CF_1	CF_2	CF_3	CF_4	⋯

Examples

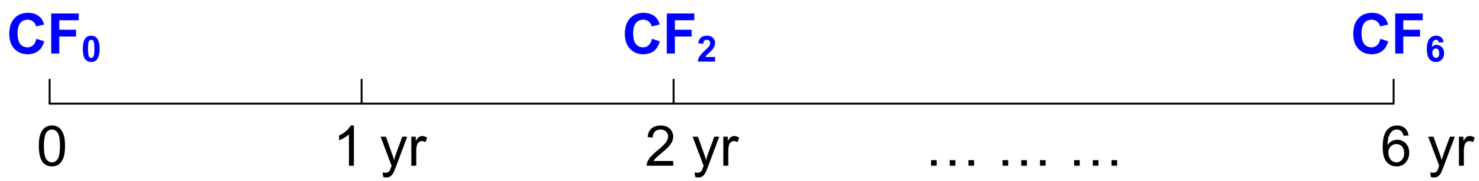
- In May 2000, the U.S. Treasury issued 30-year bonds with a coupon rate of 6 1/4 percent, paid semiannually. The principal will be repaid in May 2030, and a bond with a face value of $1000 pays $31.25 every six months until then.

- You work for Boeing. The CEO asks you to recommend whether or not to proceed with development of a new regional jet. You expect development to take 2 years, cost roughly $750 million, and you hope to get unit costs down to $32 million. You forecast that Boeing can sell 30 planes each year at an average price of $41 million.

- Firms in the S&P 500 are expected to earn, collectively, $32 this year and to pay dividends of $18. Dividends and earnings have historically grown about 3.2% annually in real terms (6.6% in nominal terms) since 1926.

Valuation

Asset



What determines the value of the asset? What factors are important?

Valuation

Suppose CF_t is riskless

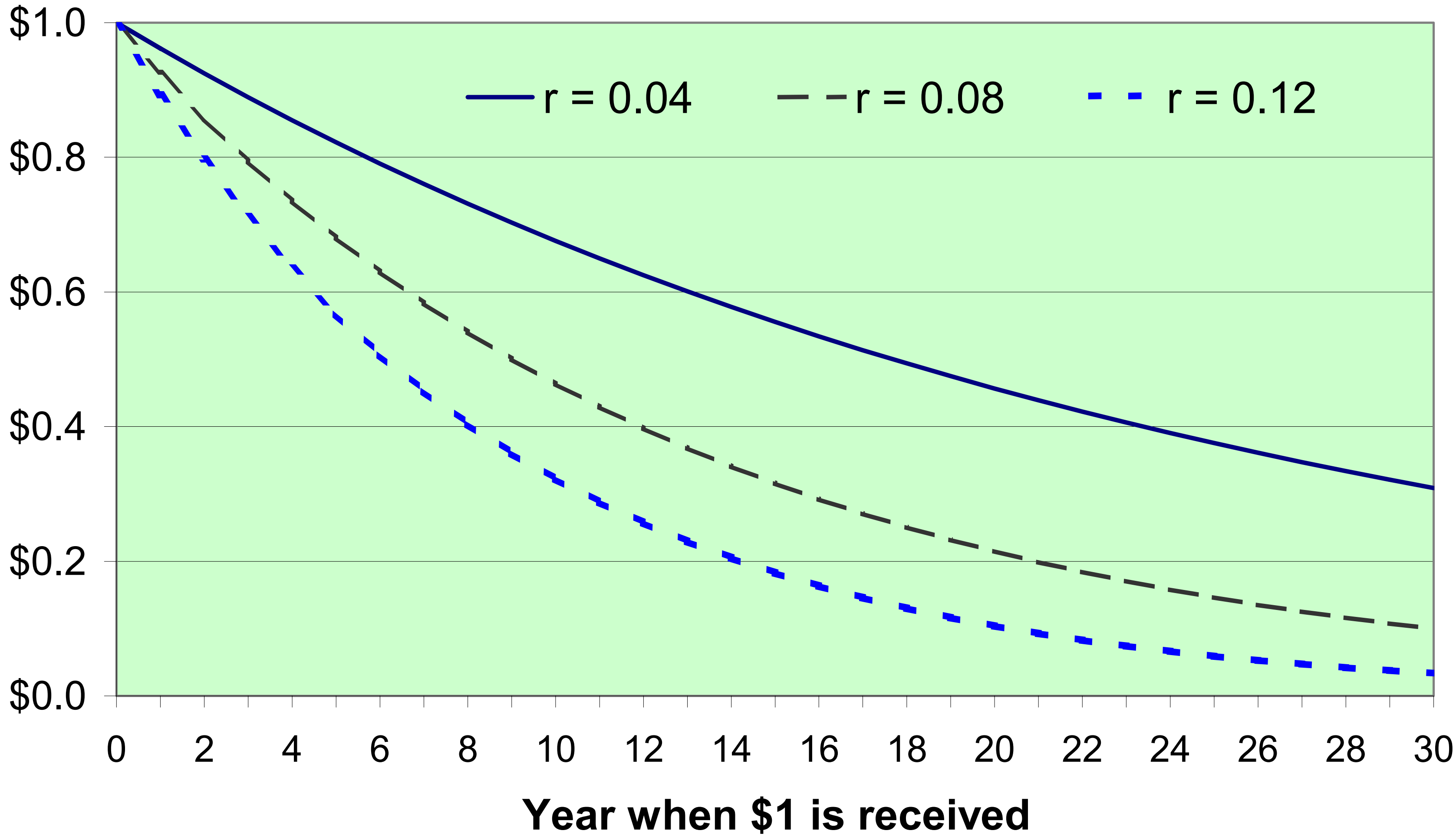
Time value of money

A \$1 received in the future is always worth less than \$1 received today.

If the interest rate is r, then the 'present value' of a riskless cashflow CF_t received in t years is:

$$\text{Present value} = \frac{CF_t}{(1+r)^t}$$

PV of $1 received in year t



Time value of money

You have \$1 today and the interest rate on riskfree investments (Treasury bills) is 5%.

How much will you have in …

1 year … $\$1 \times 1.05 = \$1.05$

2 years … $\$1 \times 1.05 \times 1.05 = \$1.103$

t years … $\$1 \times 1.05 \times 1.05 \times \ldots \times 1.05 = \$1.05^t$

These cashflows are equivalent to each other. They all have the same value.

$\Rightarrow$ \$1 today is equivalent to $\$(1+r)^t$ in t years

$\Rightarrow$ \$1 in t years is equivalent to $\$1 / (1+r)^t$ today

Example

Your firm spends $800,000 annually for electricity at its Boston headquarters. A sales representative from Johnson Controls wants to sell you a new computer-controlled lighting system that will reduce electrical bills by roughly $90,000 in each of the next three years. If the system costs $230,000, fully installed, should you go ahead with the investment?

Lighting system

Year	0	1	2	3
Cashflow	-230,000	90,000	90,000	90,000

$$\underbrace{90{,}000 \quad 90{,}000 \quad 90{,}000}_{\$270{,}000}$$

Example, cont.

Assume the cost savings are known with certainty and the interest rate is 4%.

Year	0	1	2	3
Cashflow	-230,000	90,000	90,000	90,000
÷		1.04	1.04^2	1.04^3
PV	-230,000	86,538	83,210	80,010

Net present value

$$NPV = -230{,}000 + \underbrace{86{,}538 + 83{,}210 + 80{,}010}_{\$249{,}758} = \$19{,}758$$

Go ahead.

Example, cont.

Perspective 2

Instead of investing $230,000 in the lighting system, you put it in the bank. Is this investment better or worse than investing in the lighting system?

Year	1	2	3
Beg. balance	230,000	149,200	65,168
End. balance (r = 0.04)	239,200	155,168	67,775
Withdrawal	**90,000**	**90,000**	**67,775**
Balance forward	149,200	65,168	0

The project creates value because it has a higher return than other riskfree investments.

Fundamental principle

The value of any asset or investment equals the **net present value of the expected cashflows**:

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5} + \ldots$$

Risk should be incorporated into r

The discount rate for the investment equals the rate of return that could be earned on an investment in the financial markets with similar risk.

r = ‘opportunity cost of capital’ or ‘required rate of return’

A project creates value only if it generates a higher return than similar investments in the financial market.

Example

Lighting system, cont.

Electricity prices can fluctuate, so you're not sure how much the firm will save by investing in the lighting system. Your **best guess** is that the firm will save $90,000 in each of the next three years, but the savings could be higher or lower. Risk is comparable to an investment in utility stocks, which have an expected rate of return of 7%.

$$\text{NPV} = -230{,}000 + \frac{90{,}000}{1.07} + \frac{90{,}000}{1.07^2} + \frac{90{,}000}{1.07^3} = \$6{,}188$$

Go ahead. The project is now less valuable, but it still creates value since NPV > 0.

Applications

(1) Diversifying investments

It's 1990 and you work for AT&T. Your boss, Robert Allen, asks you to evaluate a possible merger with NCR, a large computer manufacturer.

As part of the analysis, you need to come up with an appropriate discount rate for valuation. Investors generally require a 10% return on investments in AT&T.

What is the cost of capital for the merger?

10% represents the cost of capital for investments in telecom services. It is not appropriate for an investment in the computer industry. Need to estimate the cost of capital required by computer firms.

Applications

(2) Multiple divisions

You work for a large, diversified company. Last year, approximately 30% of profits came from auto parts, 30% came from electronics, and 40% came from financial services.

Your boss asks you to evaluate a proposed growth opportunity in financial services. You estimate the firm's cost of capital is 11%. Is this the appropriate discount rate to use for the proposed investment?

No. 11% is really an average cost of capital for the firm. Each division should use a separate discount rate, reflecting the risk of that division. Estimate by comparing to firms in each of the three industries.

Applications

(3) International investments

You work for Novartis AG, a large Swiss pharmaceutical company. Your investor base is predominately Swiss (79%).

The company is evaluating a possible expansion into the U.S. drug market. How should the company estimate the cost of capital for the project?

Does currency risk matter? Does the location of shareholders matter?

The cost of capital is determined by the return on similar investments, in this case an investment in a U.S. drug company. Location / currency risk shouldn't matter.

Complications

Compounding intervals

Inflation

Taxes

Currencies

Term structure of interest rates

Forecasting cashflows

Choosing the right discount rate (easy only if riskfree)

Aside: Shortcut formulas

Present value

$$PV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5} + \ldots$$

Simplifying formulas

- **Annuity**
  Level cashflow for a given number of years
- **Perpetuity**
  Level cashflow stream forever
- **Growing perpetuity**
  Cashflows grow by a fixed percent forever

Shortcut formulas

Annuity (level cashflow for t years)

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Perpetuity (level cashflow forever)

$$PV = \frac{C}{r}$$

Growing perpetuity (growing cashflow forever)

$$PV = \frac{C}{r-g}$$

Example

Firms in the S&P 500 are expected to pay, collectively, $20 in dividends next year. If growth is constant, what should the level of the index be if dividends are expected to grow 5% annually? 6% annually? Assume r = 8%.

- **Growing perpetuity**

g=5%: $$PV = \frac{20.0}{(1.08)} + \frac{21.0}{(1.08)^2} + \frac{22.05}{(1.08)^3} + \ldots = \frac{20}{0.08 - 0.05}$$

$$= \$667$$

g=6%: $$PV = \frac{20.0}{(1.08)} + \frac{21.2}{(1.08)^2} + \frac{22.47}{(1.08)^3} + \ldots = \frac{20}{0.08 - 0.06}$$

$$= \$1{,}000$$

Example

You just moved to Boston and, after seeing the affordable prices, decide to buy a home. If you borrow $800,000, what is your monthly mortgage payment? The interest rate on a 30-year fixed-rate mortgage is 5.7% (or 0.475% monthly, 5.7% / 12)

- **Annuity**

$$PV = 800{,}000 = C \times \underbrace{\left[\frac{1}{0.00475} - \frac{1}{0.00475\,(1.00475)^{360}} \right]}_{172.295}$$

C = 800,000 / 172.295 = $4,643.20

Complication 1

Inflation

How does inflation affect DCF analysis?

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5} + \ldots$$

Discounting rule

Treat inflation consistently: Discount real cashflows at the real interest rate and nominal cashflows at the nominal interest rate.

Complication 1, cont.

Terminology

Cashflows

Nominal = actual cashflows

Real = cashflows expressed in today's purchasing power

$$\textbf{real } CF_t = \textbf{nominal } CF_t \,/\, (1 + \textbf{inflation rate})^t$$

Discount rates

Nominal = actual interest rate

Real = interest rates adjusted for inflation

1 + real int. rate = (1 + nominal int. rate) / (1 + inflation rate)

Approximation: real int. rate ≈ nominal int. rate – inflation rate

Example

This year you earned $100,000. You expect your earnings to grow about **2% annually, in real terms,** for the remaining 20 years of your career. Interest rates are currently 5% and inflation is 2%. What is the present value of your income?

Real interest rate = 1.05 / 1.02 – 1 = 2.94%

Real cashflows

Year	1	2	…	20
Cashflow	102,000	104,040	…	148,595
÷	1.0294	1.0294^2	…	1.0294^{20}
PV	99,086	98,180	…	83,219

Present value = $1,818,674

Complication 2

Compounding frequency

On many investments or loans, interest is credited or charged more often than once a year.

Examples
Bank accounts – daily
Mortgages and leases – monthly
Bonds – semiannually

Implication
Effective annual rate (EAR) can be much different than the stated annual percentage rate (APR)

Example

Car loan

'Finance charge on the unpaid balance, *computed daily*, at the rate of 6.75% per year.'

If you borrow $10,000 to be repaid in one year, how much would you owe in a year?

Daily interest rate = 6.75 / 365 = 0.0185%

Day 1: balance = $10{,}000.00 \times 1.000185 = 10{,}001.85$

Day 2: balance = $10{,}001.85 \times 1.000185 = 10{,}003.70$

$\vdots$

Day 365: balance = $10{,}000.00 \times (1.000185)^{365} = 10{,}698.50$

EAR = 6.985%

Complication 2, cont.

Effective annual rate

$$\mathbf{EAR = (1 + APR / k)^k - 1}$$

APR = quoted annual percentage rate
k = number of compounding intervals each year

What happens as k gets big?
In the limit as $k \to \infty$, interest is 'continuously compounded'

$$\mathbf{EAR = e^{APR} - 1}$$

'e' is the base of the natural logarithm $\approx$ 2.7182818

Complication 2, cont.

Discounting rule

In applications, interest is normally compounded at the same frequency as payments.

If so, just divide the APR by number of compounding intervals.

Bonds
Make semiannual payments, interest compounded semiannually
Discount semiannual cashflows by APR / 2

Mortgages
Make monthly payments, interest compounded monthly
Discount monthly cashflows by APR / 12

Complication 3

Currencies

How do we discount cashflows in foreign currencies?

$$PV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5} + \ldots$$

Discounting rule

Discount each currency at its own interest rate: discount $'s at the U.S. interest rate, £'s at the U.K. interest rate, ….

This gives PV of each cashflow stream in its own currency.

Convert to domestic currency at the current exchange rate.

Currencies, cont.

Logic

You have \$1 now. How many pounds can you convert this to in one year? The current exchange rate is 1.6 \$/£ and the U.K. interest rate is 5%.

Today: \$1 = £0.625

One year: £0.625 $\times$ 1.05 = £0.6563

Implication: \$1 today is worth 0.6563 pounds in one year.

The discounting rule simply reverses this procedure. It starts with pounds in one year, then converts to \$ today.

Example

Your firm just signed a contract to deliver 2,000 batteries in each of the next 2 years to a customer in Japan, at a per unit price of ¥800. It also signed a contract to deliver 1,500 in each of the next 2 years to a customer in Britain, at a per unit price of £6.2. Payment is certain and occurs at the end of the year.

The British interest rate is $r_{£} = 5\%$ and the Japanese interest rate is $r_{¥} = 3.5\%$. The exchange rates are $s_{¥/\$} = 118$ and $s_{\$/£} = 1.6$.

What is the value of each contract?

Example

Japan

- CF_t = 2,000 × 800 = ¥1,600,000
- PV contract = $\frac{1{,}600{,}000}{1.035} + \frac{1{,}600{,}000}{1.035^2}$ = ¥3,039,511
- PV contract = 3,039,511 × (1 / $118_{¥/\$}$) = $25,759

Britain

- CF_t = 1,500 × 6.2 = £9,300
- PV contract = $\frac{9{,}300}{1.05} + \frac{9{,}300}{1.05^2}$ = £17,293
- PV contract = 17,293 × $1.6_{\$/£}$ = $27,668