ANDREW LO: Now, also, before I begin today's lecture, I want to comment a bit about what's going on in the news, because last time, on Monday, we said-- or I said-- that the Fed was going to cut rates.

[LAUGHTER]

And in fact, if you looked at the data on Monday and you looked at things like the Fed fund's future and other financial contracts, the market had priced in the fact that the Fed was going to cut at least 25 basis points, and actually a reasonable probability that it was going to cut 50. And of course, they did neither. They actually held rates steady.

But they did do something. What did they do? Anybody know? Yeah.

STUDENT: Extended [INAUDIBLE]

ANDREW LO: How large a loan? $85 billion, which, even among friends, is a lot of money.

[LAUGHTER]

Now, this is yet again an extraordinary and unprecedented measure. We know that the Fed did backstop Bear Stearns. But the Fed didn't spend any direct money on Bear Stearns. They basically got JP Morgan to buy Bear Stearns and negotiated the deal.

In this instance, the Fed is lending money to AIG, lending $85 billion. And AIG isn't even a bank. So what do you think is going on? Does that make sense? What does that tell you about what's going on in markets?

The fact that everybody thought the Fed was gonna cut rates, and they didn't-- that shows a certain kind of restraint. In fact, I think it was in this class that somebody mentioned, well, rates are already down at 2%. How much more can they cut?

I mean, if they cut 50 basis points, that leaves them very little flexibility. And also, if you think that the reason we are in this crisis is because borrowing has been so low for so long, that people have been going out making all these bad loans when they shouldn't be doing that to begin with, cutting rates is not going to really help that situation but can only encourage it. Nevertheless, there was a crisis.

Certainly over the weekend, we had some very bad news. Lehman Brothers went under, and the Fed did what? Nothing. So if the Fed did nothing for Lehman, yet they extended an $85 billion loan for AIG, something's got to be different. Right? I mean, I guess you could see whether or not Ben Bernanke has a brother-in-law working at AIG, but I don't think that's it. Yeah.

STUDENT: [? But today it was ?] reported that Barclays is actually going to go ahead and buy [? them. ?] So what changed--

ANDREW LO: Well, the announcement is that Barclays is buying some of the US operations of Lehman Brothers. They are cherry-picking the operations that they want. What Lehman tried to do over the weekend was broker a deal where Barclays would buy all of them, assume all their obligations, and allow them to keep on going as a going business concern.

Barclays couldn't do that, because they couldn't get shareholder approval quickly enough, and also ostensibly because the Fed would not backstop any losses that Lehman had hidden in its books. And in a matter of 48 hours, it's, kind of, hard to figure out all the buried bodies in an organization as complex and as large as Lehman.

But Barclays is going ahead and purchasing those units that they like, and there are many units at Lehman Brothers that are extraordinarily profitable, very good businesses with excellent people. So Barclays is going ahead with those. And by the way, there are all sorts of other sharks that are swimming around Lehman, cherry-picking various different groups.

This is part of the problem with these this kind of financial distress. We're going to actually get to this at about lecture 18. We're going to talk about financial distress, and I'm going to bring you back to Lehman Brothers and ask you to think about the problems that this company faces.

Because think about it. Now that it's been announced that Lehman is liquidating-- well, let me put it this way. Suppose you were working at Lehman Brothers, suppose that you've been there 15 years, and suppose that you were running one of the most successful proprietary trading groups at Lehman Brothers. And now, this news comes up, and it's a surprise to you.

What is your first reaction? What are you going to do? Yeah?

STUDENT: [INAUDIBLE]

ANDREW LO: Right. That's certainly one thing. You're going to take a look at what your positions are. And then, after you establish that you're OK in terms of your trading positions, what's the next thing you're going to do? What are you gonna start thinking about? Yeah?

STUDENT: Start bringing your resume?

ANDREW LO: Exactly. You're going to start looking around. So you're going to talk to lots of other people about, maybe, moving your entire group of 15 people that you've hand-picked and developed over the last 15 years. And you're gonna start talking all sorts of other counter-parties to move your entire group.

And now, Barclays decides to buy Lehman, the operations that you're a part of. But there's no slavery in the United States, at least not since the 1800s, which means that if you want to walk, you can. So if Lehman buys-- if Barclays buys Lehman and buys the group that you're in, and you're one of the most profitable parts of that, you don't have to stay.

So in addition to paying for Lehman, Barclays is also gonna have to talk to you and get you to stay, which means that they're going to have to pay you an extra bonus and all of your people bonuses to stay. So now, the price of having to keep Lehman together has just gone up dramatically, because you've got to keep all of the talent, and it's very hard to do that.

So the fact that Lehman is in trouble has caused all sorts of problems and will create additional amounts of frictions and payments that otherwise wouldn't have had to be. So I want you to keep that in the back of your minds-- the costs of financial distress. We're going to come back to that in, about, 10 lectures. OK so-- yeah, question?

STUDENT: I heard a [? quote ?] with regards to the Fed action, that the Fed decided that the problem is not the cost of money but the supply of money. So they're going to infuse capital into the market. Is that just referring to the AIG 85 billion, or is there some other way that they're infusing capital?

ANDREW LO: No. Well, that's certainly one way. But the other way is that they are allowing the other banks to borrow from them at a lower rate. So the discount window that typically banks go to borrow from the Fed-- they're making more money available in that way. And other central banks are doing the same thing-- injecting money into the system in order to calm the fears of individuals.

STUDENT: Does that lower the effect of [? interest rate? ?]

ANDREW LO: Well, we're going to see that in a minute. We're going to actually-- one of the things I want to talk about today is, exactly what do we see from market prices? Now, on Monday, I claimed that market prices was telling us, there's going to be a Fed cut.

Clearly, it was wrong. Now, that's a very good lesson, because what this is telling us is that market prices have information, but as I told you last time, they're not a crystal ball. They're not perfect, and so they can be wrong.

Apparently-- and this is now very speculative. Apparently, the Fed decided, as you pointed out, that it's not the availability of-- or rather the cost of funds. That's not what's important, but rather the availability.

In other words, they're worried about a credit crisis, a crisis of liquidity. And AIG is a very important player in that respect-- apparently, much more so than Lehman. Because Fed didn't do anything to try to keep Lehman from going under, but an $85 billion loan was what they decided was appropriate for AIG.

The reason for that-- the ostensible reason. Who knows what the real reasons may be? But the reasons that we think this happened is that AIG provides enormous amounts of insurance to a variety of other players in the credit markets. And if they go under, if they decide that they can't make good on those insurance claims, what happens is that those investors that are holding the paper that is backed by subprime assets and that are insured by AIG-- once the insurance disappears, they are obligated, a number of them, to sell those pieces of paper.

If you're a pension fund, you are obligated to hold only investment grade assets. If it turns out that for any reason those assets become lower than investment grade-- and we're going to talk about this at 4 o'clock today at that proseminar. If it falls below investment grade, by law, you are obligated to get rid of those assets.

Now, what do you think would happen to the market if everybody all at once decided to get rid of those assets?

STUDENT: [INAUDIBLE]

ANDREW LO: Right. And then, there'd be a mass panic. Right.

STUDENT: I just have-- there's something simple that I don't understand. How can the interest rate go below the inflation rate?

ANDREW LO: Well, it's not supposed to for any extended period of time. But for any short period of time, it certainly can. And what it says is that the real rate is negative or that the economy is contracting.

STUDENT: So like right now, $1 a year from now is more valuable than it [? would be ?] today. Right?

ANDREW LO: Well, if you take inflation into account, yes, in real terms, not in nominal terms. You can never have a negative nominal interest rate. Right? Unless, you know, somebody is burning dollar bills.

But let me let me hold off on that, because I want to actually-- that brings us back to the end of last lecture. What I want to do today is, I want to talk about information specifically contained in interest rates. And we're going to actually take a look at what the short-term interest rate is.

I think you'll be, kind of, surprised to see what the three-month T-bill rate is, as of today. Anybody know what it is? You've seen it? 2%? No. No. It's lower.

STUDENT: [INAUDIBLE]

ANDREW LO: Let me-- we'll see in just a minute. Let me start today's lecture by going back to where we left off last time. Last time, we talked about the pricing of pure discount bonds, bonds that pay only principal at the end and no intermediate coupon payments.

And we saw that the price today is simply equal to the face value or principal at the end of the maturity date, and then discounted back using the interest rate. And I pointed out at the end of last lecture that the interest rate can differ, depending on the horizon. So a one-year interest rate is not the same as a five-year interest rate, because the market has different expectations about how the economy will do and what the appropriate borrowing rate or the time rate of preference might be.

So in fact, for every horizon-- one year, two years, three years, five years-- we have a different interest rate. It doesn't have to be different, but in general, it does tend to be different. How do we find out what these interest rates are? Yeah.

STUDENT: The market.

ANDREW LO: Exactly. The market. The way to do it is not to think about interest rates at all, but rather to auction off pieces of paper that pay $1,000 in a year, $1,000 in two years, $1,000 in three years, and so on. And we auction off each of these pieces of paper and see what the prices we fetch are, for those pieces of paper.

Once we have the price and once we know the face value of $1,000, we can back out the interest rate. We can solve for the interest rate, r. So that's how we get the rates.

And what I want to do today is to explicate what those rates really mean. I want to show you how to read the entrails and see that these rates contain enormous amounts of information about the future. Not all of that information is good.

So sometimes, it's misleading and incorrect, but it's always useful in one form or another. Now, to do that, I want to develop a little bit of new notation and get you to think, yet again, differently about the evolution of interest rates over time. I'm going to define what's called a spot rate as the rate of interest between today and some other point in time.

And I'm going to talk about future spot rates as the interest rates between some future date, and then another date even beyond that. So to be explicit, I wanted to find new notation called capital R. Uppercase R is meant to convey a one-year spot rate of interest at a particular point in time, t. OK?

So capital R1 denotes the spot rate of interest between today and next year. Capital R3 denotes the spot rate of interest between years 2 and 3. And capital Rt denotes the one year spot rate between dates t minus 1 and t. OK? So these capital R's are always one-year rates, unlike the little r's, which can denote multi-year rates, depending on the application.

Now, there's a reason I wanted to find these big R's. It turns out that if I have a pure discount bond that pays off at year t, then I can use the one-year spot rates to compute today's price. Right? The one-year spot rate, when you accumulate that, when you multiply them together, will give you the accumulated interest over this entire t-year period.

So if I want to discount face value, F, and bring it back to year zero, I can just assume that there exists one rate. Or I can say, you know what? If there are multiple rates that differ year by year, I can use those individual rates.

So I get that first equation. Now, we don't observe them. So this is a pure fiction, in terms of what I'm writing down. It's a theory.

So I'm not telling you that we know what those big R's are. But I know that they exist, and whatever they are, this is what the price of the bond ought to be today. Any questions about that? OK.

Now, what I do observe is the price and F. Those things, I get from the marketplace and the contract for these bonds. Therefore, it turns out that as a very simple identity, this expression, this little r-- which-- I'm adding some more complicated notation to indicate when I begin and when I end in terms of my horizon-- I can simply define this little r as being equal to the geometric average of these big R's.

It's really just terminology at this point. Right? I'm simply saying that in reality, we have one-year interest rates that may change over time. And I know that the price of the bond today is equal to the future course of one-year interest rates as discounts over that period.

When I use those as discount rates, I bring back the value F, and I get today's price. I can just as well write that chain of one-year interest rates as a single number, raised to the t-th power. I can always do that.

You can think of this little r as an average, a geometric average, of the big R's. Right? So the strict definition is going to be-- little r is going to be the t-th root of the product, and then minus 1. That's what the little r is. All right?

You take that product, and you raise that to the 1 over t-th power or take the t-th root, and then subtract 1 from that-- that's what my little r is. Now, why am I going through all of this? It's because I want to show you that from a theoretical perspective, the little r, which we can observe, contains information about the future course of interest rates.

Within the little r are all the big R's-- at least today's expectations of what those big R's are going to be. So it turns out that if we look into the little r's, we can actually develop insight about what's going to be happening next year, five years from now, 30 years from now. Now, let me give you an example, just to make sure that we understand the mechanism by which these little r's and big R's are determined.

Here's a set of prices of strips. These are treasury securities, issued by the US government. And then, a third party buys them, takes the coupons, creates separate securities, and sells those separate securities, each one of which is one of these coupons.

So from our perspective, they look like pure discount bonds. There's no intermediate coupon payments for each one of these strips, and the maturity is three months, six months, one year, two years, up to 30 years. OK? And those are the prices.

So a three-month strip is currently priced-- as of August the 1st, 2001, it was priced at a little bit less than the dollar. So how do we figure out what the little r is, associated with those various prices? Well, let's take an example.

The five-year strip is priced at, about, $0.80 to the dollar. OK? So the price is 0.797, and that's equal to $1 paid five years later. So therefore, it's $1 discounted back five years.

So I'm gonna use my little r, and the zero comma five indicates that it's today's spot rate for borrowing over a five-year horizon. And it turns out that when I solve for that, I get a number that's 4.64%. That's the rate of return, the cost of capital, the yield of that five period horizon. Any questions about this? Yeah.

STUDENT: How do we do it, maybe, if it was one that was less than a year's horizon?

ANDREW LO: So if it's less than a year's horizon, then you basically have to go the other way, in terms of the power. Right?

STUDENT: [? 1 over 1/2. ?]

ANDREW LO: Yeah. Exactly. That's right. Yeah, that's it. It's just a shorter time horizon than a year.

Now, suppose that we observe a bunch of these as we do with the strips. So in other words, you've got a five-year rate, you've got a 10-year rate, you've got a two-year rate, and so on. What does that tell us about the future?

Well, let's write down the big R's. Even though we don't see them, we know that somehow, implicitly, they're there. So what are the relationships between the little r's and the big R's?

Well, you'll see something really neat emerge out of this. We'll start with a one-year strip. With a one-year strip, the little r and the big R are the same, because it's only one year.

So there's a one-year big R, a one-year little r, and when you work out the math, they're actually equal to each other. But now, when you go with two years, three years, and t years, it gets a little bit more complicated. Take a look at what happens if you take the price of the one-year, and you divide that into the price of the two-year.

These two securities-- the one-year and the two-year-- they have the same F, the same face value. They pay $1,000 at maturity. But one of them goes for one year, and one of them goes for two years.

What happens when you take P 0,1, and you divide that by P 0,2? By the way, both of those prices exist today. Right? For example, if you take a look at the strips, the price of P 0,1 is 0.967. The price of P 0,2 is 0.927. If I take the price of 1 divided by the price of 2, what do I get? Yeah.

STUDENT: [INAUDIBLE]

ANDREW LO: Exactly. I get 1 plus R2. And so if I subtract 1 from that, I get R2. So let's just go back. I don't have a calculator with me, but I'm sure all of you do.

Somebody do that division for me, will you? Can you take 0.967 and divide that by 0.927? What do you get? 0.967 divided by 0.927. What's that?

STUDENT: 1.04.

ANDREW LO: 1.04-- and then, subtract 1 from that. 4%. Actually, can you give me a few more digits of accuracy?

STUDENT: [INAUDIBLE]

ANDREW LO: What's that?

STUDENT: 4.314.

ANDREW LO: 4.314. So it turns out that in year 0, where we have all of these prices, we actually have a forecast for what big R2 is. Big R2 in this case is the borrowing cost between year 1 and year 2.

But we're sitting at year 0. So implicit in the price of a two-year bond and a one-year bond-- implicit in that is a forecast of what the price is going to be, what the yield is going to be, or what the borrowing costs is going to be between years 1 and 2. In this case, 4.3% or so.

And that's a really important observation. If you plot these little r's on a graph as a function of time, you actually get a sense of where the future big R's are going to lie. This plot, a plot of the r's as a function of time, is known as the term structure of interest rates or the yield curve, and it gives you a sense of where future interest rates are going to go.

If the curve is upward-sloping, it says that as you go out into longer maturities, your average yield, the geometric average of all the big R's-- it's getting bigger as time grows, as the time horizon grows. If it's downward-sloping, it suggests that future interest rates, future big R's, are declining.

I want to show you what the yield curve looks like today. Now, it turns out that we don't have a yield curve of strips as readily available as a yield curve that includes coupon payments. So I'm going to come back to the distinction a little bit later on.

We haven't talked about coupon bonds yet, but I just want to show you what the yield curve is. So I'm on the Bloomberg website. This is publicly available, so I don't have a particular license for it. It's the public version.

And if you click on market data, and then click on rates and bonds, you're going to get this page right here. So these are the different US treasury securities, the different horizons. These are the coupons.

For less than a year, there are no coupon payments, so these are pure discount bonds. And there's the graph. That's it. That graph-- the green line is showing you the future course of interest rates.

It's extremely low today. The scale is on the left-hand access. And by the way, these are in percent.

So where we are today, for a three-month rate, is close to zero. It's actually three basis points, three basis points for a three-month T-bill. What does that tell you? What's the relationship between price and the little r? Yeah?

STUDENT: [INAUDIBLE]

ANDREW LO: Well, that's right. But how does that yield get so low? Yes?

STUDENT: Because the price is extremely high.

ANDREW LO: The price is extremely high. That's right. Price is equal to the three-month pay-out divided by 1 plus little r. If little r ends up being really, really tiny, it's only because the price is really high. Why would the price be high?

STUDENT: Because US treasuries are the safe thing to own right now.

ANDREW LO: At least, that's what many people think, exactly. There is a really strong flight to liquidity going on in markets, as of today. And how do you know that it's as of today?

Well, take a look at the difference between the green line and the orange line. The orange line was what it was yesterday. You see, there's a difference. There's a noticeable difference on the short end that means a lot of people are out there buying treasury bills now, probably as we speak. Maybe you ought to go and buy some treasury bills. People are scared.

And they're scared because of all the things that are going on in the news, and this is exactly what the Fed is trying to stave off. So you're absolutely right. The Fed is not worried about the cost of borrowing. They're worried about whether or not there's money out there to be able to calm the fears of market participants. Yeah?

STUDENT: [INAUDIBLE] two days ago to today, [INAUDIBLE] probably the Fed didn't cut the interest rate once [INAUDIBLE]. They [INAUDIBLE].

ANDREW LO: That's right. They might have to, so that expectation actually is built into these prices. The market recognizes that, and they're worried.

But think about that. If the Fed has said that they're going to be cutting rates-- possibly cutting rates in the future, and yet, the rate stays relatively high going forward, and rates go down today, what that's telling you is that the market is being driven by a panic reaction. Now, rates are going to go up.

So the fact that you see the market determining a yield curve that's upward-sloping-- that's telling you that people expect that rates are going to go up, that rates have to go up, for one of two reasons. And in fact, you can take a look at the steepness of the yield curve as telling you what the market's expectations are for how quickly rates are going to go up and where they're going to go up. So you have to look at the x-axis a little bit differently.

These are denominated in years, so this is three months, six months, one year, two, three, four, five, up to 10-- then, 15, 20, and 30. So these are long-term rates. And you can see that the yield curve really goes up sharply after the first three months. There's a big increase in the slope, and then it becomes a little bit more gradual.

That's a sign of a short-term flight to quality or flight to liquidity. But the market expects, over time as things calm down, that interest rates will go up, for one of two reasons. Either there are inflationary pressures and that will drive rates up, or there are going to be some economic consequences of what's happening today, and that will ultimately cause rates to go up. Yeah.

STUDENT: What's happening to the interest rates outside of this case? Because I'm from Argentina, and when we have crises, like [? internal ?] interest rates go up, because the probability of default increases. And now, I see here, it's the other way around, because they are not considering it will be--

ANDREW LO: That's right. It depends on the nature of the crisis. So in certain countries where there is a financial crisis, the typical reaction of monetary authorities is to flood the market with cash, because that's their reaction to a liquidity crunch. They want to reduce the prospect of having a kind of run on the banks, so they'll flood the market with their currency.

When you do that, you encourage inflation, and that's why interest rates go up in those kinds of economies. The US, for better or for worse, has shown a certain degree of monetary restraint over the years in that while they do certainly cut interest rates-- and Alan Greenspan was very active in this respect over the last 10 or 15 years-- the fact is that there has been more measured control of monetary policy in the United States. What that means is that this is a symptom more of a short-term cash crunch.

People are just putting money in treasury bills for the short term without any expectation that the Fed is going to dramatically increase the money supply. If they did that, you would then see interest rates rise, because inflation would be much more of a problem. OK. Yes?

STUDENT: [INAUDIBLE] European Central Bank?

ANDREW LO: Yeah.

STUDENT: --normally raises the interest rates [INAUDIBLE].

ANDREW LO: That's right. They do raise it, and that's one of the reasons why the Fed did not cut it. It's because they are concerned with inflation. And so if they ended up cutting interest rates, while that might stave off certain credit crunches, that would actually encourage inflation.

STUDENT: So inflation does encourage higher interest rates, but what is the [? advantage ?] of it?

ANDREW LO: Inflation causes-- so I see the confusion, and let me make a distinction. There are two different interest rates that are going on. There is the market rate of interest, which is what this is. And then, there is the Fed's stated federal funds rate, which is what it charges its other member banks for borrowing.

The Fed is able to control what it charges to other banks. The Fed cannot control these rates. So the interest rates that you're thinking about, that-- for example, when the ECB raises rates, they do so, so as to discourage lots of borrowing and lending, and reduce the amount of money that's in circulation. And that decreases business activity, which then reduces the pressure on inflation. OK?

So that's what they do in response, but they don't control the interest rate determined by the market for treasury bills. And in these interest rates, these 30-year rates, as opposed to overnight borrowing rates and Fed funds rates, these rates give you a sense of what the market is expecting over time. [? Megan, ?] do you have a question?

STUDENT: [INAUDIBLE] As it cuts the interest rates, [INAUDIBLE] had an impact [INAUDIBLE] the actual banks that [? spread ?] over time [INAUDIBLE] to stave off that credit crunch--

ANDREW LO: Well, that's right. I think that when you think about the instruments that the Fed has for managing monetary policy, credit, and liquidity, it's actually pretty minimal. I mean, they have one variable. You know?

Imagine flying a mirror plane and you get one control. You pick the control. You want to be able to control the wheels, or the ailerons, or the--? It's very, very hard to try to manage the economy with one variable.

Now, the Fed has other policy instruments that are a little bit more complex, like the discount window, like moral suasion. The New York Fed can go to these banks and say, what are you guys doing? Are you nuts?

But what's going on now is that because the crisis has reached such an extraordinary level, they're not worrying about interest rates. They're actually trying to figure out how to stave off some kind of mass panic. And so getting directly involved with AIG, getting in discussions with Bear Stearns and JP Morgan-- they really have no choice.

And it's a signal of, sort of, how desperate times are. One last question?

STUDENT: [INAUDIBLE] My understanding is first that the Fed is not a federal organization. It's made up of a number of banks, so that it's not a government entity, but the government has appointed representatives to it. And at the same time, where does the Fed get all their money from?

ANDREW LO: Well, this is more of a question that you probably should be asking your macro instructor. I'm happy to answer it, but the macro folks may disagree with what I'm about to tell you. The Fed is a government organization. It's separate from the government in the sense that it's not a political organization, but it does have the full backing of the government, and it has powers granted to it as part of the various legal proposals that were developed to create the Federal Reserve system.

Where does the Fed get its money from? It gets its money from the treasury. So the Fed can actually engage in what are called open market operations and can actually contract or expand money supply, based upon what the treasury will allow it to do or will work with it to do. And the Fed controls the borrowing rate among all of the member banks, and actually, all of the major banks are members.

So it's not like you can start up your bank tomorrow. In order to start a bank and deal with the public, you need a bank charter, and the bank charter is issued by the government. And once you're part of that network, you are part of the Federal Reserve system.

STUDENT: Aren't they losing money from, say, the fallout of these [? banks? ?] Aren't they, in essence, losing also themselves?

ANDREW LO: Well, they're not trying to make money. That's not their objective. And if they're losing money, ultimately, it's not them that is losing money. It is-- who?

STUDENT: [INAUDIBLE]

ANDREW LO: Yeah. We're losing money. It's government-sponsored. So that's one of the reasons why, I think, the Fed has been so concerned about bailing out Lehman Brothers.

And even the bailout of Bear Stearns, which did not necessarily cost them anything-- the fact that they were willing to provide this backstop guarantee in order to make the deal happen-- that implicit insurance is a cost that we ultimately end up paying. They got a huge amount of heat for that, and that's one of the reasons why they decided to back off from the Lehman Brothers deal. It's because they would have gotten huge, huge backlash from that kind of an event.

Now, AIG-- the fact that they went and did something there tells you something about how important AIG is, or what repercussions might have come about if they had let AIG fail. So that says more, not about the Fed, but more about the situation with AIG and the specific financial transactions that they were engaged in. OK. Let me continue on. And sorry, I want to hold off questions for a little bit longer, but I do want to cover some additional material.

So this is the expression that we just described for getting a sense of future interest rates. And we saw, given today's yield curve, that there is some sense that interest rates are going to rise. But it turns out that the yield curve contains all sorts of information, not just about one-year rates, but in fact, about multi-year rates.

This is a clear example. This is one example. And it turns out that there's another example that makes this a little bit clearer, which is future rates and forward rates. These are all very confusing terminology, unless you sit down and read through it carefully, so I would encourage you all to do that after this lecture.

There's a lot of notation in this lecture, but not a lot of conceptual challenges. Because all the conceptual challenges, we derived when we talked about net present value rules. So most of this is just lots of notation and terminology. So let me describe the terminology here.

At date zero, if we focus on the price of a bond, that matures at time t minus 1. And at date zero, if we focus on the price of a bond that matures at day t, and we take the ratio of those two, then it turns out, we're getting an implicit forecast of the future one-year spot rate between t minus 1 and t. Right? That's just what we did with r2.

So this is true in general, and there's a name for this forecast. It's called, today's forward rate between dates t minus 1 and t. It is a forecast of the future spot rate between dates t minus 1 and t. OK?

We don't know what that spot rate is going to be, in general. We don't know what future interest rates are going to be. It's uncertain. But today, implicit in today's prices is a forecast of that unknown future, and we're going to call that forecast the forward rate.

That is really meant to convey that it is a rate that we observe today, and it is meant to capture the market's best guess about what the future spot rate will be. OK? So this is, I know, a little bit confusing. And just to give you a summary of all the notation and terminology we've defined today-- we have a spot rate.

A spot rate is the rate that you have to pay or that you will earn on the spot, for a period of time. So you've got the two-year spot rate today. You've also got future spot rates, which you don't know and don't observe. You also have a forward rate, which you do observe today.

And the forward rate is a rate that applies over some period in the future. And it's today's best guess of what that future rate will be. Now, we can see the implicit forecasts that are in the yield curve. The one-year spot rate today also happens to be equal to the one-year forward rate.

However, if you take a look at the two-year spot rate and compare that with the one-year spot rate, you can compute the one-year forward rate for borrowing between years 1 and 2. And similarly, if you compare the four-year spot rate with the three-year spot rate, you will be able to figure out what the forward rate is for borrowing one year between 3 and 4. OK?

Now, you might think this is complicated. Believe me. It gets even more complicated when you think about multi-year forward rates. So suppose I asked you, what is the two-year borrowing rate, three years from now?

Then, what you would do is to take a five-year bond and compare that to a three-year bond, and that would give you the two-year forward rate today, starting in year 3. OK? Lots of different rates. This is, again, why I've told you, every time you have a problem like this, draw a timeline. Otherwise, you're going to get hopelessly confused.

Now, in general, you can define forward interest rates between any two points in time, between time t1 and t2. And so the typical forward transaction is one where today, we agree to do a deal that starts at some point t1 in the future and concludes at some point t2 in the future. And that's known as a forward transaction. It's a transaction that we agree upon today, to engage in sometime in the future.

Now, I want to work through an example, because this is a bit confusing. So let me show you how this might work, and why the whole idea of forward rates and future spot rates is so important. A practical example is that you are the chief financial officer of a multinational company based in the US, and you're going to get $10 million a year from now, from operations overseas.

And it's going to come back in the form of dollars, but it's not going to come back today. It's going to come back exactly one year from today. Now, you've got to pay dividends two years from today. So you're going to use that money that's going to come in a year from now.

And then, at the end of year 2, you're going to pay it out. And so you don't want to take that money next year and fool around with it. You don't know what interest rates are going to be.

But what you'd like to be able to do is, today, lock in a rate of return between years 1 and 2. Because you know that you're going to need to get that money invested in year 1, and you'd like to be able to pay it out in year 2. And you want to do that all today.

So how do you do that? Well, you go to the financial markets, and you look at the yield curve. And you see what the one-year rate is, and what the two-year rate is.

And what you get from looking at the newspaper is, the one-year rate is 5%, and the two-year rate is 7%. Question-- is 7% a spot rate, forward rate, or future spot rate?

STUDENT: [INAUDIBLE]

ANDREW LO: It's a spot rate of what?

STUDENT: [INAUDIBLE]

ANDREW LO: Exactly. It is today's spot rate between now and two years from now. It's a two-year spot rate. Right. What you care about, though, for the example I just gave you, is what?

STUDENT: [INAUDIBLE]

ANDREW LO: Exactly. You care about the one-year spot rate in one year, the future one-year spot rate, which-- you don't know what it's going to be. That's uncertain.

But you do have the-- what rate do you have today? The forward rate, right. You have a forward rate. Because you've got the two-year spot rate, and you've got the one-year spot rate. So when you compare the two, implicitly in those two rates is the forecast of the future one-year spot rate or today's forward rate between years 1 and 2. All right. Now, let's get to brass tacks. How do you go about locking in the rates between years 1 and 2? Well, here's a really cool transaction that you can do. Today, borrow $9.524 million for a year. How do you know you can do that?

STUDENT: [INAUDIBLE]

ANDREW LO: Exactly. You've got the one-year interest rate at 5%. So if that's really a market rate, that means that you should be able to borrow at that rate. OK? So when you're borrowing money, what are you doing? You're-- are you buying a bond?

You're selling a bond. You're issuing a bond. Right. OK. So you borrowed $9.52 million dollars today. Now, in a minute, I'll explain to you why that number is so weird. Then, after you get the money today, I'm going to ask you to put it into the two-year bond.

So you got $9.52 million in cash, and you put it into a two-year bond. So let's take a look at what you've done with that transaction. The outcome looks like this. In year zero, you've borrowed 9.52, and then you've taken the proceeds and you've bought a bond at 9.52.

So in fact, your net expenditures is nothing. You borrowed money, you took that money, and you bought something else. You've loaned it out. You borrowed money for one year, and you've loaned it out for two years.

That's what you've done. So today, you actually have zero, in terms of your assets and liabilities. Now, let's see what happens next year. In one year's time, that 9.52 to magically turns into 10, but it's a negative 10, meaning you borrowed 9.52.

You've got to pay back 9.52 with interest. How much interest? 5%. That's the one-year rate. So now, you actually have to pay back $10 million. Well, it just so happens, you have $10 million.

How? From the money that's coming in from your subsidiary, that repatriation amount of money. So you take that $10 million, you pay it back, and you're done with that part of your portfolio. What do you have left?

What you have left is a bond that will pay you money in the year after that, between years 1 and 2. And there you go. You get paid $10.9 million. You've done all of this transaction today. You've locked in the rates today. OK? Yeah?

STUDENT: You locked in the one-year spot rate [INAUDIBLE]?

ANDREW LO: That's right. Well, you're locking in the forward rate, which is the forecast-- right. It's what the market expects the future one-year spot rate will be.

Now, that's a good point that you bring up, which is, let's say that in year 1, it turns out that at that point in time, the one-year spot rate is 7%. Are you happy or are you sad? Some people say said, some people say happy.

If the one-year spot rate one year from now is 7%, and you've done this deal already--

STUDENT: You're happy.

ANDREW LO: You're happy. That's right. Because, what are you getting on your portfolio? 9%.

Now, wait a minute. How do you get 9%? I thought that I told you the two-year rate was 7%. I'm purposely confusing you, so I'm hoping it works. And then, I'm going to try to un-confuse you, and I hope that works, too.

5% is the one-year spot rate. 7% is the two-year spot rate. Yeah.

STUDENT: [INAUDIBLE]

ANDREW LO: Right. That's right. The rate between years 1 and 2 is around 9%. And the reason it's got to be that way is, the 7% that I told you-- that's a two-year rate. Right? That's the average of two years.

But you know what the rate is the first year. The first year rate is 5%. So if something averages to 7% over 2 years, but the first year is 5, the second year has got to be greater than 7. Otherwise, the average can't be 7.

In fact, the second year rate-- the one year rate between years 1 and 2 is around 9%. And so that's why if when you arrive at the end of year 1, ready to borrow for one more year, and you've already locked in a 9% rate, you are pretty happy that the rate at that time is 7. However, if I told you that the rate at that time was 15, you'll be kicking yourself, because you locked in a 9% rate, and yet it's 15%.

So there's room for regret as well as celebration, depending on what the market is going to do. But the point is that in year zero, I don't know what it's going to be. And I'm not a hedge fund manager. I'm not a trader. I don't care-- I don't want to make a bet on future interest rates. I just want to get this problem solved.

And right now, today, I can actually solve my problem of figuring out how to invest my money between years 1 and 2 by doing this very simple transaction in open markets with market-determined interest rates. And I know, as long as the interest rates are not nuts, then I'm getting a reasonable deal. Yeah?

STUDENT: This is one of the main [? things that you can do. ?] Could you please [INAUDIBLE] is it even similar in the same-- at the end of year 1, I get my 10 million, put it in [INAUDIBLE], because it's pretty much [INAUDIBLE].

ANDREW LO: Yeah. But the problem is that you don't know what that one-year T-bill rate will be, whereas right now, you know that it's 9%. Right now, you know it's 9%, and it seems like a pretty good deal to go ahead and do it. If, however, you think that interest rates are going to go up much more than the market thinks, then you may want to wait.

But now, you're becoming an interest rate speculator. You're taking a risk. And as a CFO, that's generally not your job and not your level of competency. Right? You're not there trying to forecast interest rates. Yeah?

STUDENT: [INAUDIBLE]

ANDREW LO: That's right. The 9.524 is the present value of $10 million today, at the rate of 5% interest. So the way that I did this-- and you know, this is a good illustration of what I've been telling you about finance not being a spectator sport. I suspect that all of you understand the lectures that I've given so far about present value, about time value of money, and the fact that you've got to use the right exchange rate. It all is pretty straightforward.

But putting it into practice is not so easy, at least for me. I don't find this example so transparent. You have to actually spend some time thinking about it, thinking about where the money is coming from, where the money is going, how much money you have at any point in time.

But when you work it out, it all makes sense. And so I would encourage all of you to spend some time working this out. OK, question? Yeah.

STUDENT: [INAUDIBLE]

ANDREW LO: Yes, there are ways you can engage in a forward contract, of course. So you don't have to do this. But the fact is doing this is so simple. Why not?

And if it's simple, most likely, it'll be cheap. If it's complex, that's when you're going to pay for it. Right? So I'm happy to structure a derivative product for you. Let's call it a structured product that we trade over the counter, where I offer you a forward contract, one-year borrowing, with certain terms and privileges, and so on.

And by the time we're done, I'm going to charge you a transaction fee of-- oh, I don't know, maybe 5%. Versus, you buy a two-year bond and a one-year bill, and you're done. Right? So that's the difference. It's really the ease with which you can implement the strategy. All of you right now, today-- all of you can do this. You can do this. You can actually trade in these markets. Set up a brokerage account, trade these treasury instruments, and do this yourself. In fact, you can do this online. So it's very, very simple. And-- yeah?

STUDENT: [INAUDIBLE] $10 million?

ANDREW LO: Oh, yeah.

[STUDENTS LAUGHING]

Well, that's the hard part. Yeah. There's an old Steve Martin joke that says, I'm going to show you how to make a million dollars and pay no taxes. First, get a million dollars. So yes, that's the hard part. OK.

So now, this transaction today locks you into a 9% rate between years 1 and 2. And so going back to the question that [? Anan ?] raised, should you do this or should you just wait? Well, it depends.

Do you feel lucky? Do you think you can do better than 9%? I mean, today, 9% looks wonderful, but that's not with the rate you're going to get today.

In fact, if we go back to the Bloomberg site, you can see what kind of rate you would lock in today between years 1 and 2. And I promise you, it's nowhere near 9%. And so again, you might actually say-- in today's low interest rate environment, you might say, look, I think that inflation is going to heat up a great deal over the next year, and therefore what's implicit in today's forecast of future spot rates is lower than I think it ought to be.

So I'm going to hold off. That's a bet. And so you're going to take some risks. That's all. And for those people who are good at taking risks like that, they end up doing well. For those who don't, they end up losing out on good opportunities.

There's another example that I'd like you to look at and work through on your own. It's very similar to the first one, but it just gives you practice in thinking about timelines, moving money back and forth, and trying to understand how to structure the payoffs in order to satisfy certain consumption patterns. So if you look at this example and work it through, that will give you more practice on how to deal with these transactions.

Now, if there are no more questions about pure discount bonds, I want to turn to the more general case of coupon bonds. These are bonds that pay off coupons. And really, the theory behind coupon bonds is virtually identical to that of discount bonds, in the sense that you can always look at a coupon bond as a package of discount bonds. Right?

That's, sort of, the opposite of a strip. A strip takes a coupon bond and breaks it up into what look like little discount bonds. Well, if you think about what a coupon bond is, it's really just a collection of discount bonds at different maturities. That's the way to think about it.

Here's a simple example. A three-year bond with a 5% coupon is going to look like this. It's going to pay 50, 50, and then 1,050. Now, as I mentioned, there are some coupon bonds that pay semiannually. So when they say that there's a coupon of 3%, it's 3% every six months. So you have to take that into account when you're computing the present values of these objects.

How do we do it? Exactly the same way as we do for pure discount bonds. Take the coupons, each of them, and discount them back to the present, using either the big R's or the little r's. Either way, you ought to get the same answer, because the little r's are simply the geometric averages of the big R's. OK?

However, instead of using the little r's for the different payments, coupon bonds are often quoted with a single number that is a yield. So the theoretically correct way to write the price is given up there. P0 is equal to C, all the coupons, divided by the appropriate big R's.

Or we could replace every one of those big R's with the appropriate little r. By appropriate little r, I mean, little r 0,1, little r 0,2, too little r 0,3-- each of those. But we can also calculate an average of all of those little r's and just use one variable.

And to simplify notation, I'm going to give it a completely different symbol, Y, and say, , what is that single number, Y, that will give me the price of the bond? And that Y is known as the particular bond's yield. It is the single interest rate which, if interest rates were constant throughout time, would make the present value of all the coupons and principal equal to the current price.

So if you think about a mortgage, and you ask the question, if the mortgage rate is 5%, what is the value of the loan, that's exactly this expression right here. Now, obviously, when you get a fixed rate mortgage of 5.89%, you know that the interest rate is not really going to be 5.98% forever. The interest rate changes every year.

But that 5.98% is an average of the 30-year period where you're going to be borrowing that mortgage money. So you can think of this coupon bond in exactly the same way. We quote this number Y as a yield. Sometimes, we talk about yield instead of prices.

But the way that we figure out the yield is, we take this 30-year bond that pays 5% a year, we auction it off, and we figure out what the price is. Given the price, we can find the yield. Finding the yield is not so easy in this case, because in this case, unlike just taking a simple geometric average, which is what we did to calculate the little r's from the big R's-- in this case, in order to find the Y, we actually have to solve an equation that can be highly non-linear.

In fact, it's a polynomial. It's a t-th order polynomial. And for those of you high school math team jocks, you'll remember that when you've got a t-th order polynomial, first of all, you have a lot of solutions. How many solutions do you typically have? t.

And of those solutions, how many of them are guaranteed to be real numbers? Right. There's no guarantee that any of them are real. Now, you might ask, what do you mean by real?

Well, if you're asking me, you don't need to know. [LAUGHTER] Don't-- don't. It means numbers that we encounter in reality. Let's put it that way.

It turns out that there are numbers that actually don't exist in reality. They're called complex numbers. And they are quite complex, so I won't talk about them.

But these kinds of equations-- it turns out that they're not guaranteed to even have real solutions. Now, it turns out that for bonds, where the coupon payments are all positive and the principle is all positive, it turns out in that very restrictive-- and the price is positive. It turns out in those cases, you actually do get a real number-- at least, one real number.

The problem is that in some cases, you get multiple real numbers. And then, it's very, very hard to figure out which yield is the correct one to use. The only reason I'm telling you about this is because it turns out as a matter of convention, very often, people will quote these little Y yields when they talk about coupon bonds.

But the way to think about that is to think about the price, which is the present value of the coupon payments as a present discount value of the interest that really applies between today and date t. In fact, in order to do this present value calculation, you need not just one interest rate. How many interest rates do you need? t, right?

You're at year zero, and you've got payments for every single year between 1 and t. So you need interest rates that apply between 0 and 1, 0 and 2, 0 and 3, and so on. You need t interest rates or exchange rates. Right? Or exchanges between different dates.

But now, the yield is important because it allows us to quote the pseudo rate of return of this bond in a single number. And very often, people will plot the Y's as a function of the horizon of these bonds. So when I showed you that yield curve-- let me get that back. Whoops. I just had it.

Let's take a look at this again. These treasury bonds from years 2 to years 30-- those have coupon payments, and those are the coupon rates. And they have prices, and they have yields.

So what's plotted here is not the little r. It's the Y for the coupon bonds. And so the reason that they're not the same is because the yields, the little Y's, depend on the coupon payments.

And really strictly speaking, we don't care about coupon payments when we look at time to maturity. We just want to know, what is the interest rate between 0 and 1, 0 and 2, 0 and 3, 0 and 5, and so on. But this is a reasonable proxy as long as the coupons don't look too crazy and are not too different from each other.

And you can see that the coupons are all, sort of, in the same neighborhood. Some of them are 2.3% versus 4.5%, but they're not so different. So this gives us an indication of what the strip's yield curve would look like.

Here's an example of the historical yield curve for US Treasury securities, and let me just show you a plot. They move around a lot. So these yield curves tell us something about the average interest rate across various different maturities.

So if you look at the yellow line, that's a one-year yield over time. So this is not the yield curve anymore. This is a plot of the time series of one-year yields over time.

And you can see that starting when the sample began in 1982, the one-year yield for US

Treasury bills is 12%. 12% back in 1982-- and there is a point at which one of the longer maturity instruments reaches a peak of 16% or 17%.

Remember, I told you, I was looking to get a house and get a mortgage at 18%. That was a 30-year fixed rate back in the 1980s. So borrowing rates are very, very low by these historical standards.

If borrowing rates are very low, what does that tell you about credit and about the amount of cash sloshing around in the economy? Yeah?

STUDENT: [INAUDIBLE]

ANDREW LO: Exactly-- lots and lots and lots of money available. So for those of you who are thinking about entrepreneurial ventures-- if you're thinking about raising capital, you might be depressed by what's going on in markets today. But look at the interest rate and ask yourself, gee, would I rather start a company today or back in 1982?

There's a heck of a lot more money sloshing around the system today. In fact, a few years ago, I talked to a couple of MIT undergraduates. One of them came up and asked me whether I'd be willing to advise them on some internet company they wanted to start. And you know, I said, well, you know, where are you going to get your financing?

And they said, oh, don't worry about that. We've already got it all set. You know, we've got 10 of us, and each of us is able to borrow $1,000 each on credit cards, and we each have 10 credit cards, so that's $100,000.

And I'm saying, $100,000 on credit cards? I mean, you guys are going to be paying 18% a year or something on that. And they said, yeah, you know what? That's the cheapest venture financing you'll ever find.

And they're right, because those are non-recourse loans. They don't take a piece of your company. And 18% for a venture is actually pretty attractive.

Well, look at what the borrowing rate is now. Now, you can't borrow at this rate for ventures, but it indicates that there's a lot of credit out there, and there's still a lot of credit out there. In fact, the treasury and the Fed would argue there's too much credit out there.

And that's why we're in the problems that we are today. Because too many people-- people that should not have been leveraging and borrowing have done so. And now, we're feeling the pains of a contraction.

So this is the history of yield curves. And at a given point in time, we can take a look at yield curves and see how that curve changes day to day. I showed you today's yield curve, which is upward sloping.

But you know, there was a period back in 2000 where this yield curve was actually upward-sloping, and then downward-sloping. Why would the yield curve be downward-sloping? What that tells you is that there's an expectation of the market participants that interest rates in the long run have got to come down, and that there's going to be some kind of Fed policy shift possible within three years, five years, 10 years, that would make that more likely than not.

So by looking at these yield curves over different dates, you can get a sense of how the market's expectations are of the future, which I think is a tremendous ability to actually look into the future-- at least look into the future as predicted by all the market participants. This is just that yield curve that I told you about. This is not in your notes, but I was able to copy that from Bloomberg this morning. You have that in the public website.

And now, the next topic I want to take on-- very briefly, because there's a lot to be said about models of the yield curve, and I won't take up class time to do that. This is something that is a topic that's more significantly focused on in investments and in fixed-income securities. But one of the things that we'd like to be able to do is to try to model that term structure interest rates.

Is there any logic that we can come up with that explains why the yield curve is upward-sloping or backward-bending or inverted? And it turns out that there are a number of theories that have come to be proposed. I'm not going to cover any of these in any depth, but I want to at least mention the names so that you'll know that there are theories out there that will tell you whether or not the yield curve should be so steeply sloped or not.

There's something called the Expectations Hypothesis. There is something called the Liquidity Preference Hypothesis. There is a Preferred Habitat Model, Market Segmentation Model. And then, there are a whole slew of extraordinarily sophisticated and complex mathematical models, one of which-- and probably the best known of which-- was developed by our very own John Cox and Steve Ross.

The Cox-Ingersoll-Ross model of the term structure of interest rates is probably the single best known yield curve model. It's a very complicated model, but it has some pretty significant implications for whether the yield curve ought to be upward-sloping or downward-sloping, or how it changes over time. I'll give you one example of what these theories are, and I won't spend much time on it, but I want to at least leave you with some intuition for how you would go about modeling it.

So one model says that today's forward rates are in fact the best guess of what future spot rates are likely to be, and that's known as the Expectations Hypothesis. This hypothesis states that today's forward rate, which is what we do observe, is equal to the mathematical expectation today, of the future one-year spot rate. Now, you might say, well, that's not that's not much of a theory. I mean, what else could it be?

Well, it turns out that there is another theory called the Preferred Liquidity Preference Theory that says that the longer the term of borrowing, the more you're going to have to pay a premium for people to lend to you, because people would prefer liquidity. So the longer you demand the borrowing for a greater period of time, the more you have to pay-- much more so than just linearly.

So in particular, the Expectations Hypothesis suggests that the yield curve is flat. Right? There's no impact on borrowing for two years, three years, five years, 10 years. The future rate is just equal to-- today's forward rate is the expectation of the future. OK? It's a fair bet.

Liquidity Preference says that the yield curve should be upward-sloping, because it's going to be more costly for you to borrow over a three-year period than a one-year period, simply because it's going to entail somebody giving up their money for a longer period of time. They're going to be less liquid. You're going to have to bribe them to be willing to give up that liquidity. That's the Liquidity Preference Theory.

The Preferred Habitat Theory says, you know, there are preferred maturities that people have. And for those maturities, you're going to have different rates than for those that people don't prefer. So if you want a 30-year maturity-- and that's not preferred because it's too far off-- then you're gonna have to pay a premium.

On the Other Hand, 10 years is a period that a number of pension funds prefer, so you may not have to pay as much for those preferred habitats. And to give you a sense of where academics is today, none of these models work. None of them can fully explain movements in the yield curve, which, by the way, is a wonderful opportunity for all of you.

Because if you have a model that does work, then you can do extraordinarily well. You can turn very, very small forecast power into enormous amounts of wealth very, very quickly on Wall Street. Yes?

STUDENT: [INAUDIBLE]

ANDREW LO: Does he--

STUDENT: [INAUDIBLE]

ANDREW LO: You can't patent it. Right. So does he get anything out of that besides notoriety? Well, that's a good question. The question has to do with, I guess, the difference between academic endeavors and business endeavors.

As an academic, what you're trying to do is to make a name for yourself and to put out research ideas that will have an impact with your colleagues and the particular area that you're in. So if you're asking, did Professor Cox get rich off of this? The answer is, no, probably not.

Neither did Black-Scholes, actually, when they published their Black-Scholes option pricing formula. That was the same year that the Chicago Board Options Exchange started trading, and everybody just used the option pricing formula almost from the very start. Did they make money off of that? No, they didn't.

And you're right. You can't patent it. So very often, in financial firms, when they have good ideas-- so for example, I do believe there are term structure models out there that work reasonably well. They're not published.

They're kept as the Coca-Colas of financial markets. They're trade secrets. And so if you go there to work and ultimately work with the right people, you will learn such trade secrets, which, by the way, is one of the reasons why when Barclays acquires Lehman and wants to keep all of these really, really terrific people together, they're going to have to offer premia to do that. And so that's part of the cost of financial distress when you disturb the status quo.

That's a few models of the term structure of interest rates. And I want to talk about one last topic on coupon bonds before concluding this lecture. Next lecture on Monday, we're going to take up measures of interest rate risk.

We're going to look explicitly at what happens when interest rates bounce around. Before we get to that, though, I want to talk about another way to value coupon bonds, and it's exactly the idea that I said before, of using pure discount bonds to do so. If I have a pure discount bond, I can use that to price coupon bonds by building up a package of discount bonds.

The example of the strips is given here. You've got a three-year 5% bond, and it turns out that you can show that that three-year 5% bond is going to be identical to 50 one-year strips, 50 two-year strips, and 1050 three-year strips, each strip paying $1 in years 1, 2, and 3.

Any question about that claim? Anybody want to argue with me that that's not true? It's pretty obvious, right?

Well, this actually has a very dramatic implication, and let me tell you what that implication is. The implication is that the price of a three-year 5% bond better be equal to the cost of 50 one-year strips, 50 two-year strips, and 1050 three-year strips. The price of this three-year bond better be equal to what it costs to put that portfolio of strips together today. Why? Why should it be equal? Yes.

STUDENT: [INAUDIBLE]

ANDREW LO: What is arbitrage? We haven't defined that.

STUDENT: [INAUDIBLE] basically, if the prices are different, [INAUDIBLE] buy the one or sell the other, so they come together [INAUDIBLE].

ANDREW LO: Exactly. There is a way to make money if the price of that three-year bond is anything other than the cost of that package of strips. So let's do an example. Suppose that the price of a three-year bond is greater than the cost of the strips. Tell me what to do. Yeah?

STUDENT: Buy those strips, package them into a bond, and then sell it. Sell the bond to the market. Just keep doing that over and over again.

ANDREW LO: OK. So let's talk about this slowly. I first buy those strips. I buy the portfolio strips. And then, I take this bond that's trading in the open marketplace, and I sell it.

How do I sell something I don't own? Short-sell it, yes. Short-selling, which you can read about and Brealey, Myers, and Allen, is when, if you don't own the security, you can borrow it from a broker, and then sell it, and you'll collect the money from selling it.

Now, the broker, obviously, is going to want you to keep that money in that brokerage firm so that you don't run away with it. Because you've borrowed the security at some point, you've got to pay it back. You've got to return that security. Right?

So the transaction is, you buy these strips, you short-sell the bond, and what have you done? Have you made money or lost money on day one? You've made money. Why?

Because buying costs less than the amount of money you get from selling. And how do you know that? Because I just assumed that it's more expensive for you to get the three-year bond than the package of strips.

So you've made money today, but you have no further obligations, because what you end up getting from the strips-- you've bought the strips, so you're going to get their coupons or their face value. You're going to use those to pay the coupons of the bonds that you sold.

And therefore, you have no further obligations, but you have a pile of money in front of you. That's pretty cool. And if you do that a lot, that pile of money grows.

So obviously, we know it's not easy to do that. And if it's not easy to do that, that means that our assumption that the bond was greater than the cost of the strips can't be true. If you reverse the logic, you get the same kind of argument in reverse.

Therefore, the only thing that could be is that the prices are equal to each other. Next time, what we're going to do is show that a little bit of linear algebra is going to allow you to make tons of money by comparing all sorts of bonds and looking at these kinds of relationships. OK. I'll see you at 4:00, if you're interested.