Differential Evolution:
a stochastic nonlinear optimization algorithm by Storn and Price, 1996

Presented by David Craft

September 15, 2003

This presentation is based on: Storn, Rainer, and Kenneth Price. *Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces. Journal of Global Optimization* 11, 1997, pp. 341-359.

The highlights of Differential Evolution (DE)

A population of solution vectors are successively updated by addition, subtraction, and component swapping, until the population converges, hopefully to the optimum.

No derivatives are used.

Very few parameters to set.

A simple and apparently very reliable method.

DE: the algorithm

Start with *NP* randomly chosen solution vectors.

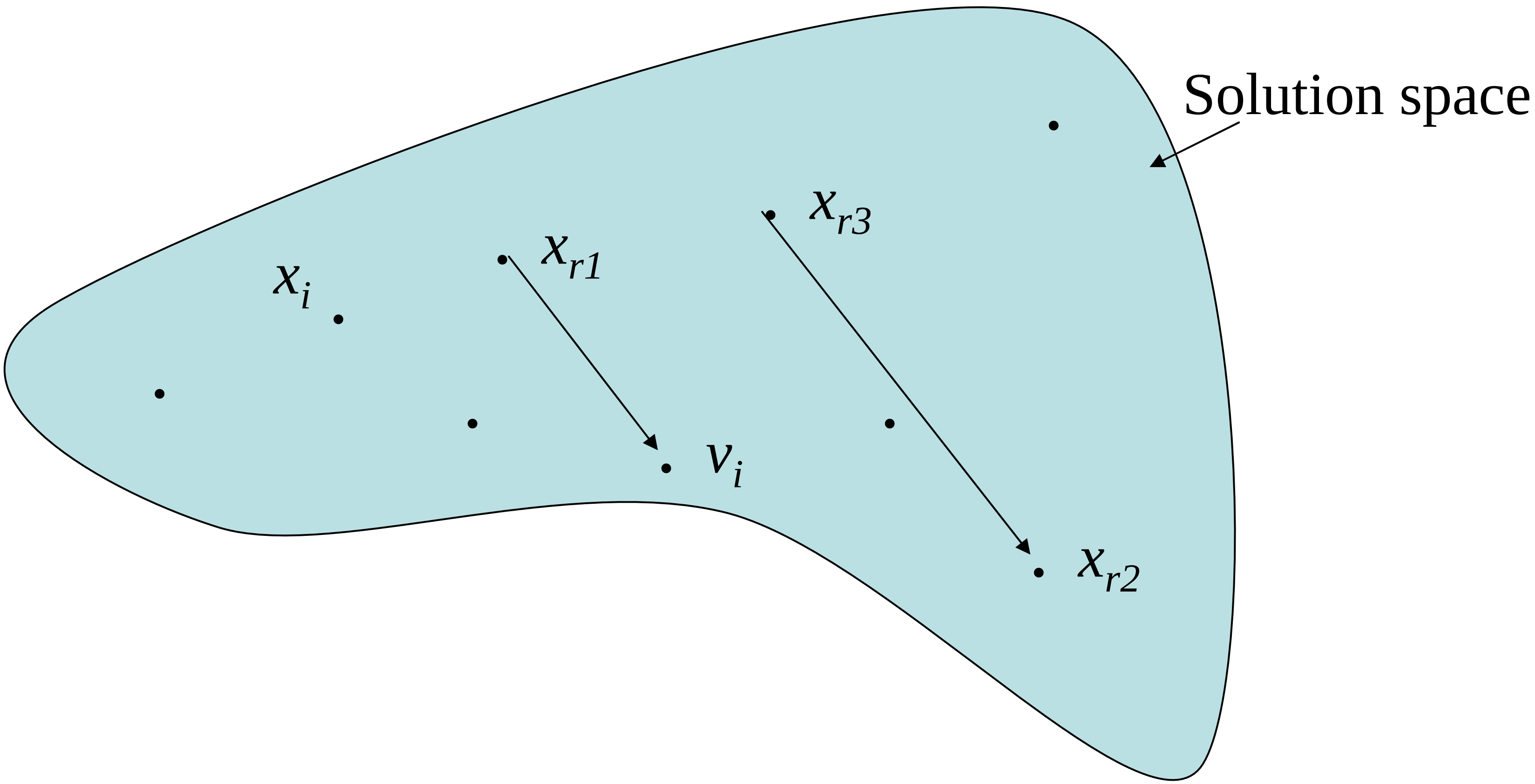
For each *i* in (1, …*NP*), form a 'mutant vector'

$$v_i = x_{r1} + F \cdot (x_{r2} - x_{r3})$$

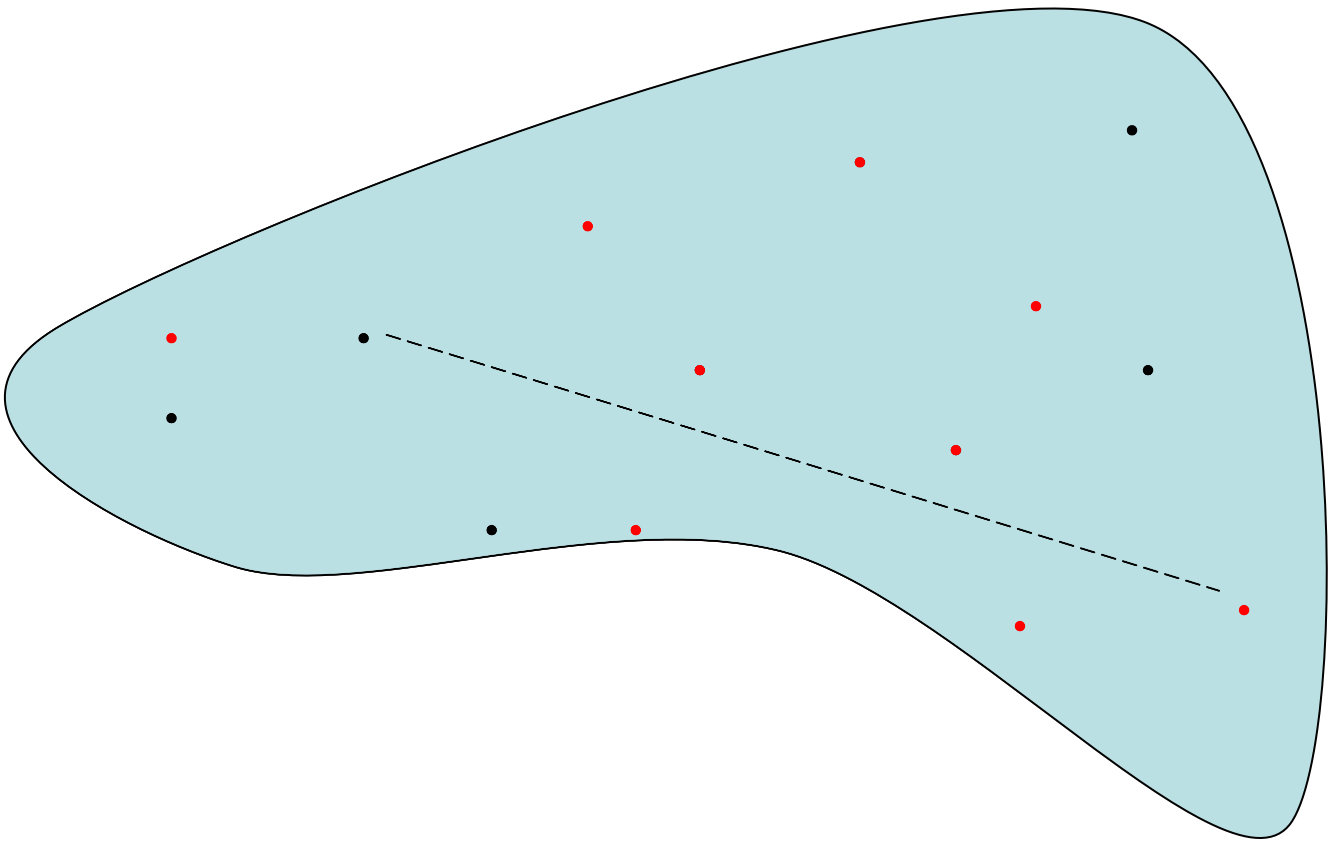
Where *r1*, *r2*, and *r3* are three mutually distinct randomly drawn indices from (1, …*NP*), and also distinct from *i*, and 0<F<=2.

DE: forming the mutant vector

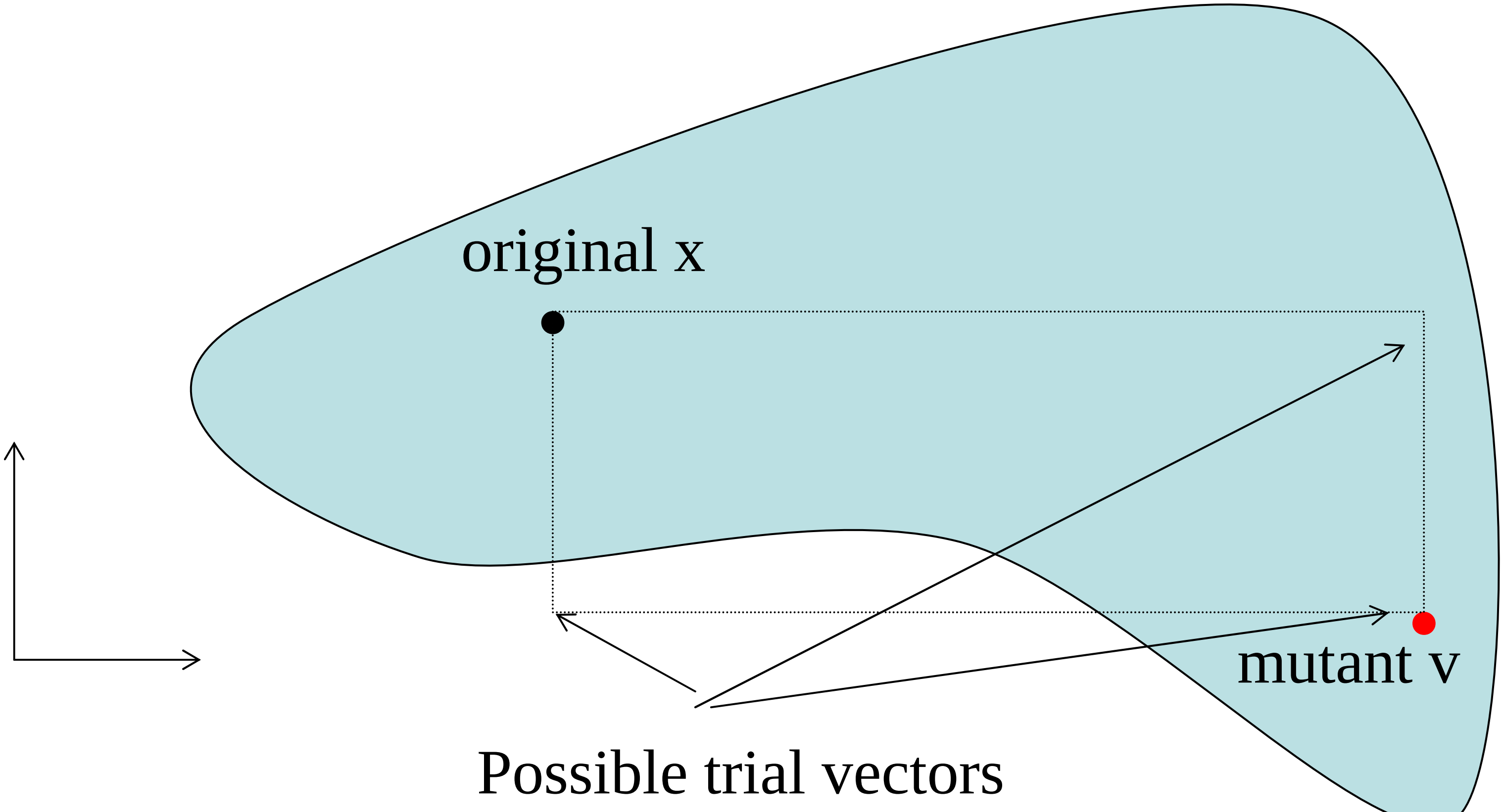
$$v_i = x_{r1} + F \cdot (x_{r2} - x_{r3})$$



DE: From old points to mutants



DE: Crossover x_i and v_i to form the trial vector



DE: Crossover x_i and v_i to form the trial vector u_i

$$x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5})$$
$$v_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5})$$
$$u_i = (\_\_, \_\_, \_\_, \_\_, \_\_)$$

For each component of vector, draw a random number in U[0,1]. Call this rand_j. Let $0 <= CR < 1$ be a cutoff. If $\text{rand}_j <= CR$, $u_{ij} = v_{ij}$, else $u_{ij} = x_{ij}$.

To ensure at least some crossover, one component of u_i is selected at random to be from v_i .

DE: Crossover x_i and v_i to form the trial vector u_i

$$x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5})$$

$$v_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5})$$

Index 1 randomly selected as definite crossover

So, for example, maybe we have

rand$_5$<=CR, so it crossed over too

$$u_i = (v_{i1}, x_{i2}, x_{i3}, x_{i4}, v_{i5})$$

DE: Selection

If the objective value COST(u_i) is lower than COST(x_i), then u_i replaces x_i in the next generation. Otherwise, we keep x_i.

Numerical verification

Much of the paper is devoted to trying the algorithm on many functions, and comparing the algorithm to representative algorithms of other classes. These classes are:

- Annealing algorithms
- Evolutionary algorithms
- The method of stochastic differential equations

Summary of tests: *DE is the only algorithm which consistently found the optimal solution, and often with fewer function evaluations than the other methods.*

Numerical verification: example

The fifth De Jong function, or “Shekel’s Foxholes”

(See equation 10 on page 348 of the *Differential Evolution* paper.)

The rest of the talk…

- Why is DE good?
- Variations of DE.
- How do we deal with constraints?
- An example from electricity load management.

Why is DE good?

- Simple vector subtraction to generate ‘random’ direction.
- More variation in population (because solution has not converged yet) leads to more varied search over solution space.

- $\Delta = (x_{r2} - x_{r3})$ [discuss: size and direction]

- Annealing versus “self-annealing”.

Variations of DE

x_{r1} : instead of random, could use **best**

$(x_{r2}-x_{r3})$: instead of single difference, could use more vectors, for more variation.

for example $(x_{r2}-x_{r3}+x_{r4}-x_{r5})$

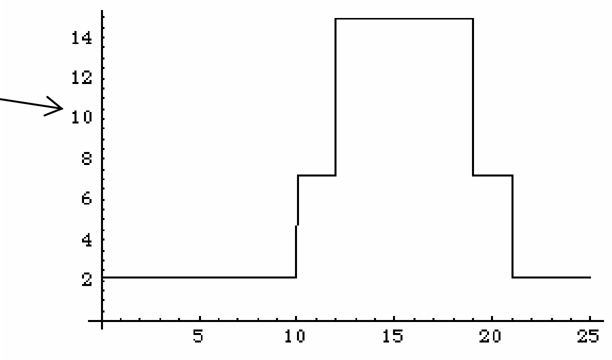
Crossover: something besides bernoulli trials…

Dealing with constraints

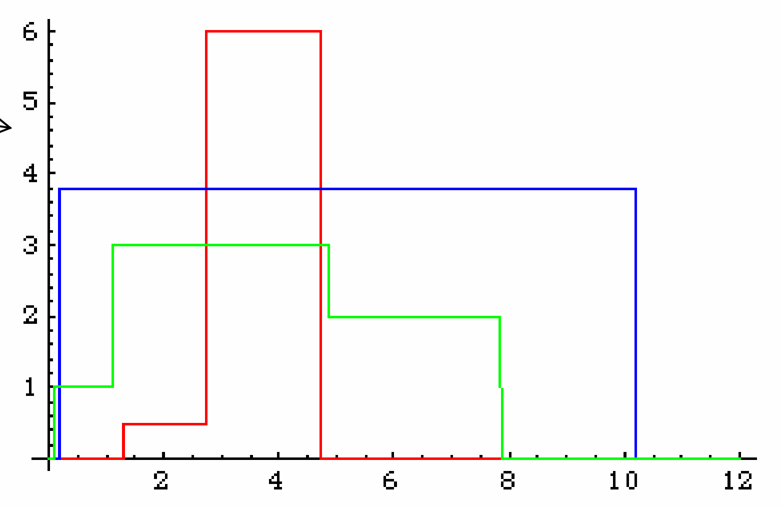
- Penalty methods for ‘difficult’ constraints.
- Simple projection back to feasible set for $l<=x<=u$ type constraints.
- Or, random value U[l,u] (when, why?)

Example: Appliance Job Scheduling

Hourly electricity prices (cents/kWh):



Power requirements for 3 different jobs (kW):



Start time constraints.

Example: Appliance Job Scheduling

Objective: find start times for each job which minimize cost.

Cost includes a charge on the maximum power used throughout the day. *This couples the problems!*

$$\min \sum_{i=1}^{J} t_i(x_i) + D(x)$$

$$s.t.\ \ a_i \le x_i \le u_i \qquad i=1,\ldots,J$$

where

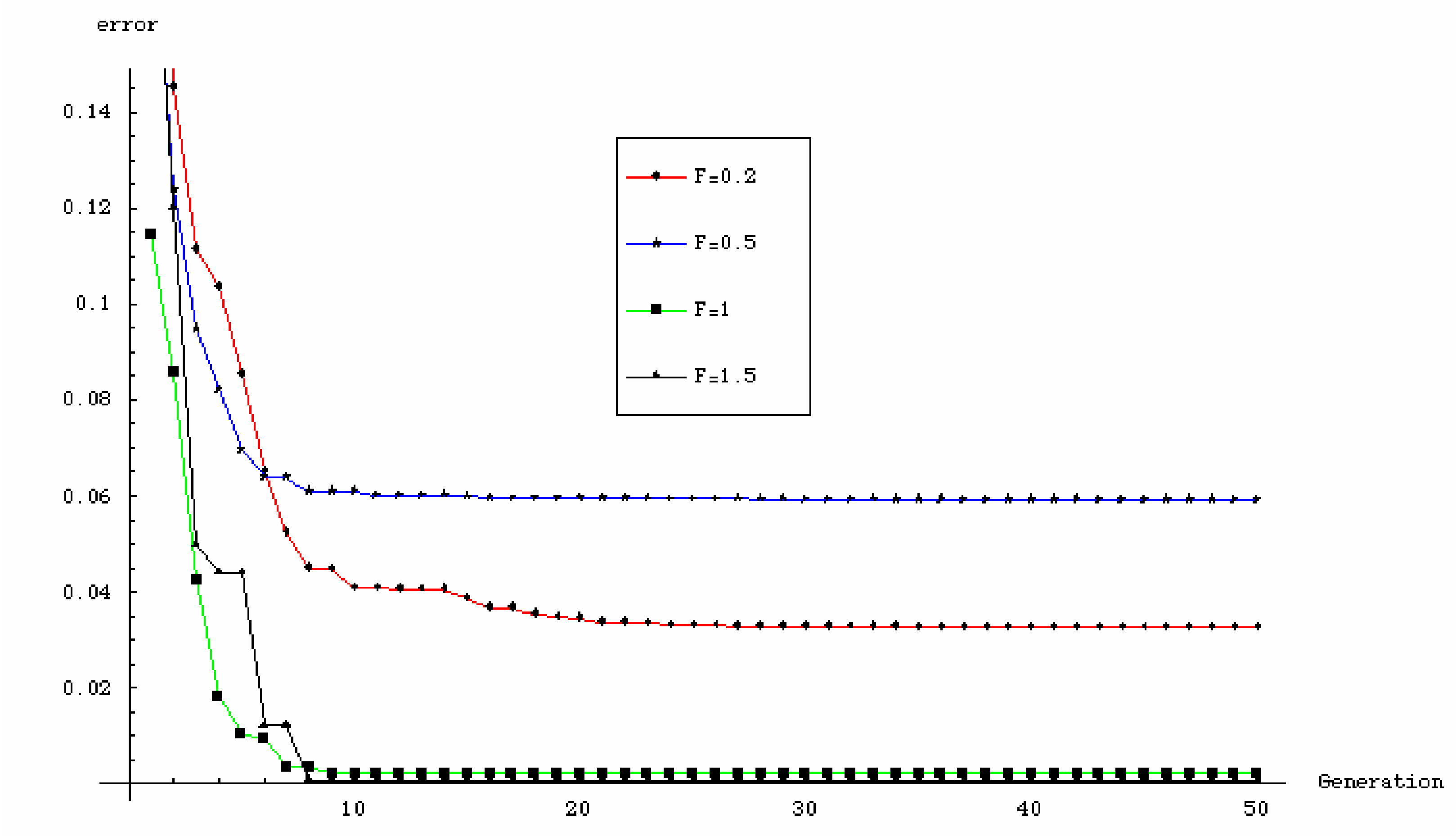
$$t_i(x_i) = \int_{x_i}^{x_i+l_i} p(t)e_i(t,x_i)\,dt$$

Cost of job *i* started at time x_i

$$D(x) = r \cdot \max_{t\in[0,T]} \sum e_i(t,x_i)$$

Demand charge

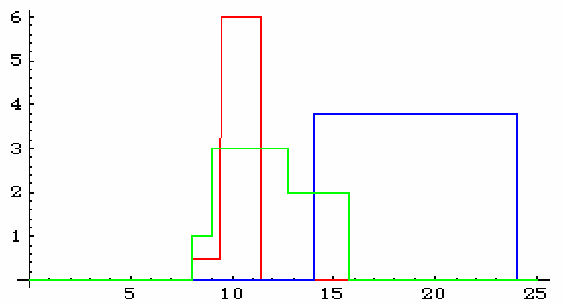
Convergence for different F



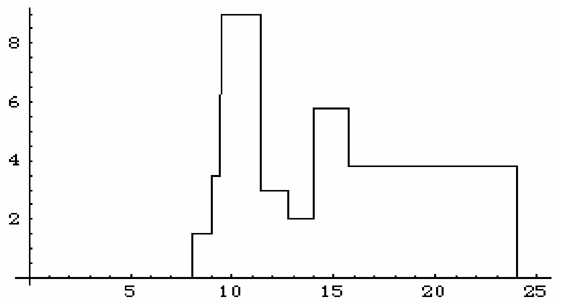
Other settings: CR=0.3, NP=6

Appliance Job Scheduling: Solution

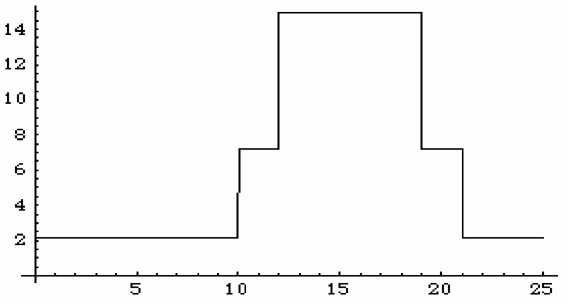
Solution



Total energy profile



Electricity price over time



Wrap-up

- DE is widely used, easy to implement, extensions and variations available, but no convergence proofs.

- More information:

  DE homepage: practical advice (e.g. start with NP=10*D and CR=0.9, F=0.8), source codes, etc.
  http://www.icsi.berkeley.edu/~storn/code.html

  DE bibliography, 1995-2002. Almost entirely DE applications.
  http://www.lut.fi/~jlampine/debiblio.htm