15.072
Midterm Exam

Date: March 22, 2006

Problem 1 *For the following questions/statements just give TRUE or FALSE answers.* ***Do not*** *derive the answers.*

Consider a G/G/1 queueing system. The arrival rate is λ and service rate is $\mu > \lambda$.

A. *Let L_{10} be the steady state number of customers in positions 1 to 10 (customer in service is assumed to be in position 1). Let S_{10} be the steady state time that a typical customer was in one of the positions 1-10. Then the Little's Law holds, namely $\mathbb{E}[L_{10}] = \lambda \mathbb{E}[S_{10}]$.*

B. *The distributional law holds for L_{10} and S_{10} when the scheduling policy is*

- ***(i)*** *First-In-First-Out*
- ***(ii)*** *Last-In-First-Out*

C. *Suppose that the system has instead two servers (that is we have G/G/2 queueing system). Then the probability that the system is empty is $1 - \rho$, where $\rho = \frac{\lambda}{2\mu}$.*

Problem 2 *Consider an M/G/1 queueing system where arrival rate $\lambda = 1$ and service time with a mixed distribution. Namely, with probability $1/2$ it is exponential with rate 2 and with probability $1/2$ it is exponential with rate 3. Assume the system operates under the First-In-First-Out policy.*

A. *Compute the traffic intensity ρ of this system.*

B. *Suppose the revenue obtained from a customer is e^{-cy} if the customer waited y time units. Compute the expected steady state revenue when $c = 1$.*

C. ***Extra credit***. *Suppose for every customer who waited y time units the cost e^{cy} is paid. What is the largest c_0 for which the expected cost is finite? Is $c_0 < 1.9$?*

Problem 3 *M/D/1 Queueing system with feedback. Namely each served customer comes back to the queue with some probability p and leaves the system with probability $1 - p$. The service time is assumed to be deterministic with value d both for the initial and returning customers. The arrival process is Poisson with rate λ.*

- *Under which conditions on λ, d, p is there a steady-state regime? Do not prove this, just provide a right answer.*
- *Compute the expected number of customers in the queue.*

  HINT: Observe that the number in the system is policy invariant for all non-preemptive work conserving scheduling policies.

Useful facts

The expected waiting time in M/G/1 queueing system under First-Come-First Serve policy is

$$\mathbb{E}[W] = \frac{\lambda \mathbb{E}[X^2]}{2(1-\rho)},$$

and the Laplace Transform of the waiting time is

$$\phi_W(s) = \frac{(1-\rho)s}{\lambda\beta(s) - \lambda + s},$$

where λ is the arrival rate, X is service time, $\rho = \lambda\mathbb{E}[X]$ and $\beta(s)$ is the Laplace Transform of the service time distribution.