



# “Location, location, location!”

## Regression Trees for Housing Data

# Boston

- Capital of the state of Massachusetts, USA
- First settled in 1630
- 5 million people in greater Boston area, some of the highest population densities in America.



# Housing Data



- A paper was written on the relationship between **house prices** and **clean air** in the late 1970s by David Harrison of Harvard and Daniel Rubinfeld of U. of Michigan.
- “Hedonic Housing Prices and the Demand for Clean Air” has been cited ~1000 times
- Data set widely used to evaluate algorithms.

# The R in CART



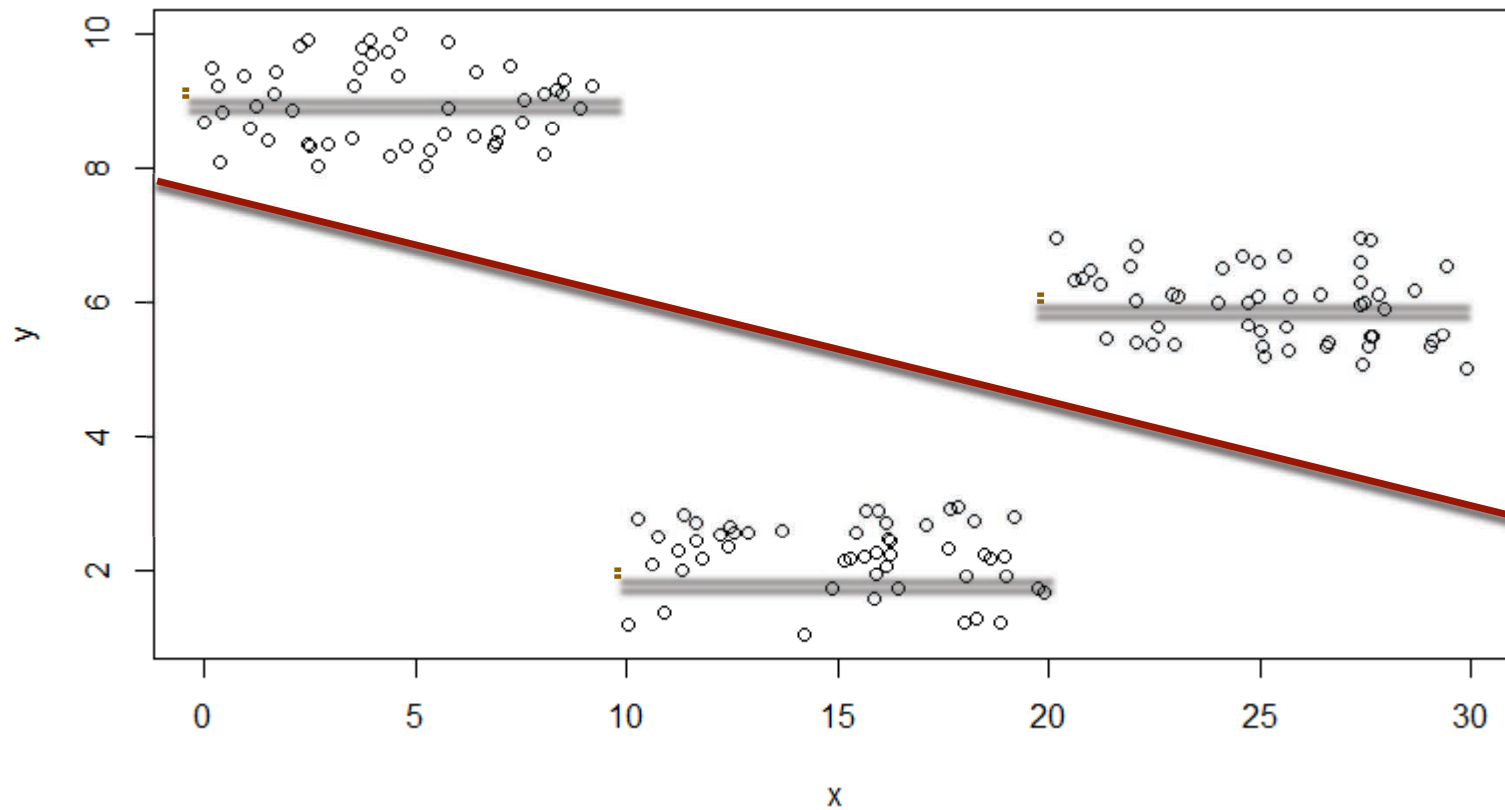
- In the lecture we mostly discussed **classification trees** – the output is a factor/category
- Trees can also be used for **regression** – the output at each leaf of the tree is no longer a category, but a number
- Just like classification trees, **regression trees** can capture **nonlinearities** that linear regression can't.

# Regression Trees



- With Classification Trees we report the average outcome at each leaf of our tree, e.g. if the outcome is “true” 15 times, and “false” 5 times, the value at that leaf is:
- With Regression Trees, we have continuous variables, so we simply report the average of the values at that leaf:

# Example



# Housing Data



- We will explore the dataset with the aid of trees.
- Compare linear regression with regression trees.
- Discussing what the “cp” parameter means.
- Apply cross-validation to regression trees.

# Understanding the data



- Each entry corresponds to a census **tract**, a statistical division of the area that is used by researchers to break down towns and cities.
- There will usually be multiple census tracts per **town**.
- **LON** and **LAT** are the longitude and latitude of the center of the census tract.
- **MEDV** is the median value of owner-occupied homes, in thousands of dollars.



# Understanding the data




- **CRIM** is the per capita crime rate
- **ZN** is related to how much of the land is zoned for large residential properties
- **INDUS** is proportion of area used for industry
- **CHAS** is 1 if the census tract is next to the Charles River
- **NOX** is the concentration of nitrous oxides in the air
- **RM** is the average number of rooms per dwelling

# Understanding the data



- **AGE** is the proportion of owner-occupied units built before 1940
- **DIS** is a measure of how far the tract is from centers of employment in Boston
- **RAD** is a measure of closeness to important highways
- **TAX** is the property tax rate per \$10,000 of value
- **PTRATIO** is the pupil-teacher ratio by town

# The “cp” parameter



- “cp” stands for “**complexity parameter**”
- Recall the first tree we made using LAT/LON had many splits, but we were able to trim it without losing much accuracy.
- Intuition: having too many splits is bad for generalization, so we should penalize the **complexity**

# The “cp” parameter

- Define **RSS**, the **residual sum of squares**, the sum of the square differences

$$RSS = \sum_{i=1}^n (y_i - f(x_i))^2$$

- Our goal when building the tree is to minimize the RSS by making splits, but we want to penalize too many splits. Define **S** to be the number of splits, and  $\lambda$  (lambda) to be our penalty. Our goal is to find the tree that minimizes


$$\sum_{Leaves} (\text{RSS at each leaf}) + \lambda S$$

# The “cp” parameter

- $\lambda$  (lambda) = 0.5

Splits	RSS	Total Penalty
0	5	5
1	$2 + 2 = 4$	$4 + 0.5 * 1 = 4.5$
2	$1 + 0.8 + 2 = 3.8$	$3.8 + 0.5 * 2 = 4.8$

# The “cp” parameter



$$\sum_{\text{Leaves}} (\text{RSS at each leaf}) + \lambda S$$

- If pick a large value of  $\lambda$ , we won't make many splits because we pay a big price for every additional split that outweighs the decrease in “error”
- If we pick a small (or zero) value of  $\lambda$ , we'll make splits until it no longer decreases error.

# The “cp” parameter

- The definition of “cp” is closely related to  $\lambda$
- Consider a tree with no splits – we simply take the average of the data. Calculate RSS for that tree, let us call it **RSS(no splits)**

$$c_p = \frac{\lambda}{\text{RSS}(\text{no splits})}$$

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15.071 Analytics Edge  
Spring 2017

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