Problem Set 1
Game Theory for Strategic Advantage (15.025)
Spring 2015

Due February 24, 2015

1) **Hotelling's Location Game.** Recall the "beach vendors" game we discussed in class, with the following modification: there are two firms ($i=1,2$), each one chooses a position from the set $S_i= \{1, 2,. . . , 10\}$. The consumers are equally distributed across these ten positions. Consumers buy from the firm whose position is closest to theirs. If the two firms are equidistant from a given position, half of the consumers go to each firm. The aim of the firms is to maximize their total sales.
Thus, for example, firm 1's payoff if both firms choose position 8 is u_1 *(8, 8) = 50.*
If instead, firm 1 chooses 7 and firm 2 chooses 8, firm 1's payoff is u_1 *(7, 8) = 70.*
[Hint: you do not need to write out the full payoff matrix!]
   a) Consider the strategy of picking location 1. Find all the strategies that *strictly* dominate strategy 1. Explain your answer. [Hint: try some guesses and see if they work.]
   b) Suppose now that there are three firms. Thus, for example, u_1*(8, 8, 8) =33.3* and u_1 *(7, 9, 9) = 73.3.* Is strategy 1 dominated, strictly or weakly, by strategy 2? How about by strategy 3? Explain.
   c) Suppose we delete strategies 1 and 10. That is, we rule out the possibility of any firm choosing either location 1 or 10, although there are still consumers at those positions. Is strategy 2 dominated, strictly or weakly, by any other strategy s_i in the reduced game? Explain.

2) **Penalty Shots Revisited.** Player 1 has to take a soccer penalty shot to decide the game. She can shoot **L**eft, **M**iddle, or **R**ight. Player 2 is the goalie. He can dive to the **l**eft, **m**iddle, or **r**ight. Actions are chosen simultaneously. The payoffs (which here are the probabilities in tenths of winning) are as follows.

		2		
		l	*m*	*r*
1	L	4,6	7,3	9,1
	M	6,4	3,7	6,4
	R	9,1	7,3	4,6

   a) For each player, is any strategy dominated by another strategy?

b) What probabilities (beliefs) must player 2 attach to player 1's strategies in order for **m** to be a best response?

c) For what beliefs about player 2's strategy is **M** a best response for player 1? [Hint: you do not need to draw a 3-dimensional picture!].

d) Suppose player 2 "puts himself in player 1's shoes" and assumes that player 1, whatever is her belief, will always choose a best response to that belief. Should player 2 ever choose **m**?

e) Show that this game does not have a (pure-strategy) Nash Equilibrium.

3) **Splitting the Dollar(s)**. Players 1 and 2 are bargaining over how to split $10. Each player i names an amount, s_i between 0 and 10 for herself. These numbers do not have to be in whole dollar units. The choices are made simultaneously. Each player's payoff is equal to her own money payoff. We will consider this game under two different rules. In both cases, if $s_1 + s_2 \leq 10$ then the players get the amounts that they named (and the remainder, if any, is destroyed).

a) In the first case, if $s_1 + s_2 > 10$ then both players get zero and the money is destroyed. What are the (pure strategy) Nash Equilibria of this game?

b) In the second case, if $s_1 + s_2 > 10$ and the amounts named are different, then the person who names the smaller amount gets that amount and the other person gets the remaining money. If $s_1 + s_2 > 10$ and $s_1 = s_2$ then both players get $5 . What are the (pure strategy) Nash Equilibria of this game?

a) Now suppose these two games are played with the extra rule that the named amounts have to be in whole dollar units. Does this change the (pure strategy) Nash Equilibria in either case?

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