15.023J / 12.848J / ESD.128J Global Climate Change: Economics, Science, and Policy
Spring 2008

Massachusetts Institute of Technology
15.023-12.848-ESD.128 Climate Change: Economics, Science and Policy
Problem Set #2

MODELING ECONOMIC GROWTH AND EMISSIONS
Due Monday, 17 March 2008

1. THE ASSIGNMENT

In this assignment you will apply a spreadsheet-based model of global CO_2 emissions to prepare 100-year projections of economic growth and carbon emissions, and to carry out studies of emissions restriction. The purpose of the exercise is to gain insight into the ways such models are constructed and applied. The model used here is simplified by ignoring the influence of government taxation and expenditure, and international trade, and by implementing a simple structure of production and input pricing.

The model is laid out in Section 2 along with parameter values. A template with most of the equations already written in is provided for download on the course website. Columns A through Q are used in this homework set. You will work with both recursive-dynamic (myopic) and forward looking versions. Analyses to be conducted are described in Section 3. Section 4 describes how the template worksheet was set up, and Section 5 provides some worksheet hints. There are nine parts: A1, A2, B1, B2, B3, C1, C2, C3 and D. You may find it convenient to include print-outs of the solved worksheets in your answers.

2. THE EMISSIONS PREDICTION MODEL

1. The Model of Economic Growth and Emissions

National output is modeled as a function of three factors: labor, capital and energy. The equations for labor and capital are similar to those discussed in class on 3 March. Carbon dioxide emissions are modeled as proportional to energy consumption, and the emissions-mitigation policy instrument is direct control of the level of energy use. Analysis of other measures that might be used in emissions control, such as taxes and subsidies, is not possible with this formulation.

Economic variables are stated on an annual basis ($\$10^{12}$ /yr). The model is solved on a 10-year time step.

1.1 GDP Growth

World GDP is represented by a single good, y, which is modeled as a function of inputs of capital, k, labor, l, and energy, e. The units of y are trillion (10^{12}) \$US. The output of the economy is represented by a constant-elasticity-of-substitution production function as follows,

$$y_t = a_t \left[\alpha\, k_t^{\rho_{KLE}} + \beta\, l_t^{\rho_{KLE}} + (1-\alpha-\beta)\, e_t^{\rho_{KLE}}\right]^{1/\rho_{KLE}} \tag{1}$$

where the elasticity of substitution, σ_{KLE}, is

$$\sigma_{KLE} = \frac{1}{1-\rho_{KLE}}.$$

The labor force grows at an annual rate λ,

$$l_t = l_0(1+\lambda)^t. \tag{2}$$

The capital stock changes over time as a result of depreciation and new investment. The magnitude of the capital stock is defined as of the first year of the time step, and the level in each subsequent time step is a function of the depreciation rate, δ, and the investment during the period, i_t,

$$k_{t+1} = n \times i_t + (1-\delta)^n k_t \tag{3}$$

where n is the number of years in each period of the model's solution. All-factor productivity change augments output at an annual rate γ.

$$a_t = a_0(1+\gamma)^t, \tag{4}$$

where this shift parameter a_t has an initial level a_0.

1.2 Income, Savings, and Investment

The output of the economy, net of the costs of supplying energy, becomes the income received by a representative global consumer. The relationship among these factors is defined by the following accounting identity:

$$\mathrm{y_t = c_t + s_t + (pe_t \times e_t)}, \tag{5}$$

where pe_t is the price of energy. Also, all savings are invested,

$$s_t = i_t. \tag{6}$$

Savings behavior is modeled differently depending on the version of the model you are working with. In the recursive-dynamic version, saving is determined as a fixed fraction, η, of income,

$$s_t = \eta\, y_t \tag{7}$$

In the forward-looking version, saving is a variable determined in the multi-period optimization, explained below.

1.3 The Cost of Energy

Energy use e_t is measured in exajoules (EJ = 10^{18}J). The marginal cost of energy (and therefore the price) is determined by the depletion of natural resources. As cumulative energy use rises, the cost of energy per EJ increases as follows

$$pe_t = \mu\left(\frac{r}{r - cume_t}\right)^{\nu} \tag{8}$$

where

- r is a constant representing the fixed amount of in-ground resources,
- $cume_t$, cumulative energy consumption, evolves according to

$$cume_{t+1} = n \times e_t + cume_t, \quad cume_0 = 0 \tag{9}$$

- μ and ν are estimated parameters.

1.4 Carbon Accounting

The carbon intensity of energy, ε, is fixed over time. Emissions of CO_2, denoted m_t, are stated in gigatons (GT = 10^9 tons) of carbon per year. They are calculated by multiplying the carbon intensity of energy by the amount of energy consumed:

$$m_t = \varepsilon \times e_t \tag{10}$$

1.5 Parameter Values

The parameter values are:

$n = 10$	(number of years per period)
$a_0 = 0.8$	(initial value of technological shift factor)
$r = 140000$ EJ	(energy resource base)
$\alpha = 0.25$	(capital share in production function)
$\beta = 0.65$	(labor share in production function)
$\delta = 0.05$	(annual depreciation rate of capital)

$\varepsilon = 0.018$ GT/EJ	(carbon coefficient on energy use)
$\mu = 0.01$	(slope parameter in energy cost function)
$\nu = 1$	(exponent parameter in energy cost function)
$\sigma = 1.25$	(elasticity of substitution)
$\rho = (\sigma - 1)/\sigma = 0.2$	(substitution parameter in production function)
$\theta = 0.03$	(annual discount rate)
$\lambda = 0.01$	(annual growth rate of labor supply)
$\gamma = 0.01$	(annual growth rate of all factor productivity change)

The initial-year values of the variables in the model are the following:

$l_0 = 22.65$	(10^9 worker-hours)
$k_0 = 130$	(10^{12} \$US)
$e_0 = 600$	(EJ)

The time $t = 0$ is the year 2000, and the spreadsheet provided for Parts A, B, and C of the homework set is already set up in 10-year periods.

3. TASKS WITH THE MODEL

You are to prepare calculations with two versions of this model, and use it to predict global emissions under various assumptions. For each version, please hand in

1. A copy of each worksheet where appropriate.
2. Plots of the time paths of consumption, carbon emissions, and atmospheric concentrations. Feel free to plot any other variables that you find of interest.
3. Comments on the solutions as requested below.

Once you have set up the worksheet, you may want to explore the effect on the results of alternative values of key parameters, e.g., γ, η, σ_{KLE}. Note that the model may not solve if you depart far from the reference parameters specified above.

Part A. Tasks with the Recursive-Dynamic Version

The Excel template incorporates a myopic version of the model, and shows a projection of global CO_2 emissions up to 2100 (column N). This version assumes that energy use grows from its year 2000 level of 600 EJ according to an elasticity relationship with GDP, y, with a one-period lag (column F). That is,

$$e_1 = 0.5\left[e_0 y_0^{\chi}\right]$$

…

$$e_t = 0.5\left[e_0 y_{t-1}^{\chi}\right]$$

where $\chi = 0.25$ is the elasticity of energy use to output.

Solve this model in separate cases for

Problem A1. A higher rate of productivity growth (γ=0.015), and

Problem A2. A lower rate of labor force growth (λ=0.005). (Return γ to its original value of 0.01 before running this case.)

Comment briefly on the way these changes influence the other variables in the model.

Part B. A Tasks with the Forward-Looking Version

Solve the model assuming that the level of energy use (the control variable) is set over all periods to maximize the present value of welfare. In any period *t*, the index describing welfare is $W_t = \ln(C_t)$. This functional form, (column K) implements an assumption that the marginal utility of consumption declines as consumption rises. Thus the function to be maximized is

$$\max PV(W_t) = \sum_t \ln c_t \left(\frac{1}{1+\theta}\right)^t \qquad (11)$$

where θ is the utility discount rate (rate of time preference), which is set to $\theta = 0.03$ on an annual basis in the worksheet. The model maximizes equation (11) by choosing the path of e_t. The initial value of this variable is fixed, with

$e_0 = 600$ (EJ).

Problem B1. Prepare a reference ("business as usual") scenario on the assumption there is no carbon constraint. (Make sure productivity growth and labor force growth are reset (γ=0.01; λ=0.01).

Problem B2. Prepare a control scenario applying an intensity target of the form announced by the Bush Administration and some developing countries. Assume the energy intensity of GDP (*m/y*) declines by 15% per decade, beginning from the intensity in the 2000 base year. (Hint: Set up the intensity constraint in with P24-P33 <= Q24-Q33 with the limit set up in the Q column.) Solver is sensitive to starting conditions here. If you get repeated errors, reload the template and start again.)

Problem B3. Construct a curve of marginal cost, defined in terms of lost (non-discounted) consumption, for global emissions reductions of up to 4 GtC, for the periods 2010 and 2020. This may be done by solving the model for reductions below reference values (from Problem **B1**) of 1.0, 2.0, 3.0, and 4.0 GtC and then plotting the *marginal* decrease in consumption. Comment briefly on the shapes of the curves. Remember that consumption is in units of trillions of dollars, and so a 0.1 change in the consumption level is actually 100 billion dollars. (Note also that, because of its simplicity, this model will yield costs higher than you will see in other analyses, and a non-zero intercept.)

Part C. Stabilization of Atmospheric CO_2 Concentration

A simple model of CO_2 accumulation (in GtC) in the atmosphere is

$$stkm_t = n \times m_t + \left(1 - \frac{1}{\tau_e}\right)^n stkm_{t-1} \tag{12}$$

where

$stkm_0$ = 806.86 GtC (carbon in atmosphere in 2000)

and

τ_e = 120 years (e-folding time of carbon in the atmosphere).

The atmospheric concentration, in ppmv, is then

$conc_t = stkm_t / 2.181$.

The template already includes two columns for the calculation of atmospheric concentrations of CO_2, and a figure to display the results is provided in the worksheet. Implement these equations and conduct the following exercises using the forward-looking model.

Problem C1. Solve for the pattern of energy use (and associated carbon emissions) that *stabilizes* atmospheric carbon concentrations at or below 550 ppmv for every period of the model. What is the yearly carbon uptake in 2100 under this scenario? Comment on the model assumption about terrestrial and ocean carbon uptake.

Problem C2. Solve the same case as **C1**, only with the discount rate reduced from 0.03 to 0.01. Comment briefly on the difference this makes to the solution. (Do not forget to rerun the solver, otherwise the optimization doesn't happen and all you do is change the net present value.)

Problem C3. Solve the **C1** case again (i.e., <= 550ppmv for all periods), only assume that because of political constraints the level of energy use cannot be reduced more rapidly than 10% per decade. Comment briefly on how the solutions differ.

Part D. How to Improve the Model

List three additional features you would add to the model to make it more realistic, commenting briefly on

1. Which you believe are the most important for understanding the climate issue?
2. What problems (data, analysis methods) would arise in implementation?
3. Are there issues that are important that this type of model would never be able to address, even with significantly more complexity?

4. SETUP OF THE SAMPLE WORKSHEET TEMPLATE

The problem is set up with 11 ten-year periods, $t^* = 0, ...,10$. For each period, t^* the value of the economic variables (y, k, l, c, i) are set at the appropriate annual levels of the first year in the 10-year calculation step. That is,

$y(t^* = 0) \equiv y(t = 0)$

$y(t^* = 1) \equiv y(t = 10)$

etc.

Thus equations (3), (9) and (12) assume that in the first year of the 10-year time step investment, energy and emissions (respectively) apply for all years of that step, so that these (annual) quantities must be multiplied by $n = 10$. Similarly, in equations (3) and (12) the annual survival factors $(1 - \delta)$ and $(1 - 1/\tau_e)$ must be updated over an entire decade, which is accomplished by raising them to the power n.

5. WORKSHEET HINTS

1. Before beginning work save a master copy of the worksheet (to be left unchanged) and set up a working version. You may need to exit and start over and this will save you having to go back to the web.
2. This note is written as if the dynamic version of the model is to be solved in Excel using Solver. Equivalent facilities should be available in other spreadsheet programs. The policies limiting emissions and/or concentrations must be entered as constraints in Solver.
3. When solving the dynamic model, bounds sometimes must be placed on values of the variables to ensure that Solver finds an optimum that in fact makes economic sense. If you run into problems try ensuring that energy use, energy prices, consumption and investment cannot be negative. Rule out these possibilities by adding the constraints $e_t \geq 0$, $pe_t \geq 0$, $c_t \geq 0$ and $i_t \geq 0$ for all t.
4. When solving the dynamic model under constraint, always re-run the unconstrained version before running with a different constraint. Solver does not always move accurately from one constrained solution to another. Also, sometimes Solver gets stuck (e.g., when confronted with a very restrictive constraint), and cannot return to the no-constraint solution. Therefore, you should always run an unconstrained version of any problem before moving from one constrained solution to another.
5. Sometimes Solver returns a solution that is not an optimum. If you run into this problem, often you can recover by limiting the zone of search for the optimal solution and performing successive solves, using the solution of one as the initial point for the next. This process should converge to a sensible answer.