Problem Set 8 Solution

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November 22, 2004

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The Solution

a)

The set of stable values is $Y_{SQ}\epsilon[4,7]$

b)

For $Y_{SQ} < 3$ or $Y_{SQ} > 7$, the outcome is 5; for $Y_{SQ}\epsilon(3,4)$, the outcome is $8 - Y_{SQ}$.

Details on the Solution

The solution is probably better seen in figures, but let me present an algebraic solution.

Let us solve the game by backward induction.

The timing of the game is as follows:

1)Committee chair chooses whether or not to propose a bill (Y_P) and chooses Y_P if it does so.

2)Committee votes on whether to send proposed bill to the floor. If it votes, not to do so, the game ends.

3)Floor leader can propose an amendment Y_A to the bill

4)Floor votes on the amendment, Y_A versus Y_P

5)The winner of stage 4) faces the status quo Y_{SQ} versus Y_A if the amendment won and Y_P versus Y_{SQ} if the amendment lost.

We know the following about the distribution of ideal points in the committee and on the floor:

	Chair	Median
Committee	6	5
Floor	5	4

Let us solve the game backward.

2.1 Stages 4) and 5)

Note that these two stages essentially amount to the median voter choosing between Y_A, Y_P and Y_{SQ} that which is least distant from its ideal point

(where we assume that the last bill proposed gets the advantage in case of a tie. (this is to ensure that a best-response for the last proposer exists)).

We will denote W_{SQ} the set of propositions that would beat Y_{SQ} in a vote on the floor ($W_{SQ} = [4 - |4 - Y_{SQ}|, 4 + |4 - Y_{SQ}|]$). Let us denote W_P the set of propositions that would beat Y_P in a vote on the floor (the range of values appears similarly).

2.2 Stage 3)

The Floor Chair wants to pick the winning proposition that is closest to his/her ideal point (5). Let us solve the problem by case.

A)$Y_{SQ} = 4$

Then no proposition (other than 4) can beat the status quo. The floor chair can propose any bill, the outcome is necessarily 4.

B)$Y_{SQ} \neq 4$; $Y_P \notin W_{SQ}$

Then the bill to beat is the status quo. Therefore, we have $Y_A = \min\{5, \max\{Y_{SQ}, 8 - Y_{SQ}\}\}$

(ie if the floor chair can beat the status quo with his/her ideal point, he/she will do so. Otherwise, then the status quo is 'too close' to 4, and the chair will propose the bill that is as close to 4 as the status quo, but on the right side of 4)

C)$Y_{SQ} \neq 4$; $Y_P \epsilon W_{SQ}$

Then the bill to beat is the bill proposed by the committee. Therefore, we have $Y_A = \min\{5, \max\{Y_P, 8 - Y_P\}\}$

(for the same reason as discussed above).

2.3 Stage 2)

Note that the committee median and the floor chair have the same ideal point. The question is whether the best bill that the floor chair can have be approved by the floor is better than the status quo. Going through the cases, we see that it is an optimal strategy for the committee median to send a bill to the floor except in the case where:

-$Y_{SQ} \epsilon (4, 5]$ and $Y_P \epsilon W_{SQ}$; then the outcome is $\max\{Y_P, 8 - Y_P\}$, which is further from 5 than Y_{SQ}

-$Y_{SQ} > 5$, $Y_P \epsilon W_{SQ}$ and $\max\{Y_P, 8 - Y_P\} \notin [10 - Y_{SQ}, Y_{SQ}]$. Then the outcome is $\max\{Y_P, 8 - Y_P\}$, which is necessarily further from 5 than the status quo.

More in details, we have:

The committee median anticipates the following results:

A)$Y_{SQ} = 4$: the outcome will be 4. The committee median may decide to send the bill to the floor (any strategy has no impact on the outcome).

B)$Y_{SQ} \neq 4$; $Y_P \notin W_{SQ}$

$Y_A = \min\{5, \max\{Y_{SQ}, 8 - Y_{SQ}\}\}$

(in the 'worst case' scenario, the floor leader will propose Y_{SQ}; otherwise, he/she will be able to pick a bill closer to 5 than the status quo. Therefore, the committee median sends the bill to the floor).

C)$Y_{SQ} \neq 4; Y_P \epsilon W_{SQ}$

C.1)If $Y_{SQ} < 4$: the outcome is $\max\{Y_P, 8 - Y_P\}$. This is better than Y_{SQ}; the committee median sends the bill to the floor.

C.2)If $Y_{SQ}\epsilon(4,5]$: the outcome is $\max\{Y_P, 8-Y_P\}$. This is certainly worse than Y_{SQ}; the committee median does not send the bill to the floor.

C.3)If $Y_{SQ} > 5$ and $\max\{Y_P, 8-Y_p\}\epsilon[10-Y_{SQ}, Y_{SQ}]$, the outcome is $\min\{5, \max\{Y_P, 8-Y_P\}\}$. The committee median sends the bill to the floor.

C.4)If $Y_{SQ} > 5$ and $\max\{Y_P, 8-Y_P\} \notin [10-Y_{SQ}, Y_{SQ}]$, the outcome is

$\max\{Y_P, 8-Y_P\}$. The committee median does not send the bill to the floor.

2.4 Stage 1)

a)If $Y_{SQ} = 4$: the outcome will be 4. The committee chair does not propose any bill.

b)If $Y_{SQ} < 4$; the comittee chair proposes $Y_P \notin W_{SQ}$ to command the outcome $\min\{5, 8-Y_{SQ}\}$.

c)If $Y_{SQ}\epsilon(4,7]$: the committee chair does not propose any bill

if $Y_{SQ}\epsilon(4,5)$: then if the committee chair proposes a bill that beats the status quo, it is rejected by the committee median. On the other hand, if he/she proposes a bill that does not beat the status quo, the outcome is Y_{SQ}, which is no improvement.

if $Y_{SQ}\epsilon[5,7]$: then if the committee chair proposes a bill that does not beat the status quo, the outcome is 5, which is further from 6 than the status quo. If he/she proposes a bill that beats the status quo such that $\max\{Y_P, 8-Y_P\} \notin [10-Y_{SQ}, Y_{SQ}]$, it is rejected by the committee median. If he/she proposes a bill that beats the status quo such that $\max\{Y_P, 8-Y_p\}\epsilon[10-Y_{SQ}, Y_{SQ}]$, the outcome is $\min\{5, \max\{Y_P, 8-Y_P\}\}$.But this is further from 6 that the status quo. Therefore, the committee chair does not propose any bill.

d)If $Y_{SQ} > 7$: the committee chair proposes $Y_P\epsilon[5, Y_{SQ}]$ to command an outcome of 5, which is an improvement over Y_{SQ}.