8.871

Solutions to problem set #5

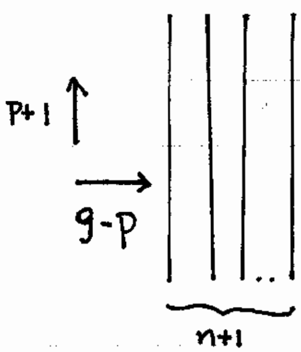
1.

The SYM theories with 16 supercharges and gauge group G in $p+1$ spacetime dimensions can be constructed as the worldvolume theory of parallel Dp-branes in various backgrounds.

The moduli space of the theory is $\boxed{\mathbb{R}^{(9-p)r}/W}$ where r is the rank of G and W is its Weyl group, corresponding to the discrete symmetries of the Dp-brane setup.

a) $G = A_n = SU(n+1)$

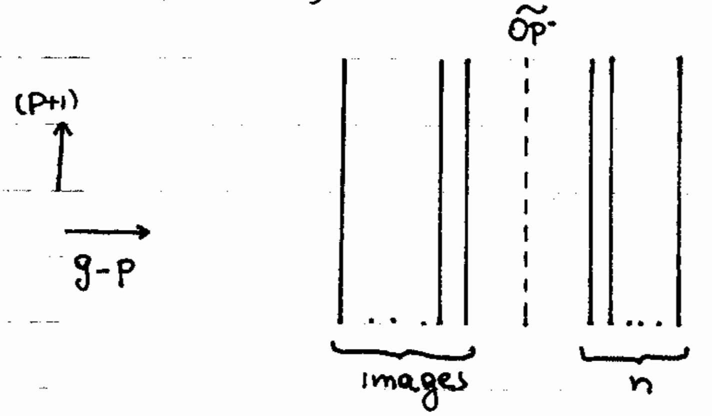
$n+1$ Dp-branes in flat space with fixed c.o.m.



$W = S_{n+1}$ = permutations of the $n+1$ Dp-branes.

b) $G = B_n = SO(2n+1)$

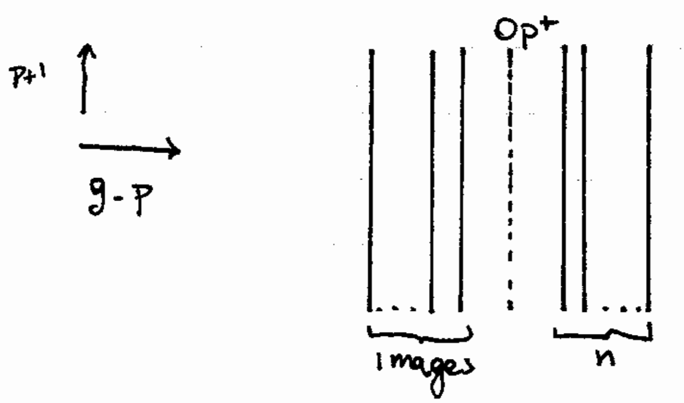
n Dp-branes in the background of an $\widetilde{Op}^-$ orientifold (which can be thought of as an Op^- with a ½ Dp-brane stuck on it)



$W = S_n \ltimes \mathbb{Z}_2^n$ = permutations of the n Dp-branes together with exchanges of a brane and its image

c) $G = C_n = Sp(n)$

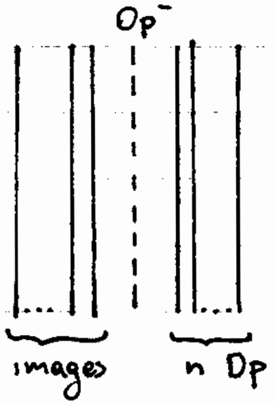
n Dp-branes in the background of an Op^+ orientifold.



W = same as in (b)

d) $G = D_n = SO(2n)$

n Dp-branes in the background of an Op^-



Here the brane picture suggests that the Weyl group is the same as in the C_n and B_n cases, i.e. $S_n \ltimes \mathbb{Z}_2^n$. We know from group theory however that

$W(D_n) = S_n \ltimes \mathbb{Z}_2^{n-1}$. The roots of D_n are $\pm e_i \pm e_j$ and the $\mathbb{Z}_2$ factors act as $e_i \to -e_i$ with

$\prod_i (\pm)_i = 1$. This restriction removes one $\mathbb{Z}_2$ and

can be seen by looking at the action on the fermion representation. The fundamental and adjoint representations are left invariant by the larger group. With 16 supercharges, all fields are in the adjoint rep., so the brane picture does not reveal the subtlety.