ast Time

SCET$_I$:
- usoft - collinear factorization
- hard - collinear factorization

[$\mathcal{L}_{qq}^{(0)}$, RPI, IR div., Running]

hard $p^\mu \sim (Q, Q, Q)$ $C = H$

collin $\sim (Q\lambda^2, Q, Q\lambda)$

usoft $\sim (Q\lambda^2, Q\lambda^2, Q\lambda^2)$

SCET$_{II}$: still to come,
- soft - collinear factorization
- Wilson coeffs

hard-collin p^μ $(Q\eta, Q, \sqrt{Q\eta})$ $C = J$ jet function

collin $p^\mu \sim (Q\eta^2, Q, Q\eta)$

soft $p^\mu \sim (Q\eta, Q\eta, Q\eta)$

Note: identification of d.o.f. is frame dependant, but relationships between d.o.f. are frame indep.

eg. boost can swap collin $\leftrightarrow$ soft

Results for observable which tie d.o.f together are "Factorization Theorems"

eg $\int [d\cdots]\ H(q^-)\ J(q^-, p^-, k^+)\ \phi(p^-)\ \phi(k^+)$

Processes

- $\gamma^*\gamma \to \pi^0$ π-γ form factor at $Q^2 \gg \Lambda^2$ for γ^*

  Breit frame $q^\mu = \frac{Q}{2}(n^\mu - \bar{n}^\mu)$, $p_\gamma^\mu = E\,\bar{n}^\mu$

  $$p_\pi^\mu = \frac{Q}{2} n^\mu + \underbrace{\left(E - \frac{Q}{2}\right)}_{m_\pi^2/2Q} \bar{n}^\mu$$

  pion = collinear in n-direction (SCET$_{II}$)

- $\gamma^* M \to M'$ M-M' (meson) form factor $Q^2 \gg \Lambda^2$ for γ^*

  M = collinear in n

  M' = " " $\bar{n}$ (say) (SCET$_I$)

- $B \to D\pi$ Matrix Elt. of 4-quark operators

  $Q = \{m_b, m_c, E_\pi\} \gg \Lambda$

  B, D are soft $p^2 \sim \Lambda^2$, π-collinear (SCET$_{II}$)

- DIS Structure Functions at $Q^2 \gg \Lambda^2$

  $e^- p \to e^- X$ and $1 - x \gg \Lambda/Q$ (ie not near endpts in Bjorken x)

  Breit frame: proton n-collinear , X - hard (SCET$_I$)

- Drell-Yan $\frac{d\sigma}{dQ^2}$ Q^2 = inv. mass of $\ell^+\ell^- \gg \Lambda^2$

  $p\bar{p} \to \ell^+\ell^- X$

  p - n-collin, $\bar{p}$ - $\bar{n}$-collin, X - hard

- $e^+e^- \to$ jets

  $\bar{p} \to$ jets • depends on observable we formulate

  $pp \to$ jets eg two jets n-collin jet

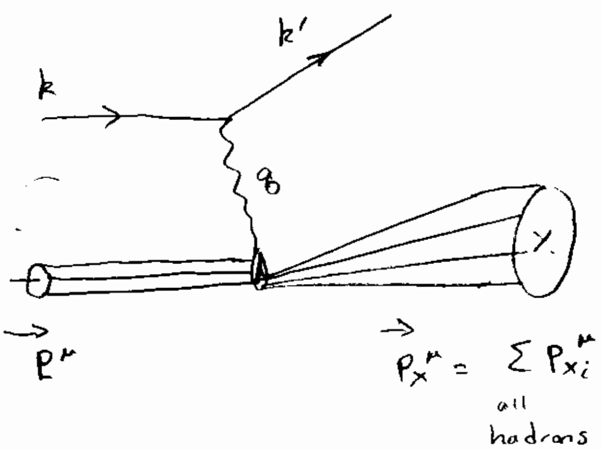
  $\bar{n}$-collin jet

etc.

DIS A rich subject, only aspects related to QCD factorization are covered here using SCET

Refs: § 1.8 of text
Aneesh M.'s review: hep-ph/9204208
Bob J.'s review: hep-ph/9602236
paper: hep-ph/0202088 (for material below)

$e^- p \to e^- X$



$Q^2 \gg \Lambda^2$

$q^2 = -Q^2$, $x = \dfrac{Q^2}{2P\cdot q}$

$P_X^\mu = P^\mu + q^\mu$

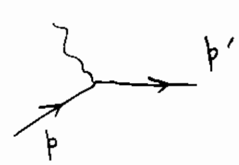
$P_X^2 = \dfrac{Q^2}{x}(1-x) + M_p^2$

<u>regions</u>

P_X^2	$\left(\frac{1}{x}-1\right)$	
$\sim Q^2$	~ 1	inclusive OPE
$\sim Q\Lambda$	$\sim \Lambda/Q$	endpt. region
$\sim \Lambda^2$	$\sim \Lambda^2/Q^2$	resonance region

$e^- p \to e^- p'$
↑ eg. excited state

<u>Parton Variables</u>



struck quark carries some fraction ξ of proton momentum

$\bar n\cdot p = \xi\, \bar n \cdot P$ ← we'll see how to formulate ξ in QCD

$p'^2 \simeq Q^2\left(\frac{\xi}{x} - 1\right)$

Frames

<u>Breit Frame</u>

$$q^\mu = \frac{Q}{2}(\bar{n}^\mu - n^\mu)$$

$$P^\mu = \frac{n^\mu}{2}\,\bar{n}\cdot P + \frac{\bar{n}^\mu m_p^2}{2\,\bar{n}\cdot P} = \frac{n^\mu}{2}\frac{Q}{x} + \cdots \quad \text{collinear}$$

$$P_X^\mu = \frac{n^\mu}{2}Q + \frac{\bar{n}^\mu}{2}\frac{Q(1-x)}{x} + \cdots \quad \text{hard}$$

Proton is made of collinear quarks and gluons

<u>Rest Frame</u>

$$P^\mu = \frac{m_p}{2}(n^\mu + \bar{n}^\mu) \quad \text{soft}$$

$$q^\mu = \frac{\bar{n}^\mu}{2}\frac{Q^2}{m_p x} - \frac{n^\mu}{2}m_p x + \cdots$$

$$P_X^\mu = \text{sum} \quad \text{"collinear"}\ P_X^2 \sim Q^2$$

Like $B \to X_c e\nu$ we can write cross-section in terms of leptonic & hadronic tensors

$$d\sigma = \frac{d^3k'}{2|\vec{k}'|}\frac{e^4}{sQ^4} L^{\mu\nu}(k,k')\,W_{\mu\nu}(P,q)$$

we'll look at spin-avg. case

$$W_{\mu\nu} = \frac{1}{\pi}\,\text{Im}\,T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{2}\sum_{\text{spin}} \langle p|\,\hat{T}_{\mu\nu}(q)\,|p\rangle$$

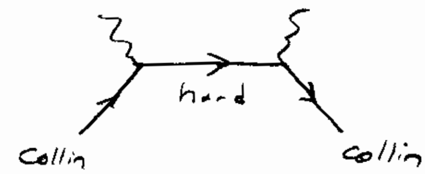
$$\hat{T}_{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\, T[J_\mu(x)\,J_\nu(0)]$$

(e.m. currents)

$$T_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) T_1(x,Q^2) + \left(P_\mu + \frac{q_\mu}{2x}\right)\left(P_\nu + \frac{q_\nu}{2x}\right) T_2(x,Q^2)$$

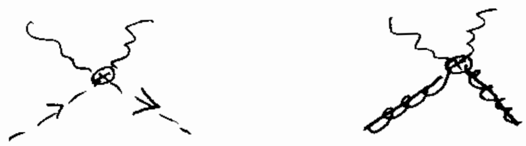
satifies current conservation, P, C, T, etc.

Want imaginary part of forward scattering



First Match onto SCET Ops.

at L.O. :



$$\hat{T}^{\mu\nu} = \frac{g_\perp^{\mu\nu}}{Q}\left(O_1^{(i)} + \frac{O_1^g}{Q}\right) + \frac{(n^\mu+\bar{n}^\mu)(n^\nu+\bar{n}^\nu)}{Q}\left(O_2^{(i)} + \frac{O_2^g}{Q}\right)$$

<u>$O(\lambda^2)$ operators</u>

flavor = u, d, ...

$$O_j^{(i)} = \bar{\xi}_{n,p'}^{(i)} W \frac{\bar{n}\!\!\!/}{2} C_j^{(i)}(\bar{P}_+, \bar{P}_-) W^\dagger \xi_{n,p}^{(i)}$$

$$O_j^{(g)} = \mathrm{tr}\left[W^\dagger B_\perp^\lambda W \, C_j^g(\bar{P}_+, \bar{P}_-) W^\dagger B_{\perp\lambda} W\right]$$

where $ig B_\perp^\lambda \equiv [i\bar{n}\cdot D_c , iD_{\perp c}^\lambda] \sim \lambda \sim \xi_n$

$\bar{P}_\pm = \bar{P}^\dagger \pm \bar{P}$

$O_j^{(i)}$ will lead to quark, anti-quark p.d.f.'s

O_j^g " " gluon p.d.f.'s

Quark contribution in detail :

$$O_j^{(i)} = \int d\omega_1 \, d\omega_2 \; C_j^{(i)}(\omega_+, \omega_-) \left[(\bar{\xi}_n W)_{\omega_1} \frac{\bar{n}\!\!\!/}{2} (W^\dagger \xi_n)_{\omega_2}\right]$$

$\uparrow \delta(\omega_1 - \bar{P}^\dagger)$ $\qquad \uparrow \delta(\omega_2 - \bar{P})$

$\omega_\pm = \omega_1 \pm \omega_2$

- coord space $f_{i/p}(z) = \int dy\, e^{-i 2 z \bar{n}\cdot p\, y} \langle p| \bar{\xi}(y)\, W(y,-y)\, \bar{n}\!\!\!/\, \xi(y) |p\rangle$

parton distn for quark i in proton p

$$\bar{f}_{i/p}(z) = - f_{i/p}(-z) \quad \text{for anti-quark}$$

mom. space $\langle p_n| (\bar{\xi}_n W)_{\omega_1} \bar{n}\!\!\!/ (W^\dagger \xi_n)_{\omega_2} |p_n\rangle = 4\bar{n}\cdot p \int_0^1 d\xi\, \delta(\omega_-)$

$$* \left[\delta(\omega_+ - 2\xi\bar{n}\cdot p) f_{i/p}(\xi) - \delta(\omega_+ + 2\xi\bar{n}\cdot p) \bar{f}_{i/p}(\xi) \right]$$

recall: positive $\omega_1 = \omega_2$ gives particles; negative $\omega_1 = \omega_2$ gives anti-particles

- $(\bar{\xi}_n W)_\omega\, \bar{n}\!\!\!/\, (W^\dagger \xi_n)_\omega$ is a number operator for collinear quarks with momentum ω

($\bar{\xi}_n W$: a parton)

[If we tried to couple usoft or soft gluons to this op. its a singlet so they decouple, more later]

Charge Conjugation

$$C_j^{(i)}(-\omega_+, \omega_-) = - C_j^{(i)}(\omega_+, \omega_-)$$

- relates Wilson-Coeff for quarks & anti-quarks at operator level
- Only need matching for quarks
- δ-functions set $\omega_- = 0$, $\omega_+ = 2\xi\bar{n}\cdot p = 2Q\frac{\xi}{x}$

Relate basis

$$\frac{1}{\pi} \text{Im}\, T_1 = \int [d\omega] \; \frac{-1}{Q} \left(\frac{1}{\pi} \text{Im}\, C_1(\omega) \right) \langle O^{(i)}(\omega) \rangle$$

$$\frac{1}{\pi} \text{Im}\, T_2 = \int [d\omega] \left(\frac{4x}{Q} \right)^2 \frac{1}{Q} \frac{1}{\pi} \text{Im} \left(C_2(\omega) - \frac{C_1(\omega)}{4} \right) \langle O^{(i)}(\omega) \rangle$$

Define $H_j(z) = \frac{\text{Im}}{\pi} C_j(2Qz, 0, Q^2, \mu^2)$

$\omega_+ \;, \omega_-$ — do $\omega_\pm$ with δ-functions

$$T_1(x, Q^2) = \frac{-1}{x} \int_0^1 d\xi \; H_1^{(i)}\left(\frac{\xi}{x} \right) \left[f_{i/p}(\xi) + \bar{f}_{i/p}(\xi) \right]$$

$$T_2(x, Q^2) = \frac{4x}{Q^2} \int_0^1 d\xi \left(4 H_2^{(i)}\left(\frac{\xi}{x} \right) - H_1^{(i)}\left(\frac{\xi}{x} \right) \right) \left[f_{i/p}(\xi) + \bar{f}_{i/p}(\xi) \right]$$

- this is factorization for DIS (to all order in α_s) into
  computable coefficients H_i
  universal non-pert. functions $f_{i/p}$, $\bar{f}_{i/p}$
  (show up in many processes)
- Coefficients C_j were dimensionless and can only have $\alpha_s(\mu) \ln(\mu/Q)$ dependence on Q
  → Bjorken scaling

[Analysis valid to LO in $\frac{\Lambda^2}{Q^2}$]

- $H_i(\mu)$ $f_{i/p}(\mu)$ traditionally this μ-dependence is called the "factorization-scale" $\mu = \mu_F$
  & one also has "renorm. scale" $\alpha_s(\mu = \mu_R)$

In SCET the μ is just the ren. scale in SCET. We have new UV divergences associated with running of p.d.f., along with running for $\alpha_s(\mu)$.

- Tree Level Matching
  (upon which a lot of intuition is based)

find just $g_\perp^{\mu\nu}$ ie $C_2 = 0$

↳ Callan-Gross relation that $W_1 / W_2 = Q^2/4x^2$

$$C_1(\omega_+) = 2 e^2 Q_i^2 \left[\frac{Q}{(\omega_+ - 2Q) + i\epsilon} - \frac{Q}{-(\omega_+ + 2Q) + i\epsilon} \right]$$

↑ charges

$$H_1 = -e^2 Q_i^2 \; \delta\left(\frac{\xi}{x} - 1\right)$$

gives parton-model interpretation $\xi = x$

Comments on DIS

- contrast ∞ set of ops in text
- power of $SCET_I / SCET_{II}$ not really needed, no soft (think of it as $SCET_I$ for example)

- do eg. first -

Soft-Collinear Interactions (SCET$_{II}$)

Recall $q = q_s + q_c \sim Q\,(\lambda, 1, \lambda)$ [components $+\;-\;\perp$]

$q^2 = Q^2\lambda \gg (Q\lambda)^2$

offshell w.r.t s, c

On-shell modes $q^\mu \sim Q\,(\lambda, 1, \sqrt{\lambda})$ are hard-collinear

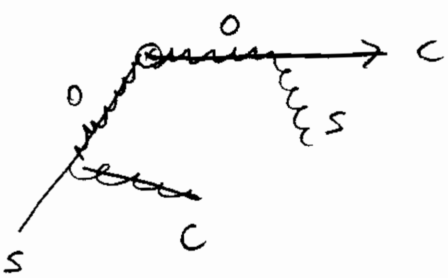
compared to collinear $q^\mu \sim Q(\lambda^2, 1, \lambda)$

Integrating out these fluctuations builds up a soft Wilson line S_n (analogous to $Y(n\cdot A_{us})$ but with soft fields)

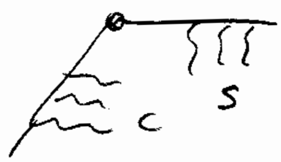
Toy eg. heavy-to-light soft-collin current $\bar{\xi}_n \Gamma h_v$

s = soft, c = collinear

0 = offshell



adding more gives

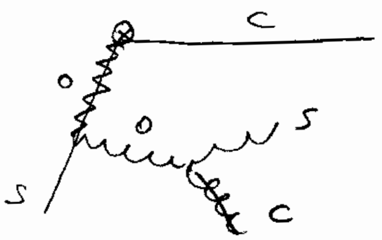


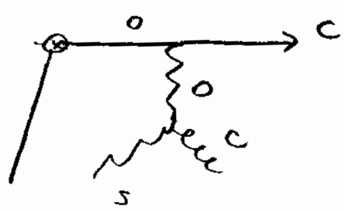
$\bar{\xi}_n S_n^\dagger \Gamma W h_v$

$S_n^\dagger[n\cdot A_{us}]$

$W[\bar{n}\cdot A_c]$

In QCD need 3-gluon, 4-gluon vertices too; these flip order of $S^\dagger$ & W





$(\bar{\xi}_n W)\,\Gamma\,(S_n^\dagger h_v)$

collinear gauge invariant; soft gauge invariant

[can be extended to all orders]

this is soft-collinear factorization