8.851 Homework 6

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Problem 1) Renormalization of $c_F(\mu)$

Draw the diagrams needed to compute the anomalous dimension of the coefficient $c_F(\mu)$ which appears in the $\mathcal{O}(1/m_Q)$ magnetic moment HQET Lagrangian $\mathcal{L}_F^{(1)}$. Discuss whether the kinetic energy Lagrangian $\mathcal{L}_K^{(1)}$ mixes with $\mathcal{L}_F^{(1)}$ under renormalization. Argue that the anomalous dimension vanish in the abelian case (and therefore is proportional to the adjoint Casmir C_A) without computing any integrals. (Hint: think about Coulomb gauge.)

Problem 2) Heavy-to-Light Currents in HQET

Consider the $\mathcal{O}(1/m_Q)$ heavy-to-light vector currents

$$\begin{aligned} O_1 &= \bar{q}\,\gamma^\mu i\not{D} Q_v\,, \qquad & O_4 &= \bar{q}\,(-iv\cdot\overleftarrow{D})\gamma^\mu Q_v\,,\\ O_2 &= \bar{q}\,v^\mu i\not{D} Q_v\,, \qquad & O_5 &= \bar{q}\,(-iv\cdot\overleftarrow{D})v^\mu Q_v\,,\\ O_3 &= \bar{q}\,iD^\mu Q_v\,, \qquad & O_6 &= \bar{q}\,(-i\overleftarrow{D}^\mu)Q_v\,, \end{aligned} \tag{1}$$

with coefficients B_1 to B_6. Using reparameterization invariance determine which of these coefficients are fixed by the coefficients C_1 and C_2 of the leading order vector heavy-to-light currents $\bar{q}\gamma^\mu Q_v$ and $\bar{q}v^\mu Q_v$.

Problem 3) OPE for $B \to X_c e\bar{\nu}$

Use the results of Sec. 6.2 of your text to derive the λ_2 terms appearing in the double differential decay rate in Eq. (6.57). Explain why the λ_1 terms in Eq. (6.57) include one proportional to $\delta'(z)$ while the λ_2 terms do not. (You are not being asked to derive the λ_1 coefficients explicitly, just those for λ_2.)