

8.851 Homework 2

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Problem 1) Field Redefinitions

In class we considered the following Lagrangian for a scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + \eta g_1\phi^6 + \eta g_2\phi^3\partial^2\phi \quad (1)$$

where $\eta \ll 1$. We then made a field redefinition $\phi \rightarrow \phi'$ which eliminated the last term, giving a new Lagrangian \mathcal{L}' .

- Show at $\mathcal{O}(\eta)$ that the field redefinition gives the same answer for \mathcal{L}' as applying the $\eta = 0$ equations of motion to the last term. Determine the new λ' and g_1' parameters in terms of those in the original Lagrangian.
- Demonstrate the equivalence of the 4-point and 6-point functions to $\mathcal{O}(\eta)$ by explicit computation of the tree level Feynman graphs with \mathcal{L} and \mathcal{L}' . Use your result from a).
- Draw the diagrams you would need to compute to demonstrate the equivalence of the 4-point functions at one-loop and $\mathcal{O}(g_2\eta)$. Roughly sketch how the equivalence would work by taking zero momenta on the external lines and looking at the form of the loop integrals (do not worry about the symmetry factors needed to show it explicitly).
- With the original field redefinition determine the terms induced at $\mathcal{O}(\eta^2)$ in \mathcal{L}' , and then show that a second field redefinition can be used to again eliminate the terms proportional to the equations of motion.

Problem 2) Running Quark Masses and Renormalons

Consider QCD with n_f flavors.

- Compute the quark mass anomalous dimension at one-loop in $\overline{\text{MS}}$. Solve to get the running $\overline{\text{MS}}$ mass $\bar{m}(\mu)$. (For a bonus, also compute the relation between the pole mass and $\overline{\text{MS}}$ mass at one-loop.)

- b) The Higgs decay rate for $H \rightarrow c\bar{c}$ is proportional to m_c^2 , making it quite sensitive to the charm mass. Suppose we know the $\overline{\text{MS}}$ mass $\bar{m}_c(\mu = \bar{m}_c) = 1.4 \text{ GeV}$, and want to know what value to use for the Higgs decay at $\mu = m_H = 500 \text{ GeV}$. By matching and running in effective theories determine $\bar{m}_c(m_H)$. Why is this choice, $\mu \simeq m_H$, appropriate?

In b), why not use the charm pole mass? (It's independent of μ .) Let's look at a toy example that indicates why. Perturbation series in quantum field theory are most often asymptotic expansions. A series is asymptotic to an observable $f(\alpha_s)$ if $|f(\alpha_s) - \sum_{n=0}^N f_n \alpha_s^n| < K_{N+1} \alpha_s^{N+1}$ for some numbers K_N and α_s in a finite region. If at large n we have $f_n \sim a^n n!$ for some $a > 0$ then often the truncation error $K_N \propto a^N N!$ also. The series for $f(\alpha_s)$ decreases until $n = N^* \sim 1/(a\alpha_s)$ and then starts to diverge. The convergence is improved for the Borel transform $\tilde{f}(t)$ where

$$\tilde{f}(t) = f(0)\delta(t) + \sum_{n=0}^{\infty} \frac{f_n}{n!} t^n, \quad f(\alpha_s) = \int_0^{\infty} dt e^{-t/\alpha_s} \tilde{f}(t). \quad (2)$$

If the integral exists the series is Borel summable and gives a definition for the sum of the series. Now in perturbation theory the value of the pole mass has an intrinsic ambiguity, while the $\overline{\text{MS}}$ mass is a short distance mass and does not. To see this problem with m_c^{pole} lets consider a toy example of a set of diagrams which have an ‘‘infrared renormalon’’.

For the fermion self energy let \mathcal{G} be the diagrams proportional to $\alpha_s [n_f \alpha_s]^k$, $k \geq 1$ (draw them). We'll consider how they contribute to the pole mass.

- c) Read section 4.6 of your text up to Eq.(4.74). Use Eq.(4.74) to compute the Borel transform of the series from \mathcal{G} for contributions to the pole mass from $\Sigma(\not{p}=m)$. You should get Eq.(4.78) of the text. (Subtract by hand the pole at $u = 0$ which corresponds to an ultraviolet renormalization.)
- d) The pole at $u = 1/2$ is called an infrared renormalon. It makes the inverse Borel transform in Eq.(2) ambiguous, and demonstrates that the pole mass is sensitive to infrared momenta. Consider the ambiguity in deforming the contour of the integral in Eq.(2) to avoid the $u = 1/2$ pole. Compute the size of the ambiguity, δm_{pole} , and demonstrate that it is $\sim \Lambda_{\text{QCD}}$.