8.851 Homework 1

Iain Stewart, February 11, 2003

Problem 1) Calculate the muon decay rate assuming $M_W \gg m_\mu \gg m_e$.

Problem 2) Matching with Massive Electrons

Consider QED with electrons and photons. For photon momenta q^μ much less than m_e we can integrate out the electrons.

a) Calculate the one-loop photon vacuum polarization diagram with dimensional regularization and $\overline{\rm MS}$, and expand $\Pi(q^2)$ in q^2/m_e^2.

b) Explain why the first term in the expansion motivates matching onto a theory without electrons at a scale $\mu \sim m_e$ rather than $\mu \sim 1\,\text{TeV}$.

c) Write down a Lagrangian with a gauge invariant photon operator that reproduces the second term in the expansion. Use your calculation from part a) to determine the Wilson coefficient of the operator at this order in α.

d) What QED symmetry(s) forbid dimension-6 operators with three field strengths from ever appearing?

e) At dimension-8, operators are generated which give light-by-light scattering. Determine the number of α's in their coefficients. Then use dimensional analysis in the low energy effective theory to numerically estimate the size of the $\gamma\gamma \to \gamma\gamma$ cross section for $10\,\text{keV}$ photons.

Problem 3) Right Handed Neutrinos

Consider adding three right-handed singlet neutrinos N_R to the standard model. A Majorana mass term is allowed, so

$$\mathcal{L}_N = \bar{N}_R\, i \not{\partial}\, N_R - \frac{1}{2}\bar{N}_R^c M N_R - \frac{1}{2}\bar{N}_R M^* N_R^c\,, \tag{1}$$

where $N_R^c = C\bar{N}^T$ is the charge conjugate field, $C = i\gamma_2\gamma_0$, and M is a complex symmetric Majorana mass matrix.

a) Write down the most general dimension-4 operators that couple N_R to the fields in the standard model, making use of the gauge symmetries.

b) Transform the N_R fields to three Majorana mass eigenstates $N_i = N_i^c$, $i = 1, 2, 3$ with real masses M_i.

c) Count the total number of physical parameters in M and the coefficients of the operators in part a). Hint: Consider the $G = U(3) \times U(3) \times U(3)$

flavor symmetry of the free L_L, e_R, and N_R kinetic terms. This symmetry is broken by the mass and Yukawa matrices, so the number of physical parameters can be obtained by subtracting the number of parameters in G from the number in the original matrices. How many of the parameters are CP-odd phases? For the ambitious, repeat the counting for n families of light leptons and n' right-handed neutrinos.

d) Take the masses M_i large compared to the electroweak scale and integrate out the right handed neutrinos at tree level. Show that the leading term reduces to the form of the dimension-5 standard model operator we discussed in class.