differential cross section for this process can be written as

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq^2 dY dB_a^+ dB_b^+} = \sum_{ij} H_{ij}(q^2,\mu)\int dk_a^+ dk_b^+ Q^2 B_i[\omega_a(B_a^+ - k_a^+), x_a, \mu] B_j[\omega_b(B_b^+ - k_b^+), x_b, \mu]$$
$$\times\, S_{i\,\mathrm{hemi}}(k_a^+, k_b^+, \mu)\left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}, \frac{\sqrt{B_{a,b}\omega_{a,b}}}{Q}\right)\right] \tag{12.40}$$

where $\omega_{a,b} = x_{a,b}E_{cm}$ andB_i is defined as our "Beam Function."

$$B_q(\omega b^+, \omega/\hat{p}^-, \mu) = \frac{\theta(\omega)}{\omega}\int \frac{dy^-}{4\pi} e^{ib^+ y/2} \langle p_n(\hat{p}^-)|\, |\bar{\chi}_n(y^-\frac{n}{2})\delta(\omega - \bar{\mathcal{P}})\frac{\bar{n}\!\!\!/}{2}\chi_n(0)|\, |p_n(\hat{p}^-)\rangle \tag{12.41}$$

We recll the definitions of jet function

$$\langle 0|\, |\bar{\chi}_{n,\omega}(y^-\frac{n}{2})\frac{\bar{n}\!\!\!/}{2}\chi_n(0)|\, |0\rangle \tag{12.42}$$

and pdf

$$\langle p|\, |\bar{\chi}_{n,\omega}(0)\frac{\bar{n}\!\!\!/}{2}\chi_n(0)|\, |p\rangle \tag{12.43}$$

We see that the Jet Function is a mix of both. The proton is a collinear field in SCET$_{II}$ and the jet is collinear in SCET$_I$. Matching SCET$_I$ to SCET$_{II}$ gives us

$$B_i(t,x,\mu) = \sum_i \int_x^1 \frac{d\xi}{\xi}\mathcal{I}_{ij}(t, \frac{x}{\xi}, \mu) f_j(\xi,\mu)\left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{t}\right)\right] \tag{12.44}$$

$$b_a^\mu = (\xi - x)E_{cm}\frac{n_a}{2} + b_a^+\frac{\bar{n}_a}{s} + b_{a\perp} \tag{12.45}$$

At tree level the Beam Function is simply

$$B_i(t,x,\mu) = \delta(t) f_i(x,\mu) \tag{12.46}$$

as in the pdf case we can write the RGE for the beam function

$$\mu\frac{d}{d\mu}B_i(t,x,\mu) = \int dt' \gamma_i(t-t',\mu) B_i(t',x,\mu) \tag{12.47}$$

Like the jet function B_i is independent of mass evolution. The RGE sums $ln^2(t/\mu)$, is independent of x and has no mixing.

A More on the Zero-Bin

A.1 0-bin subtractions with a 0-bin field Redefinition

A.2 0-bin subtractions for phase space integrations

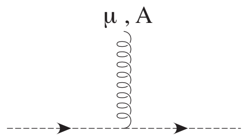
B Feynman Rules with a mass

If we add a mass the collinear Lagrangian becomes

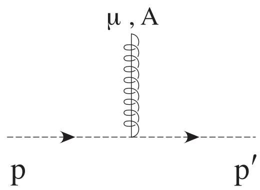
$$\mathcal{L}_{\xi\xi}^{(0)} = \bar{\xi}_n(x)\left[in\cdot D + (i D\!\!\!/_\perp^c - m)\frac{1}{i\bar{n}\cdot D^c}(i D\!\!\!/_\perp^c + m)\right]\frac{\bar{n}\!\!\!/}{2}\xi_n(x)\,, \tag{B.1}$$

and the modified Feynman rules are shown in Fig. 12.

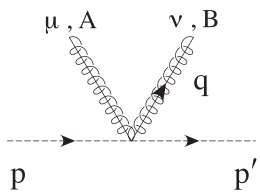
$(\tilde{p}, p_r) \qquad = \; i\,\frac{\not n}{2}\,\frac{\bar n\!\cdot\! p}{n\!\cdot\! p_r\,\bar n\!\cdot\! p + p_\perp^2 - m^2 + i\epsilon}$



$$= ig\,T^A\, n_\mu\, \frac{\bar{\not n}}{2}$$



$$= ig\,T^A \left[n_\mu + \frac{\gamma_\mu^\perp(\not p_\perp + m)}{\bar n\!\cdot\! p} + \frac{(\not p'_\perp - m)\gamma_\mu^\perp}{\bar n\!\cdot\! p'} - \frac{(\not p'_\perp - m)(\not p_\perp + m)}{\bar n\!\cdot\! p\,\bar n\!\cdot\! p'}\bar n_\mu \right] \frac{\bar{\not n}}{2}$$



$$= \frac{ig^2\,T^A\,T^B}{\bar n\!\cdot\!(p-q)} \left[\gamma_\mu^\perp\gamma_\nu^\perp - \frac{\gamma_\mu^\perp(\not p_\perp + m)}{\bar n\!\cdot\! p}\bar n_\nu - \frac{(\not p'_\perp - m)\gamma_\nu^\perp}{\bar n\!\cdot\! p'}\bar n_\mu + \frac{(\not p'_\perp - m)(\not p_\perp + m)}{\bar n\!\cdot\! p\,\bar n\!\cdot\! p'}\bar n_\mu \bar n_\nu \right] \frac{\bar{\not n}}{2}$$
$$+ \frac{ig^2\,T^B\,T^A}{\bar n\!\cdot\!(q+p')} \left[\gamma_\nu^\perp\gamma_\mu^\perp - \frac{\gamma_\nu^\perp(\not p_\perp + m)}{\bar n\!\cdot\! p}\bar n_\mu - \frac{(\not p'_\perp - m)\gamma_\mu^\perp}{\bar n\!\cdot\! p'}\bar n_\nu + \frac{(\not p'_\perp - m)(\not p_\perp + m)}{\bar n\!\cdot\! p\,\bar n\!\cdot\! p'}\bar n_\mu \bar n_\nu \right] \frac{\bar{\not n}}{2}$$

Figure 12: Order λ^0 Feynman rules as in Fig. 6, but with a collinear quark mass.

C Feynman Rules for the Wilson line W

Results for the Feynman rules for the expansion of the W Wilson line are also useful

$$W = 1 - \frac{gT^A\,\bar n\!\cdot\!\varepsilon_n^A(q)}{\bar n\!\cdot\! q} + \dots\,,$$
$$W^\dagger = 1 + \frac{gT^A\,\bar n\!\cdot\!\varepsilon_n^A(q)}{\bar n\!\cdot\! q} + \dots\,, \tag{C.1}$$

where here the momentum q is incoming and ε_n^A is the gluon-polarization vector.

D Feynman Rules for Subleading Lagrangians

In this subsection Feynman rules are given for the subleading quark Lagrangians involving two collinear quarks

$$\mathcal{L}_{\xi\xi}^{(1)} = (\bar\xi_n W)\, i\not D_{us}^\perp \frac{1}{\bar n\!\cdot\!\mathcal{P}} \big(W^\dagger i\not D_c^\perp \frac{\bar{\not n}}{2}\xi_n\big) + \big(\bar\xi_n i\not D_c^\perp W\big) \frac{1}{\bar n\!\cdot\!\mathcal{P}} i\not D_{us}^\perp \big(W^\dagger \frac{\bar{\not n}}{2}\xi_n\big)$$
$$\mathcal{L}_{\xi\xi}^{(2)} = (\bar\xi_n W) i\not D_{us}^\perp \frac{1}{\bar n\!\cdot\!\mathcal{P}} i\not D_{us}^\perp \frac{\bar{\not n}}{2}\big(W^\dagger \xi_n\big) + \big(\bar\xi_n i\not D_c^\perp W\big) \frac{1}{\bar n\!\cdot\!\mathcal{P}^2} i\bar n\!\cdot\! D_{us} \frac{\bar{\not n}}{2}\big(W^\dagger i\not D_c^\perp \xi_n\big)\,, \tag{D.1}$$

and for the mixed usoft-collinear Lagrangians from Eq. (**??**),

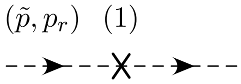
$$\begin{aligned}
\mathcal{L}_{\xi q}^{(1)} &= \bar{\xi}_n \frac{1}{i\bar{n}\cdot D_c} ig\not{B}_c^{\perp} W q_{us} + \text{h.c.} \ , \\
\mathcal{L}_{\xi q}^{(2a)} &= \bar{\xi}_n \frac{1}{i\bar{n}\cdot D_c}\, ig\not{M}\, W\, q_{us} + \text{h.c.}\,, \\
\mathcal{L}_{\xi q}^{(2b)} &= \bar{\xi}_n \frac{\bar{\not{n}}}{2} i\not{D}_\perp^c \frac{1}{(i\bar{n}\cdot D_c)^2}\, ig\not{B}_\perp^c W\, q_{us} + \text{h.c.}\ . \qquad \text{(D.2)}
\end{aligned}$$

All Feynman rules for $\mathcal{L}_{\xi q}^{(i)}$ involve at least one collinear gluon. From $\mathcal{L}_{\xi q}^{(1)}$ we obtain Feynman rules with zero or one $A_n^\perp$ gluons and any number of $\bar{n}\cdot A_n$ gluons. The one and two-gluon results are shown in Fig. 15. For $\mathcal{L}_{\xi q}^{(2a)}$ we have Feynman rules with zero or one $\{n\cdot A_n, A_{us}^\perp\}$ gluon and any number of $\bar{n}\cdot A_n$ gluons. The one and two-gluon results are shown in Fig. 16. Finally, for $\mathcal{L}_{\xi q}^{(2b)}$ one finds Feynman rules with zero, one, or two $A_n^\perp$ gluons and any number of $\bar{n}\cdot A_n$ gluons. In this case the one and two gluon Feynman rules are shown in Fig. 17.

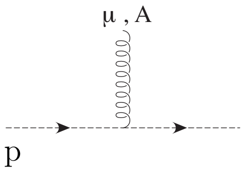
Finally, for the subleading terms in the mixed usoft-collinear gluon action we find

$$\begin{aligned}
\mathcal{L}_{cg}^{(1)} &= \frac{2}{g^2}\,\text{tr}\Big\{\big[iD_0^\mu, iD_c^{\perp\nu}\big]\big[iD_{0\mu}, WiD_{us\,\nu}^{\perp}W^\dagger\big]\Big\}\,, \qquad \text{(D.3)} \\
\mathcal{L}_{cg}^{(2)} &= \frac{1}{g^2}\,\text{tr}\Big\{\big[iD_0^\mu, WiD_{us}^{\perp\nu}W^\dagger\big]\big[iD_{0\mu}, WiD_{us\,\nu}^{\perp}W^\dagger\big]\Big\} \\
&\quad + \frac{1}{g^2}\,\text{tr}\Big\{W\big[iD_{us}^{\perp\mu}, iD_{us}^{\perp\nu}\big]W^\dagger\big[iD_{c\mu}^{\perp}, iD_{c\nu}^{\perp}\big]\Big\} + \frac{1}{g^2}\,\text{tr}\Big\{\big[iD_0^\mu, in\cdot D\big]\big[iD_{0\mu}, Wi\bar{n}\cdot D_{us}W^\dagger\big]\Big\} \\
&\quad + \frac{1}{g^2}\,\text{tr}\Big\{\big[WiD_{us}^{\perp\mu}W^\dagger, iD_c^{\perp\nu}\big]\big[iD_{c\mu}^{\perp}, WiD_{us\nu}^{\perp}W^\dagger\big]\Big\}\,,
\end{aligned}$$

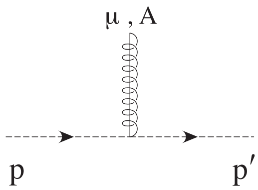
where $iD_0^\mu = i\mathcal{D}^\mu + gA_n^\mu$.



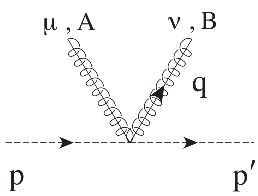
$$= i\,\frac{\bar{n}\!\!\!/}{2}\,\frac{2p^{\perp}\cdot p_r^{\perp}}{\bar{n}\cdot p}$$



$$= ig\,T^A\,\frac{\bar{n}\!\!\!/}{2}\,\frac{2p_{\perp}^{\mu}}{\bar{n}\cdot p}$$



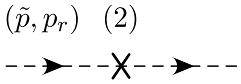
$$= ig\,T^A\,\frac{\bar{n}\!\!\!/}{2}\left[\frac{\gamma_\mu^\perp p\!\!\!/_r^\perp}{\bar{n}\cdot p}+\frac{p\!\!\!/_r'^\perp\gamma_\mu^\perp}{\bar{n}\cdot p'}+\frac{\bar{n}^\mu p\!\!\!/_r^\perp p\!\!\!/^\perp}{\bar{n}\cdot q\,\bar{n}\cdot p}-\frac{\bar{n}^\mu p\!\!\!/'^\perp p\!\!\!/_r'^\perp}{\bar{n}\cdot q\,\bar{n}\cdot p'}-\frac{\bar{n}^\mu p\!\!\!/_r'^\perp p\!\!\!/^\perp}{\bar{n}\cdot q\,\bar{n}\cdot p'}+\frac{\bar{n}^\mu p\!\!\!/'^\perp p\!\!\!/_r^\perp}{\bar{n}\cdot q\,\bar{n}\cdot p}\right]$$



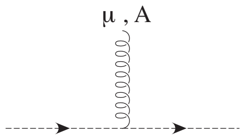
$$= \frac{ig^2\,T^A\,T^B}{}\,\frac{\bar{n}\!\!\!/}{2}\left[\gamma_\mu^\perp\gamma_\nu^\perp\cdots\right]$$

$$+\,\frac{ig^2\,T^B\,T^A}{}\,\frac{\bar{n}\!\!\!/}{2}\left[\gamma_\nu^\perp\gamma_\mu^\perp\cdots\right]$$

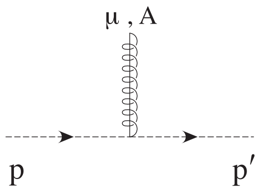
Figure 13: Order λ^1 Feynman rules with two collinear quarks from $\mathcal{L}_{\xi\xi}^{(1)}$.



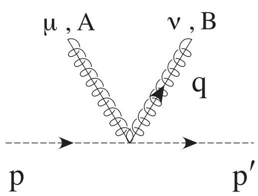
$$= i\,\frac{\bar{n}\!\!\!/}{2}\,\frac{p_{r\perp}^2}{\bar n\cdot p}$$



$$= ig\,T^A\,\frac{\bar{n}\!\!\!/}{2}\left[\frac{2p_r^{\perp\mu}}{\bar n\cdot p}-\frac{\bar n^\mu p_\perp^2}{(\bar n\cdot p)^2}\right]$$

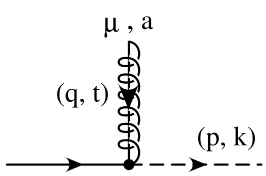


$$= ig\,T^A\,\frac{\bar{n}\!\!\!/}{2}\left[\frac{\bar n^\mu p_{r\perp}^2}{\bar n\cdot p}-\frac{\bar n^\mu p_{r\perp}^{\prime 2}}{\bar n\cdot p'}-\frac{\gamma_\mu^\perp p\!\!\!/_\perp \bar n\cdot p_r}{(\bar n\cdot p)^2}-\frac{p\!\!\!/'_\perp\gamma_\mu^\perp\bar n\cdot p_r}{(\bar n\cdot p')^2}-\frac{\bar n^\mu p\!\!\!/'_\perp p\!\!\!/_\perp\bar n\cdot p_r}{\bar n\cdot q(\bar n\cdot p)^2}+\frac{\bar n^\mu p\!\!\!/'_\perp p\!\!\!/_\perp\bar n\cdot p_r}{\bar n\cdot q(\bar n\cdot p')^2}\right]$$

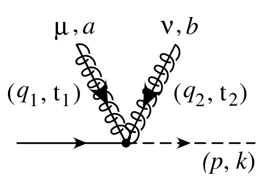


$$= \frac{ig^2\,T^A\,T^B}{}\,\frac{\bar{n}\!\!\!/}{2}\left[\gamma_\mu^\perp\gamma_\nu^\perp\cdots\right]$$
$$+\frac{ig^2\,T^B\,T^A}{}\,\frac{\bar{n}\!\!\!/}{2}\left[\gamma_\nu^\perp\gamma_\mu^\perp\cdots\right]$$

Figure 14: Order λ^2 Feynman rules with two collinear quarks from $\mathcal{L}_{\xi\xi}^{(2)}$.



$$= \qquad ig\,T^a\left[\gamma_\mu^\perp-\bar n_\mu\,\frac{q\!\!\!/_\perp}{\bar n\cdot q}\right]$$



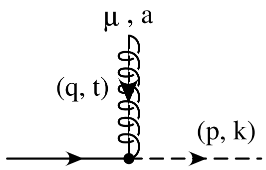
$$= \qquad ig^2\,\frac{T^bT^a}{\bar n\cdot q_1}\left[\frac{\bar n_\mu\bar n_\nu\,p\!\!\!/^\perp}{\bar n\cdot p}-\gamma_\nu^\perp\,\bar n_\mu\right]$$
$$+\,ig^2\,\frac{T^aT^b}{\bar n\cdot q_2}\left[\frac{\bar n_\mu\bar n_\nu\,p\!\!\!/^\perp}{\bar n\cdot p}-\gamma_\mu^\perp\,\bar n_\nu\right]$$

Figure 15: Feynman rules for the subleading usoft-collinear Lagrangian $\mathcal{L}_{\xi q}^{(1)}$ with one and two collinear gluons (springs with lines through them). The solid lines are usoft quarks while dashed lines are collinear quarks. For the collinear particles we show their (label,residual) momenta. (The fermion spinors are suppressed.)

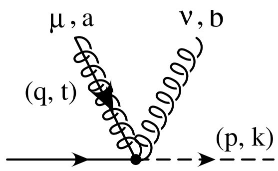
D.1 Feynman rules for J_{hl}

Here we give Feynman rules for the $\mathcal{O}(\lambda)$ heavy-to-light currents $J^{(1a)}$ and $J^{(1b)}$ in Eq. (**??**) which are valid in a frame where $v_\perp = 0$ and $v\cdot n = 1$.

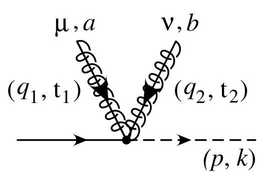
For the subleading currents the zero and one gluon Feynman rules for $J^{(1a)}$ and $J^{(1b)}$ are shown in Figs. 18 and 19 respectively. (From the results in the previous sections the Feynman rules for the currents



$$= \quad ig\,T^a \frac{\not{\bar n}}{2}\Big(n_\mu - \frac{\bar n_\mu\, n\cdot t}{\bar n\cdot q}\Big)$$

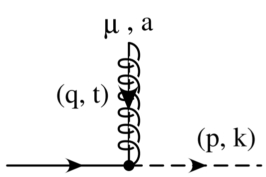


$$= \quad \frac{-g^2 f^{abc}T^c}{\bar n\cdot q}\,\frac{\not{\bar n}}{2}\,\bar n_\mu n_\nu$$

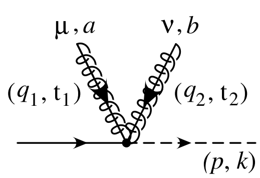


$$= \quad ig^2\,\frac{T^aT^b}{\bar n\cdot q_2}\Big[-n_\mu\bar n_\nu + \bar n_\mu\bar n_\nu\,\frac{n\cdot(t_1+t_2)}{\bar n\cdot p}\Big]\frac{\not{\bar n}}{2} + ig^2\,\frac{T^bT^a}{\bar n\cdot q_1}\Big[-n_\nu\bar n_\mu + \bar n_\mu\bar n_\nu\,\frac{n\cdot(t_1+t_2)}{\bar n\cdot p}\Big]\frac{\not{\bar n}}{2}$$

Figure 16: Feynman rules for the $O(\lambda^2)$ usoft-collinear Lagrangian $\mathcal{L}^{(2a)}_{\xi q}$ with one and two gluons. The spring without a line through it is an usoft gluon. For the collinear particles we show their (label,residual) momenta, where label momenta are $p, q, q_i \sim \lambda^{0,1}$ and residual momenta are $k, t, t_i \sim \lambda^2$. Note that the result is after the field redefinition made in Ref. [**?**].



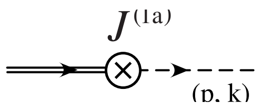
$$= \quad ig\,\frac{T^a}{\bar n\cdot q}\,\frac{\not{\bar n}}{2}\Big[\not{q}_\perp\gamma^\perp_\mu - \bar n_\mu\,\frac{q_\perp^2}{\bar n\cdot q}\Big]$$



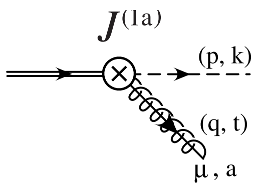
$$= \quad ig^2\,\frac{T^aT^b}{\bar n\cdot q_2}\,\frac{\not{\bar n}}{2}\Big[\gamma^\perp_\mu\gamma^\perp_\nu - \frac{\not{p}_\perp}{\bar n\cdot p}(\gamma^\perp_\mu\bar n_\nu + \gamma^\perp_\nu\bar n_\mu) - \frac{\gamma^\perp_\mu\,\bar n_\nu\,\not{q}_{2\perp}}{\bar n\cdot q_2} + \bar n_\mu\bar n_\nu\Big(\frac{p_\perp^2}{(\bar n\cdot p)^2} + \frac{\not{p}_\perp\,\not{q}_{2\perp}}{\bar n\cdot p\,\bar n\cdot q_2}\Big)\Big] + \big[(a,\mu,q_1,t_1)\leftrightarrow(b,\nu,q_2,t_2)\big]$$

Figure 17: Feynman rules for the $O(\lambda^2)$ usoft-collinear Lagrangian $\mathcal{L}^{(2b)}_{\xi q}$ with one and two gluons. For the collinear particles we show their (label,residual) momenta, where label momenta are $p, q, q_i \sim \lambda^{0,1}$ and residual momenta are $k, t, t_i \sim \lambda^2$.

with $v_\perp \neq 0$ and $v\cdot n \neq 1$ can also be easily derived.) For $J^{(1a)}$ the Wilson coefficients depend only on the total λ^0 collinear momentum, while for $J^{(1a)}$ the coefficients depend on how the momentum is divided between the quark and gluons. The $J^{(1a)}$ current has non-vanishing Feynman rules with zero or one $A_n^\perp$ gluon and any number of $\bar n\cdot A_n$ gluons. The possible gluons that appear in the $J^{(1b)}$ currents are similar, but the current vanishes unless it has one or more collinear gluons present.

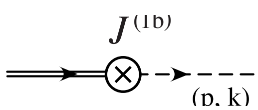


$$= \quad -i\, B_i^{(d)}(\bar n\cdot\hat p)\, \frac{p_\alpha^\perp\, \Upsilon_i^{(d)\alpha}}{\bar n\cdot p}$$

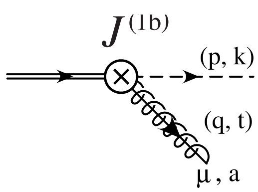


$$= \quad -i\, B_i^{(d)}\big(\bar n\cdot(\hat p+\hat q)\big)\, \frac{g\, T^a}{\bar n\cdot(p+q)} \left[\Upsilon_i^{(d)\mu} + \frac{\bar n^\mu\, p_\alpha^\perp \Upsilon_i^{(d)\alpha}}{\bar n\cdot q}\right]$$

Figure 18: Feynman rules for the $O(\lambda)$ currents $J^{(1a)}$ in Eq. (**??**) with zero and one gluon (the fermion spinors are suppressed). For the collinear particles we show their (label,residual) momenta, where label momenta are $p, q \sim \lambda^{0,1}$ and residual momenta are $k, t \sim \lambda^2$. Momenta with a hat are normalized to m_b, $\hat p = p/m_b$ etc.



$$= \quad 0$$



$$= \quad i\, B_i^{(d)}\ \ \bar n\cdot\hat p, \bar n\cdot\hat q\big)\, \frac{g\, T^a}{m_b} \left[\Theta_i^{(d)\mu} - \frac{\bar n^\mu\, q_\alpha^\perp \Theta_i^{(d)\alpha}}{\bar n\cdot q}\right]$$

Figure 19: Feynman rules for the $O(\lambda)$ currents $J^{(1b)}$ in Eq. (**??**) with zero and one gluon. For the collinear particles we show their (label,residual) momenta, where label momenta are $p, q, q_i \sim \lambda^{0,1}$ and residual momenta are $k, t \sim \lambda^2$. Momenta with a hat are normalized to m_b, $\hat p = p/m_b$ etc.

E Integral Tricks

Feynman parameter tricks:

$$
\begin{aligned}
a^{-1}\,b^{-1} &= \int_0^1 dx\ \big[a+(b-a)x\big]^{-2} \\
a^{-n}\,b^{-m} &= \frac{\Gamma(n+m)}{\Gamma(n)\Gamma(m)} \int_0^1 dx\ \frac{x^{n-1}(1-x)^{m-1}}{[a+(b-a)x]^{n+m}} \\
a^{-1}\,b^{-1}\,c^{-1} &= 2\int_0^1 dx \int_0^{1-x} dy\ \big[c+(a-c)x+(b-c)y\big]^{-3} \\
&= 2\int_0^1 dx \int_0^1 dy\ x\big[a+(c-a)x+(b-c)xy\big]^{-3} \\
a_1^{-1}\cdots a_n^{-1} &= (n-1)! \int_0^1 dx_1\cdots dx_n\ \delta\Big(\sum x_i - 1\Big)\ \Big(\sum x_i a_i\Big)^{-n} \\
(a_1^{m_1}\cdots a_n^{m_n})^{-1} &= \frac{\Gamma(\sum m_i)}{\Gamma(m_1)\cdots\Gamma(m_n)} \int_0^1 dx_1\cdots dx_n\ \delta\Big(\sum x_i - 1\Big)\ \Big(\sum x_i a_i\Big)^{-n} \prod x_i^{m_i-1}
\end{aligned}
\tag{E.1}
$$

To get the fourth line from the third we let $x' = 1-x$ and $y' = y/x$.

Georgi parameter tricks (when one or more propagators are linear in loop momenta):

$$
\begin{aligned}
a^{-1}\,b^{-1} &= \int_0^\infty d\lambda\ \big[a+b\lambda\big]^{-2} \\
a^{-q}\,b^{-1} &= q\int_0^\infty d\lambda\ \big[a+b\lambda\big]^{-(q+1)} = 2q\int_0^\infty d\lambda\ \big[a+2b\lambda\big]^{-(q+1)} \\
a^{-q}\,b^{-p} &= \frac{2^p\,\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \int_0^\infty d\lambda\ \lambda^{p-1}\big[a+2b\lambda\big]^{-(p+q)} \\
a^{-1}\,b^{-1}\,c^{-1} &= 2\int_0^\infty d\lambda\,d\lambda'\ \big[c+a\lambda'+b\lambda\big]^{-3} = 8\int_0^\infty d\lambda\,d\lambda'\ \big[c+2a\lambda'+2b\lambda\big]^{-3}
\end{aligned}
\tag{E.2}
$$

F QCD Summary

The $SU(N_c)$ QCD Lagrangian without gauge fixing

$$\mathcal{L} = \bar\psi(i\not{D} - m)\psi - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A}\,, \qquad G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - gf^{ABC}A^B_\mu A^C_\nu \tag{F.1}$$
$$D_\mu = \partial_\mu + igA^A_\mu T^A\,, \qquad [D_\mu, D_\nu] = igG^A_{\mu\nu}T^A\,.$$

The equations of motion and Bianchi

$$(i\not{D} - m)\psi = 0\,, \qquad \partial^\mu G^A_{\mu\nu} = gf^{ABC}A^{B\mu}G^C_{\mu\nu} + g\bar\psi\gamma_\nu T^A\psi\,, \qquad \epsilon^{\mu\nu\lambda\sigma}(D_\nu G_{\lambda\sigma})^A = 0. \tag{F.2}$$

Color identites

$$[T^A, T^B] = if^{ABC}T^C\,, \qquad \mathrm{Tr}\big[T^A T^B\big] = T_F\delta^{AB}\,, \qquad \bar T^A = -T^{A*} = -(T^A)^T\,,$$
$$T^A T^A = C_F\mathbf{1}\,, \qquad f^{ACD}f^{BCD} = C_A\delta^{AB}\,, \qquad f^{ABC}T^B T^C = \frac{i}{2}C_A T^A\,,$$
$$T^A T^B T^A = \Big(C_F - \frac{C_A}{2}\Big)T^B\,, \qquad d^{ABC}d^{ABC} = \frac{40}{3}\,, \qquad d^{ABC}d^{A'BC} = \frac{5}{3}\delta^{AA'}\,, \tag{F.3}$$

where $C_F = (N_c^2 - 1)/(2N_c)$, $C_A = N_c$, $T_F = 1/2$, and $C_F - C_A/2 = -1/(2N_c)$. The color reduction formula and Fierz formula are

$$T^A T^B = \frac{\delta^{AB}}{2N_c}\mathbf{1} + \frac{1}{2}d^{ABC}T^C + \frac{i}{2}f^{ABC}T^C\,, \qquad (T^A)_{ij}(T^A)_{k\ell} = \frac{1}{2}\,\delta_{i\ell}\delta_{kj} - \frac{1}{2N_c}\,\delta_{ij}\delta_{k\ell}\,. \tag{F.4}$$

Feynman gauge rules, fermion, gluon, ghost propagators, and Fermion-gluon vertex

$$\frac{i(\not{p} + m)}{p^2 - m^2 + i0}\,, \qquad \frac{-ig^{\mu\nu}\delta^{AB}}{k^2 + i0}\,, \qquad \frac{i}{k^2 + i0}\,, \qquad -ig\gamma^\mu T^A\,. \tag{F.5}$$

Triple gluon and Ghost Feynman rules in covariant gauge for $\{A^A_\mu(k), A^B_\nu(p), A^C_\rho(q)\}$ all with incoming momenta, and $\bar c^A(p)A^B_\mu c^C$ with outgoing momenta p:

$$-\,gf^{ABC}\big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\big]\,, \qquad gf^{ABC}p^\mu\,. \tag{F.6}$$

Triple gluon Feynman rule in bkgnd Field covariant gauge $\mathcal{L}_{gf} = -(D^A_\mu Q^A_\mu)^2/(2\xi)$ for $\{A^A_\mu(k), Q^B_\nu(p), Q^C_\rho(q)\}$ with A^A_μ a bkgnd field:

$$-\,gf^{ABC}\Big[g^{\mu\nu}\Big(k - p - \frac{q}{\xi}\Big)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}\Big(q - k + \frac{p}{\xi}\Big)^\nu\Big]\,. \tag{F.7}$$

Lorentz gauge:

$$\mathcal{L} = -\frac{(\partial_\mu A^\mu)^2}{2\xi}\,, \qquad D^{\mu\nu}(k) = \frac{-i}{k^2 + i0}\Big(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\Big)\,, \tag{F.8}$$

where Landau gauge is $\xi \to 0$. Coulomb gauge:

$$\vec\nabla\cdot\vec A = 0\,, \qquad D^{\mu\nu}(k) = \frac{-i}{k^2 + i0}\left(g^{\mu\nu} - \frac{[g^{\nu 0}k^0k^\mu + g^{\mu 0}k^0k^\nu - k^\mu k^\nu]}{\vec k^2}\right),$$
$$D^{00}(k) = \frac{i}{\vec k^2 - i0}\,, \qquad D^{ij}(k) = \frac{i}{k^2 + i0}\Big(\delta^{ij} - \frac{k^i k^j}{\vec k^2}\Big). \tag{F.9}$$

Running coupling with $\beta_0 = 11C_A/3 - 4T_F n_f/3 = 11 - 2n_f/3$:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{2\pi}\alpha_s(\mu_0)\ln\frac{\mu}{\mu_0}} = \frac{2\pi}{\beta_0\ln\frac{\mu}{\Lambda_{\rm QCD}}}\,, \qquad \frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + \frac{\beta_0}{2\pi}\ln\frac{\mu}{\mu_0}\,. \tag{F.10}$$

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