8.811 Particle Physics
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Fall, 2004

Assignment 1,
Due in class at 11:00AM, Sept. 28

1. Prob. 2.3 in Q&L.

2. Prob. 2.6 in Q&L. Save the rotation matrices for computing angular distributions of various cross sections later.

3. Use the symmetric spin-flavor wave functions of the ground state baryons to find the ratio of the magnetic dipole moments of a proton and a neutron.

4. Prob. 2.21 in Q&L.

5. Prob. 2.25 in Q&L.

6. Prob. 2.27 in Q&L.

2.3 Use isospin invariance to show that the reaction cross sections σ must satisfy:

$$\frac{\sigma(pp \to \pi^+ d)}{\sigma(np \to \pi^0 d)} = 2$$

given that the deuteron, d, has isospin $I = 0$ & the π has isospin $I = 1$

Hint: You may assume that the reaction rate is:

$$\sigma \sim |\text{amplitude}|^2 \sim \sum_I |\langle I', I_3' | A | I, I_3 \rangle|^2$$

where I & I' are the total isospin quantum #'s of the initial & final states, respectively, & $I = I'$ & $I_3 = I_3'$.

The initial states:

$$|pp\rangle = |I=1, I_3=1\rangle$$

$$|np\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}} |1,0\rangle - \tfrac{1}{\sqrt{2}} |0,0\rangle \qquad \text{by C.G. table}$$

The final states:

individually:

$$|\pi^+\rangle = |I=1, I_3=1\rangle$$
$$|d\rangle = |0,0\rangle$$
$$|\pi^0\rangle = |1, 0\rangle$$

so the final states:

$$|\pi^+ d\rangle = |1, 1\rangle$$

$$|\pi^0 d\rangle = |1, 0\rangle$$

Thus the cross sections:

$$\sigma(pp \to \pi^+ d) \sim |\langle \pi^+ d | A | pp \rangle|^2 = |\langle 1, 1 | A | 1, 1 \rangle|^2$$

$$\sigma(np \to \pi^0 d) \sim \tfrac{1}{2} |\langle 1,0 | A | 1,0 \rangle|^2 + \tfrac{1}{2} |\underbrace{\langle 1,0 | A | 0,0 \rangle}_{0}|^2$$

And, since this is a nuclear reaction, the rate is indep. of I_3, so

$$|\langle 1,1 | A | 1,1 \rangle|^2 = |\langle 1,0 | A | 1,0 \rangle|^2 \equiv A_1$$

Thus our ratio:

$$\frac{\sigma(pp \to \pi^+ d)}{\sigma(np \to \pi^0 d)} = \frac{A_1}{\tfrac{1}{2} A_1} = 2 \qquad \blacksquare$$

Problem 2 Q&L: 2.6

$$d^j_{m'm} = \langle jm' | e^{-i\vartheta \frac{1}{2}\cdot\sigma_2} | jm \rangle$$

$j = \frac{1}{2} \Rightarrow \qquad |\frac{1}{2}, +\frac{1}{2}\rangle = |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\qquad\qquad |\frac{1}{2}, -\frac{1}{2}\rangle = |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_2^{\,2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$$e^{-i\frac{\vartheta}{2}\sigma_2} = I + \left(-i\frac{\vartheta}{2}\sigma_2\right) + \frac{1}{2!}\left(-i\frac{\vartheta}{2}\sigma_2\right)^2 + \frac{1}{3!}\left(-i\frac{\vartheta}{2}\sigma_2\right)^3 + \frac{1}{4!}\left(-i\frac{\vartheta}{2}\sigma_2\right)^4 + \cdots$$

$$= I - i\frac{\vartheta}{2}\sigma_2 + \frac{-1}{2!}\left(\frac{\vartheta}{2}\right)^2 \cdot I + \frac{+i}{3!}\left(\frac{\vartheta}{2}\right)^3\sigma_2 + \frac{1}{4!}\left(\frac{\vartheta}{2}\right)^4 I + \cdots$$

$$= \left(1 - \frac{1}{2!}\left(\frac{\vartheta}{2}\right)^2 + \frac{1}{4!}\left(\frac{\vartheta}{2}\right)^4 - \frac{1}{6!}\left(\frac{\vartheta}{2}\right)^6 + \cdots\right) I - i\left(\frac{\vartheta}{2}\sigma_2 - \frac{1}{3!}\left(\frac{\vartheta}{2}\right)^3\sigma_2 + \frac{1}{5!}\left(\frac{\vartheta}{2}\right)^5\sigma_2 + \cdots\right)$$

$$= \cos\left(\frac{\vartheta}{2}\right) I - i\,\sigma_2 \cdot \sin\left(\frac{\vartheta}{2}\right)$$

$$= \begin{pmatrix} \cos\frac{\vartheta}{2} & -\sin\frac{\vartheta}{2} \\ \sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2} \end{pmatrix}$$

So, $d^{1/2}_{++} = d^{1/2}_{--} = \cos\frac{\vartheta}{2}$

and $d^{1/2}_{+-} = -d^{1/2}_{-+} = -\sin\frac{\vartheta}{2}$

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For $j = 1$ we have :

$$|1,+1\rangle = |e_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|1,0\rangle = |e_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|1,-1\rangle = |e_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

I'll find the components of the matrix representation of $\hat{J}_2$ in this basis.

$$\left.\begin{aligned} \hat{J}_+ &= \hat{J}_1 + i\hat{J}_2 \\ \hat{J}_- &= \hat{J}_1 - i\hat{J}_2 \end{aligned}\right\} \quad 2i\hat{J}_2 = \hat{J}_+ - \hat{J}_- \Rightarrow \quad \hat{J}_2 = \frac{1}{2}(-i)\left[\hat{J}_+ - \hat{J}_-\right]$$

$$\langle e_1|\hat{J}_2|e_1\rangle = 0 = \langle e_2|\hat{J}_2|e_2\rangle = \langle e_3|\hat{J}_2|e_3\rangle$$

$$\langle e_1|\hat{J}_2|e_2\rangle = \frac{1}{2}(-i)\cdot\left\{\langle e_1|\hat{J}_+|e_2\rangle - \langle e_1|\hat{J}_-|e_2\rangle\right\}$$

$$= -\frac{1}{2}i\cdot\hbar\sqrt{1(1+1)-0(0+1)} = -\frac{\hbar}{\sqrt{2}}i = \frac{-1}{\sqrt{2}}i \qquad (\hbar \equiv 1)$$

$$\langle e_1|\hat{J}_2|e_3\rangle = 0$$

$$\langle e_2|\hat{J}_2|e_3\rangle = \frac{-i}{2}\left\{\langle e_2|\hat{J}_+|e_3\rangle - \langle e_2|\hat{J}_-|e_3\rangle\right\}$$

$$= \frac{-i}{2}\sqrt{1(1+1)-(-1)(-1+1)} = -\frac{1}{\sqrt{2}}i$$

So, $J_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & -i \\ 0 & +i & 0 \end{pmatrix}$, $J_2^{\,2} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

$$J_2^{\,3} = \frac{1}{2^{3/2}}\begin{pmatrix} 0 & -2i & 0 \\ +2i & 0 & -2i \\ 0 & 2i & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ +i & 0 & -i \\ 0 & i & 0 \end{pmatrix} = J_2$$

$$e^{-i\vartheta J_2} = I - i\vartheta J_2 + \frac{-1}{2!}\vartheta^2 J_2^{\,2} + \frac{i}{3!}\vartheta^3 J_2 + \frac{+1}{4!}\vartheta^4 J_2^{\,2} + \frac{-i}{5!}\vartheta^5 J_2 + \cdots$$

$$= \left(I - \frac{1}{2!}\vartheta^2 J_2^{\,2} + \frac{1}{4!}\vartheta^4 J_2^{\,2} - \frac{1}{6!}\vartheta^6 \cdot J_2^{\,2} + \cdots\right) + (-i) J_2 \left(\vartheta - \frac{\vartheta^3}{3!} + \frac{\vartheta^5}{5!} - \frac{\vartheta^7}{7!} + \cdots\right)$$

$$= \begin{pmatrix} \left(1 - \frac{1}{2}\frac{1}{2!}\vartheta^2 + \frac{1}{2}\frac{1}{4!}\vartheta^4 \cdots\right) & 0 & -(1-1)\frac{1}{2} + \frac{1}{2}\frac{1}{2!}\vartheta^2 - \frac{1}{2}\frac{1}{4!}\vartheta^4 + \cdots \\ 0 & 1 - \frac{1}{2!}\vartheta^2 + \frac{1}{4!}\vartheta^4 - \cdots & 0 \\ -\frac{(-1+1)}{2} + \frac{1}{2}\frac{1}{2!}\vartheta^2 - \frac{1}{2}\frac{1}{4!}\vartheta^4 \cdots & 0 & 1 - \frac{1}{2}\frac{1}{2!}\vartheta^2 + \frac{1}{2}\frac{1}{4!}\vartheta^4 \cdots \end{pmatrix} - i \cdot \sin\vartheta \cdot J_2$$

$$= \begin{pmatrix} \frac{1}{2}\cos\vartheta + \frac{1}{2} & 0 & -(-1+\cos\vartheta)\frac{1}{2} \\ 0 & \cos\vartheta & 0 \\ -(-1+\cos\vartheta)\frac{1}{2} & 0 & \frac{1}{2}\cdot\cos\vartheta + \frac{1}{2} \end{pmatrix} - \sin\vartheta \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 0 & +1 & 0 \\ -1 & 0 & +1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} + \frac{1}{2}\cos\vartheta & -\frac{1}{\sqrt{2}}\sin\vartheta & -(-1+\cos\vartheta)\frac{1}{2} \\ +\frac{1}{\sqrt{2}}\sin\vartheta & \cos\vartheta & -\frac{1}{\sqrt{2}}\sin\vartheta \\ -(-1+\cos\vartheta)\frac{1}{2} & +\frac{1}{\sqrt{2}}\sin\vartheta & \frac{1}{2} + \frac{1}{2}\cos\vartheta \end{pmatrix}$$

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So, $d^1_{11} = d^1_{-1-1} = \frac{1}{2}(1+\cos\vartheta)$, $d^1_{00} = \cos\vartheta$

$d^1_{10} = -d^1_{-10} = -d^1_{01} = +d^1_{0-1} = \frac{-1}{\sqrt{2}}\sin\vartheta$, $d^1_{1-1} = d^1_{-11} = \frac{1}{2}(-\cos\vartheta + 1)$

Problem 3

We are going to find $\frac{\mu_p}{\mu_n}$, where, for the $|p\uparrow\rangle$:

$$\mu_p = \langle p\uparrow | \, Q_1 \frac{e}{2m} (\sigma_z)_1 + Q_2 \frac{e}{2m} (\sigma_z)_2 + Q_3 \frac{e}{2m} (\sigma_z)_3 \, | p\uparrow \rangle$$

$$\mu_p = \frac{e}{2m} \cdot \langle p\uparrow | \, Q_1 (\sigma_z)_1 + Q_2 (\sigma_z)_2 + Q_3 (\sigma_z)_3 \, | p\uparrow \rangle$$

where $|p\uparrow\rangle = \frac{1}{\sqrt{2}} \Big(\sqrt{\frac{1}{6}} (udu + duu - 2uud) \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)$

$+ \frac{1}{\sqrt{2}} (udu - duu) \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \Big) \Rightarrow$

$$|p\uparrow\rangle = \frac{3}{\sqrt{18}} \Big\{ \frac{1}{6} (u\uparrow d\downarrow u\uparrow + u\downarrow d\uparrow u\uparrow - 2u\uparrow d\uparrow u\downarrow + d\uparrow u\downarrow u\uparrow + d\downarrow u\uparrow u\uparrow$$
$$- 2d\uparrow u\uparrow u\downarrow - 2u\uparrow u\downarrow d\uparrow - 2u\downarrow u\uparrow d\uparrow + 4u\uparrow u\uparrow d\downarrow) +$$
$$+ \frac{1}{2} (u\uparrow d\downarrow u\uparrow - u\downarrow d\uparrow u\uparrow - d\uparrow u\downarrow u\uparrow + d\downarrow u\uparrow u\uparrow) \Big\} \Rightarrow$$

$$|p\uparrow\rangle = -\frac{1}{\sqrt{18}} \Big\{ u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + u\uparrow d\uparrow u\downarrow + u\downarrow d\uparrow u\uparrow$$
$$- 2u\uparrow d\downarrow u\uparrow + d\uparrow u\downarrow u\uparrow + d\uparrow u\uparrow u\downarrow - 2d\downarrow u\uparrow u\uparrow \Big\}$$

By interchanging $d \leftrightarrow u$ we get

$$|n\uparrow\rangle = -\frac{1}{\sqrt{18}} \Big\{ d\uparrow d\downarrow u\uparrow + d\downarrow d\uparrow u\uparrow - 2d\uparrow d\uparrow u\downarrow + d\uparrow u\uparrow d\downarrow + d\downarrow u\uparrow d\uparrow$$
$$- 2d\uparrow u\downarrow d\uparrow + u\uparrow d\downarrow d\uparrow + u\uparrow d\uparrow d\downarrow - 2u\downarrow d\uparrow d\uparrow \Big\}$$

So, $$\mu_p = \frac{e}{2m}\,\frac{1}{18}\cdot\Big[\left(\tfrac{2}{3}-\tfrac{2}{3}-\tfrac{1}{3}\right)+\left(-\tfrac{2}{3}+\tfrac{2}{3}-\tfrac{1}{3}\right)+4\left(\tfrac{2}{3}+\tfrac{2}{3}+\tfrac{1}{3}\right)$$
$$+\left(\tfrac{2}{3}-\tfrac{1}{3}-\tfrac{2}{3}\right)+\left(-\tfrac{1}{3}\right)+4\left(\tfrac{4}{3}+\tfrac{1}{3}\right)$$
$$+\left(-\tfrac{1}{3}\right)+\left(-\tfrac{1}{3}\right)+4\left(\tfrac{1}{3}+\tfrac{4}{3}\right)\Big] \Rightarrow$$

$$\mu_p = \frac{e}{2m}\left[\frac{1}{18}\right]\left[12\left(2\tfrac{2}{3}+\tfrac{1}{3}\right)+6\left(-\tfrac{1}{3}\right)\right]$$
$$= \frac{e}{2m}\,\frac{1}{18}\cdot\left(12\cdot\frac{5}{3}+(-2)\right) = \frac{e}{2m}$$

to find μ_n, I can just put $\frac{2}{3}$ wherever I have $-\frac{1}{3}$ and $-\frac{1}{3}$ wherever I have $\frac{2}{3}$:

$$\mu_n = \frac{e}{2m}\left(\frac{1}{18}\right)\left(12\left(-2\tfrac{1}{3}-\tfrac{2}{3}\right)+6\left(\tfrac{2}{3}\right)\right)$$
$$= \frac{e}{2m}\,\frac{1}{18}\,(-16+4) = \frac{e}{2m}\,\frac{-12}{18} = \frac{e}{2m}\,\frac{-2}{3}$$

so, $\frac{\mu_p}{\mu_n} = -\frac{3}{2}$, $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$

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3] Use the symmetric spin-flavour wave fcns of the gnd state baryons to find the ratio of the magnetic dipole moments of a proton & a neutron

Magnetic moment, for pt. particle i:

$$\mu_i = Q_i \left(\frac{e}{2m_i}\right)$$

assume:
$m_u = m_d$

where:
$Q_u = +\frac{2}{3}$, $Q_d = -\frac{1}{3}$

We want the ratio:

$$\frac{\mu_p}{\mu_n} = \frac{\sum_{i=1}^{3} \langle p\uparrow | \mu_i (\sigma_3)_i | p\uparrow \rangle}{\sum_{i=1}^{3} \langle n\uparrow | \mu_i (\sigma_3)_i | n\uparrow \rangle}$$

{where we're summing over the 3 quarks in the baryon.}

The wavefcns:

$$|p\uparrow\rangle = \sqrt{\tfrac{1}{18}}\left(u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{cyclic perm.}\right)$$

(from: eq. 2.71, p.54 of text)

for the neutron, we just swap the u & d quarks & get:

$$|n\uparrow\rangle = \sqrt{\tfrac{1}{18}}\left(d\uparrow d\downarrow u\uparrow + d\downarrow d\uparrow u\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{cyclic}\right)$$

Thus the magnetic moments...

$$\mu_p = \tfrac{1}{18}\left[(\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d) + 4(\mu_u + \mu_u - \mu_d)\right] \times \overset{\text{permutations}}{3}$$

$$= \tfrac{1}{6}\left[8\mu_u - 2\mu_d\right] = \tfrac{1}{3}(4\mu_u - \mu_d)$$

$$\mu_n = \tfrac{1}{18}\left[(\mu_d - \mu_d + \mu_u) + (-\mu_d + \mu_d + \mu_u) + 4(2\mu_d - \mu_u)\right] \times 3$$

$$= \tfrac{1}{6}\left[8\mu_d - 2\mu_u\right] = \tfrac{1}{3}(4\mu_d - \mu_u)$$

with $m_u = m_d$, $\mu_u = -2\mu_d$ thus:

$$\frac{\mu_p}{\mu_n} = \frac{4(-2\mu_d) - \mu_d}{4\mu_d + 2\mu_d} = \frac{-9\mu_d}{6\mu_d} = \boxed{-\frac{3}{2} = \frac{\mu_p}{\mu_n}}$$

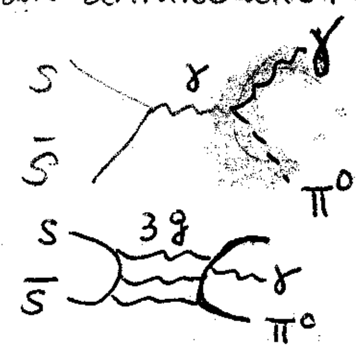
2.21 Assuming (2.54):

$$\phi \approx s\bar{s} \qquad \omega = \sqrt{\tfrac{1}{2}}\,(u\bar{u} + d\bar{d})$$

show that the quark model forbids the decay $\phi \to \pi^0 \gamma$ and predicts that

$$\frac{\text{Rate}(\omega \to \pi^0 \gamma)}{\text{Rate}(\rho \to \pi^0 \gamma)} = \left(\frac{\mu_d - \mu_u}{\mu_d + \mu_u}\right)^2 = 9$$

The decay $\phi \to \pi^0 \gamma$ is forbidden b/c it must occur through an annihilation process: $\phi \to 1$ gluon is forbidden because color



if you then pop 2 quarks (i.e. $u\bar{u}$), then they have some momentum, so they go flying apart, emiting a γ.

The ω & ρ decays into $\pi^0\gamma$ require a quark spin flip (ω, ρ have $S = 1$; π^0 has $S = 0$) & hence involve the quark magnetic moment operator.

The spin-flavour wavefcns for the particles in question:

$$\omega = \tfrac{1}{\sqrt{2}}(u\uparrow\bar{u}\uparrow + d\uparrow\bar{d}\uparrow) \qquad (\text{in the } M_J = +1 \text{ state})$$

$$\rho = \tfrac{1}{\sqrt{2}}(u\uparrow\bar{u}\uparrow - d\uparrow\bar{d}\uparrow) \qquad (\quad '' \quad)$$

$$\pi^0 = \tfrac{1}{2}(u\uparrow\bar{u}\downarrow - u\downarrow\bar{u}\uparrow - d\uparrow\bar{d}\downarrow + d\downarrow\bar{d}\uparrow)$$

Using the information from EX 2.20:

Amplitude$(\omega \to \pi^0\gamma) =$

$$\sum_{i=1,2} \langle \pi^0 | \mu_i \vec{\sigma}_i \cdot \vec{\varepsilon}_R^{\,*} | \omega(M_J = +1) \rangle = -\sqrt{2} \sum_{i=1,2} \langle \pi^0 | \mu_i (\sigma_-)_i | \omega(M_J = 1) \rangle$$

where: $\vec{\varepsilon}_R = \frac{1}{\sqrt{2}}(1, i, 0)$ is the polarization vector for the emitted (helicity $= 1$ γ) & $\sigma_- = \frac{1}{2}(\sigma_1 - i\sigma_2)$ is the "step down" operator, which "flips" the quark spin.

2.21, cont

For the $\omega \to \pi^0 \gamma$ transition:

$$-\sqrt{2}\sum_{i=1,2} \langle \pi^0 | \mu_i (\sigma_-)_i | \omega(M_J = 1) \rangle =$$

$\omega \to \pi^0$

the transitions

$$u\uparrow\bar{u}\uparrow \to u\uparrow\bar{u}\downarrow \Rightarrow -\mu_u$$
$$u\uparrow\bar{u}\uparrow \to -u\downarrow\bar{u}\uparrow \Rightarrow -\mu_u$$

$$d\uparrow\bar{d}\uparrow \to -d\uparrow\bar{d}\downarrow \Rightarrow \mu_d$$
$$d\uparrow\bar{d}\uparrow \to +d\downarrow\bar{d}\uparrow \to \mu_d$$

put together w/ the normalization factors:

$$(-\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)(-2\mu_u + 2\mu_d) = \mu_d - \mu_u$$

from the π^0 wavefcn

from the ω

For the $\rho^0 \to \pi^0 \gamma$: (the $u\bar{u}$ part is the same)

$$-d\uparrow\bar{d}\uparrow \to -d\uparrow\bar{d}\downarrow \Rightarrow -\mu_d$$
$$-d\uparrow\bar{d}\uparrow \to +d\downarrow\bar{d}\uparrow \Rightarrow -\mu_d$$

$$-2\mu_u - 2\mu_d$$

with the normalization:

$$\text{Amplitude}(\rho \to \pi^0 \gamma) = \mu_u + \mu_d$$

Thus comparing the rates (Amplitude^2):

$$\frac{\text{Rate}(\omega \to \pi^0\gamma)}{\text{Rate}(\rho \to \pi^0\gamma)} = \left(\frac{\mu_u - \mu_d}{\mu_u + \mu_d}\right)^2 = 9$$

where I used: $\mu_i = \frac{Q_i e}{2m_i}$ with the assumtion that $m_u = m_d$.

Thus: $\mu_u = -2\mu_d$

$$\Rightarrow \left(\frac{\mu_u - \mu_d}{\mu_u + \mu_d}\right)^2 = \left(\frac{-2\mu_d - \mu_d}{-2\mu_d + \mu_d}\right)^2 = \left(\frac{-3\mu_d}{\mu_d}\right)^2 = 9$$

2.25 the leptonic decay of neutral vector ($J^{PC} = 1^{--}$) mesons can be pictured as proceeding via a virtual γ

$$V(q\bar{q}) \rightarrow \gamma \rightarrow e^+e^-$$

The V-γ coupling $\propto$ the charge of the quark, q. Neglecting possible dependence on the meson mass, show that the leptonic decay widths are in the ratios:

$$\rho : \omega : \phi : \psi = 9 : 1 : 2 : 8.$$

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the quark compositions:

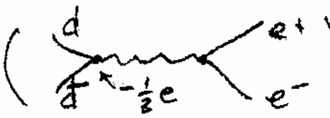
$$\rho = \tfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad \phi = s\bar{s}$$

$$\omega = \tfrac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad \psi = c\bar{c}$$

The amplitudes...

A(

) $= \frac{2}{3}\not{c} \equiv A_u$ — some constant

A(

) $= -\frac{1}{3}\not{c} \equiv A_d$

Give us the amplitudes:

$$A(\rho \to e^+e^-) = \frac{1}{\sqrt{2}}(A_u - A_d) = \frac{1}{\sqrt{2}}\not{c}\left(\frac{3}{3}\right)$$

$$A(\omega \to e^+e^-) = \frac{1}{\sqrt{2}}(A_u + A_d) = \frac{1}{\sqrt{2}}\not{c}\left(\frac{1}{3}\right)$$

$$A(\phi \to e^+e^-) = A_d = -\frac{1}{3}\not{c}$$

$$A(\psi \to e^+e^-) = A_u = \frac{2}{3}\not{c}$$

The decay rates $R = |A|^2$:

$$R(\rho \to e^+e^-) = \frac{1}{2}|\not{c}|^2 \qquad R(\omega \to e^+e^-) = \frac{1}{18}|\not{c}|^2$$

$$R(\phi \to e^+e^-) = \frac{1}{9}|\not{c}|^2 \qquad R(\psi \to e^+e^-) = \frac{4}{9}|\not{c}|^2$$

Thus the ratios: $\rho : \omega : \phi : \psi = 9 : 1 : 2 : 8$

2.27 The hadronic decay widths of η_c & $\psi(3.1)$ are estimated using:

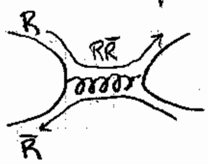
$$\eta_c\,(c\bar{c}) \longrightarrow ng \longrightarrow \text{hadrons}$$

$$\psi\,(c\bar{c}) \longrightarrow n'g \longrightarrow \text{hadrons}$$

where g is a gluon & n, n' are integers. Show that the minimum values of $n=2$ & $n'=3$.

The 8 gluons are: $R\bar{G}$, $G\bar{R}$, $R\bar{B}$, $B\bar{R}$, $G\bar{B}$, $B\bar{G}$, $\frac{R\bar{R}+G\bar{G}-2B\bar{B}}{\sqrt{6}}$, $\frac{R\bar{R}-G\bar{G}}{\sqrt{2}}$

Neither the η_c nor the ψ can decay via a single g b/c



cancels w/ the $B\bar{B}$ & $G\bar{G}$ states b/c amplitude:

$R\bar{R} \propto \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}$
$B\bar{B} \propto -2/\sqrt{6}$
$G\bar{G} \propto \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}$ add 'em up $= 0$

What about $2g$'s?
The quantum #s of our states (J^{PC}): $\eta_c = 0^{-+}$, $\psi = 1^{--}$

If we think of g's as "coloured γ's", then we have $C(\text{one } g) = -1$
$\Rightarrow C(2g\text{'s}) = +1$

thus η_c with $J^{PC} = 0^{-+}$ can decay via $2g$'s
but ψ with 1^{--} cannot b/c it must have $C = -1$

$3g$'s is an allowed decay for $\psi\,(1^{--})$ b/c $3g$'s has C-parity of -1, which is the same as the ψ.

$\Rightarrow n = 2$, $n' = 3$