8.512 Theory of Solids II
Spring 2009

Lecture 8: Mott's Variable Range Hopping

Mott variable range hopping theory describes the low temperature behavior of the resistivity in strongly disordered systems where states are localized. Consider two states located a distance R apart. The state on the left is at energy E_1 and the state on the right is at energy E_2. Suppose $W = E_2 - E_1 > 0$. An electron can hop from left to right by the absorption of a phonon with energy ω. The hopping rate is given by

$$\frac{1}{\tau_R} = \omega_0 e^{-2R/\xi} f_1(1-f_2)n(W), \tag{1}$$

where ω_0 is the attempt frequency which is given by a typical phonon frequency, ξ is the localization length, $n(W)$ is the Bose factor, and $f_i = \frac{1}{e^{(E_i-\mu_i)/kT}+1}$ is the Fermi factor for state $i = 1, 2$ with chemical potential μ_1 and μ_2. The hopping rate for right to left is

$$\frac{1}{\tau_L} = \omega_0 e^{-2R/\xi} f_2(1-f_1)(n(W)+1) \tag{2}$$

by the emission of phonons.

The current is

$$I = eR\left(\frac{1}{\tau_R} - \frac{1}{\tau_L}\right). \tag{3}$$

Expanding to linear or in $\mu_2 - \mu_1$, we find

$$I = \frac{eR\omega_0}{kT} e^{-W/kT} e^{-2R/\xi}(\mu_1 - \mu_2). \tag{4}$$

Setting $\mu_1 - \mu_2 = eV$ we find the expression for the conductance to be

$$G = \frac{e^2 R\omega_0}{kT}\left\{e^{-W/kT} e^{-2R/\xi}\right\}. \tag{5}$$

The key insight of Mott is that G should be determined by optimizing the competition between the overlap term $e^{-2R/\xi}$, which favors short hops, and the energy activation $e^{-W/kT}$, which favors long hops. With longer hops, one has a better chance of reducing the activation energy W. We estimate W by the typical energy level spacing in volume with radius R

$$W = \frac{3}{4\pi R^3 N(0)} \tag{6}$$

where $N(0)$ is the density of states at the Fermi energy. Maximizing the $\{\quad\}$ in Eq.(5) using Eq.(6), we find

$$\bar{R}^4 = \frac{9\xi}{4\pi N(0)kT} \tag{7}$$

and the conductance

$$G \approx e^{-\left(\frac{T_0}{T}\right)^{1/4}} \tag{8}$$

where

$$kT_0 = \frac{1}{N(0)\xi^3} \tag{9}$$

is the average energy level spacing in a volume ξ^3. Equation (8) is the famous Mott variable hopping law. It is observed in doped semiconductors at low temperatures.