PROFESSOR: So over the last lecture, we have talked about coherence within an atom, coherence between two levels, coherence between three levels. And today, in the last class, we want to talk about coherence between atoms.

So this is now, I think, for the first time in this course that we really have more than one atom. Well, maybe we discussed some collision or broadening, or we discussed [INAUDIBLE] interaction between two atoms. But usually, atomic physics is one atom at a time. But now we want to understand one important phenomenon which happens when we have many atoms.

And the phenomenon is called superradiance. So I left something good for the end. And superradiance has in common the word super with superconductivity and superfluidity. And it really represents that the atom, many atoms act together. And the word super also means coherence among atoms-- superfluidity and superconductivity have macroscopic wave function where all the atoms, the meta waves are coherent. The phenomenon of superradiance, as we will see, has not so much to do with coherent atoms. It has more to do with coherent photons. So it's more-- some people regard superradiance as a laser without mirrors. But you'll see where the story leads us to.

So just to set the stage-- for many atoms, we should first talk about single atoms. And all that is described in this landmark paper by [INAUDIBLE] in 1954, which is posted on our website. So if you have a single atom prepared in the excited state, it decays to the ground state, and we want to characterize the system by emission rate as a function of time. So the emission rate initially is gamma, the natural language of the excited state. And then, of course, the emission rate decays because we don't have any atoms left.

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Similarly, the probability to be in the ground state is zero initially, and then with an exponential approach, it eventually goes to unity after a while, after we have only atoms in the gamma state. So this is rather straightforward. But now we want to bring in a second atom. And I'm asking, what happens when we have not the one atom, but two atoms? One is in the ground state. One is excited.

So pretty much what we have added to the original situation with one excited atom was we've brought in one ground state atom, which naively you would think does nothing. But that's not the case. What happens is-- and I assume just for review-- we will drop the assumption later, but we assume for now that all the atoms are within one optical wavelength.

What we then realize is for two atoms-- and I will show you that in its full beauty-- that the initial rate of light which comes out of the system is the same. So the extra ground state atom does not change the initial emission rate, but it goes down faster. And if we ask what is the probability that the atom is in the ground state, we find that it's only one half. So in other words, normalized [INAUDIBLE] system, we have a ground and excited state atom, and what comes out is only half a photon. Half of the atoms do not decay.

So it's not the same rate and the same decay. Something profoundly has happened. And this is what you want to understand.

So let me give you the correct answer. The rate of emission is a function of time for this situation. We start out with gamma, but then the emission decays, not with gamma but with 2 gamma. And the probability that both atoms are in the ground state-- or that the second atom, so to speak, is in the ground state-- will only asymptotically go to 1/2. And it does so exponentially-- but again, with the time constant, which is two times faster than for the single-atom system. So we have the same initial emission rate, but only probability of 1/2 to emit at all.

So in order to understand it, we have to look at an atom in the excited state and atom in the ground state. And we want to write down the wave function as a

superposition of a symmetrized and antisymmetrized wave function. I should tell you, I'm going very slowly for two atoms. And then once I've introduced the concept for two atoms, with a few pen strokes, we can immediately discuss in atoms. So all the phsyics, all the understanding what goes on in superradiance is already displayed for two atoms.

So we want to have a superposition of symmetric and antisymmetric wave function. The symmetric one is a normalized wave function which is ge plus eg. And we call that the superradiant wave function, for reasons which will become clear in a moment. And if you have a minus sign here, the antisymmetric combination, we call this subradiant wave function.

Now, what happens is, we have to consider-- so we have symmetrized the wave function. Well, I didn't really tell you why, but it's always good to symmetrize. Symmetry is, if you can use it, something good. And the reason why I symmetrized it is because I want look at the interaction Hamiltonian.

And if I look at the interaction Hamiltonian-- the one we have seen many, many times but now for two atoms-- we will immediately realize that this interaction Hamiltonian is symmetric. So therefore, if the Hamiltonian is symmetric, it's a really good starting point to have the wave function for the atoms expanded in a symmetric basis.

And since I want to emphasize that the whole story I'm telling you today has nothing to do with the kind of second quantization-- it is about spontaneous emission, but it's not involving any subtlety of spontaneous emission and field quantization-- I want to write down the interaction Hamiltonian both in a classical and a quantum mechanical way.

In the classical way, we have the dipole moment d1. We have the dipole moment d2. And the atoms talk to the electric field at position RNT. And now you realize where some of the assumptions are important, since the atoms are localized to within a wavelength, they rarely talk to the same electric field. There are no phase factors. In about 55 minutes or so, we introduce phase factors for extended

samples. But for now, we don't.

And therefore, what the atoms couple with is with a dipole moment, which is a sum of the two dipole moments. So this is classical or semi-classical. So what enters in the Hamiltonian is only the sum of the operators for the two atoms. And the same happens in the QED Hamiltonian. And actually, I will get a little bit more mileage out of the QED Hamiltonian, as you will see in a moment.

Because with the QED Hamiltonian we describe the atomic system-- so at first atom one-- with the raising and lowering operator with the atoms interacting with a and a [INAUDIBLE]. And then I have to add the term where the index one and two are exchanged. So we are introducing here-- that's convenient for two-level atom-- the spin notation sigma plus and sigma minus are the raising and lowering operator which flip the atom from the ground to the excited state and vice versa.

But the important part now is-- and this is where, actually, everything comes from in superradiance-- that the coupling involves not the individual spins, little sigma plus, sigma 1 and sigma 2-- it only involves the sum of the individuals. i equals 122, and later we extend the sum to n. So therefore, what matters for the interaction of the atoms with the electromagnetic field is the sum of all the atomic spin operators. And the sum is, of course, symmetric against exchange.

So therefore, when we are asking what is the coupling of the state which I called the superradiant state, the one where we had symmetrized eg plus ge, or we ask, what is the coupling of the subradiant state to-- well, the state where both atoms are in the ground state. Well, now we can use symmetry. The left-hand side is symmetric. The operator is symmetric. And now only the symmetric state will couple. The antisymmetric state will not couple.

So therefore, the subradiant state, eg minus ge, cannot decay. That's why we call it subradiant. I think a better word would be non-radiant, but non is definitely subradiant. And for the matrix element eg and ge, we find that we have actually an enhancement of the coupling by a factor of square root 2.

So now we pretty much know what we have to do. You want to use the symmetry of-- let's assume we consider ground and excited state of each atom as spin 1/2. But now we want to look at the total spin, the total pseudo angular momentum of the two atoms, and later we extend it to n atoms. So we want to use now the power of the angular momentum description. And that goes like follows. We have four states of two atoms. And this is gg, ge, eg, and ee. And if I denote with ground state spin down, excited state spin up, I'm talking about 2 spin 1/2 states. And 2 spin 1/2 states can couple to s equals 1, total s equals 1, and total spin s equals 0.

And that's what I've done here. I've arranged the states ee, the symmetric superradiant state, the ground state, and the subradiant state. A variation energy level diagram-- here we have 0 excitation energy, here we have 1 excitation energy, and here we have two excitations energies of the atom. But I've also labeled now the spin labels for the combined system. Those symmetrized states correspond to a spin equals 1. It's a triplet letter with three different magnetic quantum numbers. m equals plus 1 means everything is highly excited. m equals minus 1 means we are in the total ground state. And here we have the simulate state, which is the antisymmetric state or the subradiant state.

And our interaction Hamiltonian is the total spin plus minus. It is the raising and the lowering operator. And you know that the raising and lowering operator for the spin is only making transitions within a manifold of total s. It just changes the end quantum number by plus minus 1. So the Hamiltonian cannot do anything to the simulate state, because there is no other simulate state to couple.

But within the triplet manifold, the sigma plus sigma minus operator is creating transitions between the different end states. And the coupling constant, which for an individual atom was little g is now factor of square root 2 enhanced. And we will see in a few minutes, that for n atoms, it's square root n enhanced. And if any speak, that's where the word super in superradiance comes from.

Yeah, actually let me just quickly add the diagram for the single atom. The single atom has only an excited state, a ground state. It corresponds to s equals 1/2. And

we have magnetic quantum numbers off plus 1/2 and minus 1/2. And the coupling due to the light atom interaction goes with the coupling constant g.

So the key message we have learned here is that when we have several atoms within an optical wavelength, we should use for their description symmetrized and antisymmetrized states. Or when we generalize to more than two atoms, we should just add the total angular momenta by treating each atom as pseudo spin 1/2. And it is this angular classification which tells us how the radiation proceeds. Because the coupling to the electromagnetic field is only involving the lowering and raising operators for the total spin. And this only acts on a manifold where the total spin s is conserved. And what we get is transitions with delta in plus minus 1.

So the question, have those effects been observed? Yes, they have, actually. And they're important for a lot of research. But just for two atoms, the simplest observation is when you take two atoms-- let's say two sodium atoms-- bring them very close, and you form a sodium 2 molecule.

And to some extent, in four states where the molecule are binding is not completely changing the electronic structure, we can regard the sodium 2 molecule as consisting of two sodium atoms. And indeed, if you do spectroscopy of the sodium 2 molecule, you find some molecular states which are very long lived, like the subradiant states, which do not radiate at all, but then you find states which have a spontaneous emission rate which is about two times faster than the spontaneous emission rate. So you find that you can understand some radiative properties of molecules by assuming that they are related to the sub and superradiant state of the two atoms which form this molecule.

So an example here is sodium 2 molecule. A state where the gamma molecule is approximately 2 times gamma sodium, or other states where it's very small. OK. Now we understand the basic four of superradiance in two atoms. And therefore, we can now generalize it to end particles. But before I use the spin algebra to describe end particles, I want to glean some intuition where we just consider-- and this takes us back to the beginning of the course-- where we consider end spins in a magnetic

field.

And I really invite you to think now completely classically. We'll describe it quantum mechanically in a moment. But I've often said in this course, if in doubt, if you have a classical description and a quantum mechanical and they seem to contradict, usually there is more truth in the classical description. It's so much easier to fool yourself with the formalism of quantum mechanics.

So let's take end spins in a magnetic field and ask what happens. So we have end spins. So these are now real spins. They have a real magnetic moment. These are tiny little bar magnets. And we do pi over 2 pulse. And after we've done a pi over 2 products, the spins are aligned like this. Let's assume we had our magnetic field. And now what happens is these spins will precess at the line of frequency. So now you have your end spins. They precess together. And if you have a magnetic moment which oscillates, the classical equation of electromagnetism tells you that you have now a system which radiates.

But compared to a single atom, the dipole moment is now n times the single atom dipole moment. So therefore, what do we expect for the radiated power? Well, if the electromagnetic radiation by an oscillating electric or an oscillating magnetic dipole moment scales with a dipole moment squared, therefore, we would expect that the power radiated is proportional to n squared. And that means I have to take the perfect of n [INAUDIBLE]. This means this is n times higher than if you assume you have n individual particles, and each of them emits electromagnetic radiation.

what I'm telling you is if you scatter n spins through your laboratory, you excite them. Pi over 2 pulse, they radiate. They radiate a power which is proportion to n. But if you put them all together, localize them better than the wavelengths, their radiated power is proportional to n square, which is an n times enhancement.

So the way how I put it for n spins-- and this is a situation of nuclear magnetic resonance-- this is the completely natural picture. But if I would have asked you the question-- let's take n atoms which are excited and put them close together, you say, well, each atom does spontaneous emission, and if you have n atoms, we get n

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times c intensity you would have gotten a different result. So we are so accustomed to look at spins in NMR as a coherent system, look that all the spins add up to one giant antenna, to one giant oscillating dipole moment, whereas for atoms, we are so much used to saying each atom is its own particle and thus its own thing.

So for n excited atoms, they are usually regarded as independent. However-- and this is the message of today-- there shouldn't be a difference. All 2 level systems are equivalent. Side remark-- for NMR in spins, it is much, much easier to observe the effect, because the condition that all the spins are localized within one wavelength is always fulfilled if the wavelength is meter or kilometers. But if you have atoms which radiate at the optical wavelengths, this condition becomes nontrivial. That is partially responsible for the misconception that you treat the two-level system which is a spin in your head differently from the two-level system which is an atom.

So the important difference here is lambda. And we have to compare it with a sample size. And usually, the sample size is much larger in the optical domain, and is much, much smaller in the NMR domain. However-- and that's what we'll see during the remainder of this class-- some of the dramatic consequences of superradiance will even survive under suitable conditions in the extended samples. So when we have samples of excited atoms much, much larger than the optical wavelengths, we can still observe superradiance.

So therefore, for pedagogical reasons, I first complete the focus on the case that everything is tightly localized. We derive some interesting equations, and then we see how they are modified when we go to extended samples. But I want to say, the intuition from spin systems, the intuition from classical precession and nuclear magnetic resonance, will help us what happens for electronically excited atoms. So we want to use this other spin 1/2 system as a powerful analogy to guide us.

So before I start with the angular momentum formalism, I want to emphasize that what are the ingredients here. Well, we're talking about coherence-- coherent radiation, coherence between atoms-- and we'll talk about radiation. And the

important part here is the following. That when we talk about radiation, we have the situation that all atoms interact with a common radiation field.

In other words, all the spins, all the atoms have to emit their photons into the same mode of the electromagnetic field. And therefore, you may be right in some limit that the atoms are independent, but not the photons they emit. They go into the same mode. And therefore, the emitted photons cannot be treated independently.

And that's why the classical picture is so powerful for that. Because in the classic picture, we do a coherent summation of the field amplitudes. So we have constructive interference. The superposition principle of field amplitudes build into our equations and deeply engraved in our brains. And that's why when we use classical arguments, we automatically account for that the photons interfere, that the photons are emitted into the same mode of the electromagnetic field.

And eventually, this leads to the phenomenon that we have coherence and enhancement when we look at spontaneous emission for n atoms which are sufficiently localized.

So let me also discuss what we have assumed here. Number one is, we have assumed we have a localization of the sample smaller than the optical wavelengths. The other thing-- and this is really important-- we are talking here about a collective phenomenon where n atoms act together and do something. They develop the phenomenon of superradiance.

They decay much, much faster than any individual atom could do by itself. But nevertheless, we have not assumed-- or we have actually excluded in our description-- that there is any direct interaction between the atoms. The atoms have no [INAUDIBLE] interaction. They're not forming molecules. They're not part of a solid with shared electrons. The atoms are, in that sense, non-interacting. And therefore, in a way, as long as they are just atoms, independent.

Finally-- and I want you to think about it-- you can think about already for two atoms before we generalize it to n atoms. Think about it. What was really the assumption

about the atoms? Do the atoms have to be bosons to be in this symmetric state? Can they be fermions? Or can they be even distinguishable particles?

If the two atoms where one would be a sodium atom and one would be a rubidium atom-- but let's just say we live in a world where sodium and rubidium atoms emit exactly the same color of light. Would we have been sub and superradiant state for two atoms, one of which is sodium and one of which is rubidium?

Yes?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Exactly.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yes. And that confuses many people. It is the indistinguishability of the photons they have emitted. It is the common mode where the photons are emitted. The atoms can be distinguished.

Also, we've made the assumption that the atoms are localized to within an area which is much smaller than lambda. But you could imagine you have a solid state matrix and you have one atom here, one atom there, and you can go with a microscope and distinguish them. So therefore, the moment you can distinguish them because they are pinned down in a lattice-- or if you don't like a lattice, take two microscopic ion traps a few nanometers apart, and you tightly hold onto two ions-- it doesn't matter whether they are bosons or fermions. It only matters whether your bosons or fermions when the atomic wave functions overlap and you have to symmatrize it.

As long as you have two atoms which are spatially separated, it doesn't matter whether they are bosons or fermions. And that also means they can be completely different atoms. You can already call the Boson A, Boson B. Now you can call it sodium and rubidium, and they can have different numbers of nucleus. They can be different numbers of neutrons in the nucleus. It could be different isotopes of the

same atom. The whole collective phenomenon comes when they emit a photon into the same mode.

OK. So now we want to treat to a formula treatment for end particles. So we have now the individual pseudo spins one half. We perform now with some overall end particles. We get the total spin s. The total spin s quantum number has to be smaller or equal to n over 2, because we have m spin 1/2 systems. The end quantum number is 1/2 times the difference of the atoms which are in the excited state minus the atoms which are in the ground state. And this is, of course, trivial. Trivially must be smaller than s. Because m is this z component of s. And we are now describing the system by the eigenstates s and n of the collective spin.

So that means we have the following situation. We have a manifold-- we want to show now all the energy levels. We have a manifold which has a maximum spin n over 2. The next manifold has n over 2 minus 1. And the last one has-- let's assume we have an odd number of particles-- x equals 1/2. So here, we have now n energy levels. We can go from all the n atoms excited to all the n atoms being de-excited. In the following manifold, we have s is one less. And therefore, we have a letter of states which is a little bit shorter. And eventually, for s equals 1/2, we have only two components plus 1/2 and minus 1/2.

So those levels interact with the electromagnetic field. The operator of the electromagnetic field, we have already derived that, involves the sum of all of the little sigma pluses, sigma i pluses, and we call the sum of all of them s plus and s minus. And the matrix element is now for spontaneous emission. You have a state with sm. s minus is the lowering operator for the n particle system, so it goes from a state with a certain number of atomic excitations to one excitation less, and that means this is the act of emitting spontaneously one photon. The operator s minus stays within the s manifold, so we state in the same letter, which is characterized by the quantum number s. But we lower the end quantum number by one. The end quantum number is a measure of the number of excitations.

And we know from general spin algebra that this matrix element is s minus m plus 1

times s plus n. There are, of course, pre-factors like the dipole matrix element of a single atom. But I always want to normalize things to a single atom. And by just using the square root, if you have a single particle, which is in the s to m equals 1/2 state, then you see that it this square root is just 1. So therefore, for when I discuss now the relative strengths of the transitions between those eigenstates, I've always normalized to a single particle. For a single particle, the transition matrix element is 1.

OK. So therefore, what we want to discuss now is, we want to discuss the rate, which is the matrix element squared, or the intensity of the observed radiation relative to a single particle. So the intensity-- and this is what we are talking about-- is now the square root of the square root, or the square of the square root, which is s minus 1 plus m times s plus m. Pretty much, this is the complete description of superradiance for strongly localized atoms. It's all in this one formula. Once we learned how to classify the states, we can just borrow all the results from angular momentum, addition, and angular momentum operators.

So I want to use this formula for the intensity and look at which is the most superradiant state, the state where all the particles are symmetric. And this is a state with s equals n where the spin is n over 2. So I'm looking now at the letter of m states, and I want to figure out what happens. So the maximum m state is m equals s, all the atoms are excited. And now the first photon gets emitted.

So just put s equals m equals n half into the formula for the intensity, and you find that the intensity gives us this expression is just n. So we have n excited atoms, and they initially emit with an intensity which is n. And this is the same as for n completely independent atoms.

So nothing really special to write home about. But now we should go further down the ladder, and let's look at the state which has m equals zero. Well, then the intensity of the matrix element squared for the transition, which goes to m equals zero, has an intensity which is n over 2. I will just look. s is n over 2, and m is zero. So we have the question whether we have odd or even number of particles, but it

doesn't really matter.

What dominates is always the big factor n over 2. So what we find out is that we have an enhancement, huge enhancement over independent atom, because this intensity goes with n squared, and this proportionality to n squared, this is a hallmark of superradiance. So this is what is characteristic for superradiance. We have an n times enhancement relative to the a singular atom.

So this is one important aspect. Now, in the classical picture, that should come very naturally. If you have all the spins aligned and they start the [INAUDIBLE] procession, there is not a lot of oscillating dipole moment. But when half of the spins are de-excited, they are now in the XY plane. Now you have this giant antenna which oscillates and radiates. So it's clear that at the beginning, the effect is less pronounced, and if you're halfway down the Bloch sphere, then you would expect this n times enhancement.

But now let's go further down the ladder and ask what happens when we arrive at the end. So I'm asking now, what is the intensity when the last photon gets emitted. There is only one excitation in the system. And the answer is, it's not one like an independent atom. If you inspect the square root expression, you find it's n. So we have one excitation in the system, but it's completely symmetrized.

And therefore, we have an n times enhancement. And I want to show you where it comes about. So there's only one particle excited. And here, we have an n times enhancement.

By the way, the states with the classification s and m are called the [INAUDIBLE] state. And this state here, which has a single excitation but it radiates n times faster than a single atom, is a very special [INAUDIBLE] state. And there is currently an effort in Professor [INAUDIBLE] lab to realize in a very well-controlled way this special [INAUDIBLE] state in the laboratory. There are non-classical states, because they're not behaving as you would maybe naturally assume a system with a single excitation to behave.

So let's maybe try to shed some light on it. One way how you can intuitively understand superradiance is really with a classical antenna picture that you have n spins which form a microscopic dipole moment which oscillates. And this is a very nice picture to understand the n times enhancement when we have half of the atoms excited and the other half de-excited. Let me now give you a nice argument which explains why a single excitation in this system now leads to an n times enhanced decay.

The situation is that the initial state for this last photon is, we have an excited atom, and all the atoms are in the ground state. However, we could also have in this nomenclature the second atom excited. Or we could have the third one excited, and so on. So therefore, what we have is-- because we are in the left-most [INAUDIBLE] which has the maximum s spin quantum number of n over 2, that means everything is fully symmetrized. So therefore, we have to fully symmetrize by summing over the n possibilities.

And our final state is, of course, the fully symmetrized [INAUDIBLE] state. And now you realize that you have a coherent summation over-- you have n contributions. So therefore, the matrix element has n contributions compared to single atom. The normalization is only square root n. So therefore, the matrix element is square root n times larger than for an individual atom.

So by simply having one atom excited and n minus 1 atom not excited, but if you now have the fully symmetrized state, you don't know for fundamental reasons which atom is excited. You have a superposition state where the excitation can be with either of the atom. This state, which has a similar quantum of excitation, radiates n times faster than a single atom would.

Let me make a side remark. Maybe some of you remember when we went to [INAUDIBLE] QED, we had just proudly quantized the electromagnetic field, and we discussed the vacuum Rabi splitting. And I told you that the vacuum Rabi splitting is if the cavity is not empty but is filled with n atoms, because of the matrix element of the a dagger operator, you get an enhancement of the vacuum Rabi splitting, which

is square root n, then photon number.

But then I showed you the important first observation, the pioneering research at Cal Tech by JF Kimball and Gerhard Rempe, and they didn't vary the photon number. They varied the atom number. And when they had more flux in the atomic beam, the Rabi splitting became larger and larger.

Well, we've just learned that when we have n atoms, that the matrix element for emitting the photon is square root n times enhanced. So if you put n atoms in a cavity filled with little n photons, the Rabi splitting between the two modes has square root n plus 1 in the photon number and square root big n in the atom number. So the effect I've shown you in the demonstration of the vacuum Rabi splitting is this scaling with the atom number. This actually can be understood as a superradiant effect.

OK. So that's pretty much what I wanted to tell you about the basic phenomenon of superradiance. Now I want to discuss two more things. One is superradiance in an extended sample. Ad we have time for that. But I also want to discuss with you the question. Let's assume we have the same system, and we just convinced ourselves, yes, it's superradiant. Photons are emitted n times faster.

Now, what would you think will happen when we are not looking for spontaneous emission, but we shine a laser light on it, and we are asking for induced emission? Or the other way around-- we ask-- and you know that from Einstein's treatment, it's completely recipocal-- where we are asking the question, what happens to the absorption process? So is a stimulated emission process or an absorption process, are they also enhanced n times?

I don't know. Do you have any opinions about that?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yes. It's a subtle way of counting. I've shown you that certain matrix element-- especially the matrix element when the spin is in the middle, is at 90 degrees-- that we have matrix elements which are n times enhanced. And of course, if you ask for

absorption or stimulated emission, we are talking to a system which has an n times enhanced matrix element. And you would say, things go n times faster. Why don't you hold this thought for a moment?

Now let me just wear another hat and let me say that we have assumed that we have n independent spins that are closely next to each other, but they're not interacting. And now I take these n spins, and for stimulated emission and absorption, we can just use a picture of Rabi oscillation. On the first Rabi cycle, we emit. On the next Rabi cycle, we absorb. So if I take now-- and why don't you think not about these pseudo spins electronically, atoms with [INAUDIBLE] excitation-- just think of real spins which have a magnetic moment, and you drive them to the magnetic field.

So now you have your n little spins. You apply a magnetic field to them, time-dependent magnetic field, and the time-dependent magnetic field is now driving the spin in Rabi oscillations. And the external drive field talks to one spin, talks to the next, talks to all of them. But each of the spins does exactly the same Rabi oscillation it would do if all the other atoms where not present.

So the picture is, you have an external field. All the atoms couple to the external field. But the coupling of each atom to this external field is exactly the same as of a single atom coupling to the external field. And the Rabi frequency for each atom is exactly the Rabi frequency you would get for a single atom. So therefore, based on this picture, I would expect that I have my end spins, and these can now be real spins with a magnetic moment, or can be atoms in the electronically excited state. I coherently drive them with a drive field, and they will do Rabi oscillations. But the frequency or Rabi oscillations will be the same as for a single atom.

OK. We've just held the thought that their matrix elements in the Dickey states, which are square root n times larger, and the Dickey state seem to suggest something to us which would say there should be an enhancement, whereas the analysis in independent atoms, which are driven by an external field, also seems compelling. So now we have to reconcile the two approaches.

Is the question clear? We want to figure out-- we have matrix elements in the Dickey state which suggest enhancement, but the simple picture of n independent atoms driven by an external field would say there is no enhancement.

OK. So let me just write down more formally what I explained. When we have an initial state, which is all the atoms are in the ground state to the power n, and the state n would evolve when it is driven in a state phi of t, so now the exact wave function for our n particles is nothing else than the time-dependent solution of Schrodinger's equation for the single particle. So this is single particle to the power n.

So therefore, this is pretty much a mathematical proof unless I've made a mistake, which I haven't. So it takes exactly half a single atom Rabi period to completely invert the population exactly s for a single atom. So that's the result.

However, if you describe the system by Dickie states, you have matrix element which are matrix elements which are proportional to n. However, if you want to-- I've described it just as a two-level system. Each atom does Rabi oscillation. I've said, OK, the system of n atoms is just n individual systems. But if you insist to describe it as a collective spin, then we have the Dickie states. Then we have the n times enhancement of the matrix element. But then we also have to go through n states.

So we have n steps in the Dickie letter ladder. And one can say now-- and this is the exact argument-- you have n steps. You take each step n times faster, but the total time is the same. So n times 1 over n is 1. OK. But now when we talk about spontaneous emission, we are not driving the system with an external field. It's really driven by the system itself, which emits photons into the empty mode.

Spontaneous emission, each step is proportional to the matrix element squared, because we're talking about [INAUDIBLE] spontaneous emission. So this is proportional to n square. And if you say that we have n steps, well, then we have n squared over n. Then we have a speed up. Each step is n squared faster. We divide by n, and we get the superradiant speed of which is n.

So superradiance is something that you observe in spontaneous emission, but you cannot absorb it in a driven system. Because in a driven system, you can say you have a classical external field. And this external talks to one atom, to n atoms exactly in the same way. It is really the interference of spontaneously emitted photons which is at the heart of superradiance.

As a side remark, we are talking here about coherent effect, which is n times enhanced. And you can actually regard that as a kind of bosonic enhancement in the emission of photons, because the photons are bosons. When Bose-Einstein condensation was discovered and people were thinking about basic experiments, of course, one thing which was on our mind is, we wanted to show that there are processes in the Bose-Einstein condensate we are n times enhanced. For fermions, they would be suppressed. This is the flip side. Big enhancements for bosons, complete suppression for fermions.

And we found that, for instance, the formation of the condensate had an n times enhancement. There was a stimulation factor. But we also thought you should actually-- there may be ways where you can observe suppression of light scattering or enhancement of light scattering. But we thought about it with two laser beam [INAUDIBLE] scattering, and the idea seemed compelling.

And then we said, no, wait a moment. If you use laser beams, everything is stimulated. You can observe bosonic enhancement and fermeonic suppression only when you have spontaneous events. If you drive it in a unitary time evolution, you will not be able to see quantum statistical suppression or enhancement.

And the same thing as we have seen here-- when you have a stimulated system, everything is undergoing a unitary time evolution, and the unitary time evolution for n atoms is the same as for a single atom. You need the element of spontaneous emission.

So I'm not proving it to you, but I'm just making as a remark-- what we have seen here, that the superradiance only shows up in spontaneous emission and not when we drive the system-- a driven system is a unitary evolution. And the same

conclusion which we just got for superradiance also applies if you want to observe fermionic suppression or pulsonic enhancement in quantum gases. It needs an element of spontaneous scattering or spontaneous emission. Yes?

AUDIENCE: If we think of it in terms of interference of photons, how does that tie in here? Because if the stimulated [INAUDIBLE] photons are still interfering, then you can get emission [INAUDIBLE].

PROFFESOR: The quick answer is, you have a classical field which you use for-- you have a laser field for stimulate emission for absorption. There are so many photons in the laser field that the few photons which your system emits do not matter. They are really talking to a classical field, and it doesn't matter whether the other n minus 1 atoms have emitted a photon, because you have zillions of photons in your laser field, and they determine the dynamics of the system.

OK. Super radiance would not be as important as it is if it could not observed in extended samples. So now I want to use the last 10 minutes to show you what is kept and what has to be dropped when we talk about extended samples. So let's-- it doesn't really matter, but for pedagogical reasons, let's assume we have an extended sample which is much, much longer than the optical wavelengths along the long axis of the cigar and smaller along the short axis. The second condition that the cigar is smaller than along the long axis does not really matter.

No, I'm not making this assumption. So it's a cigar, and let's just assume that everything is-- just saw a contradiction in my notes. So anyway, we have now a system which is-- let's have a Cuban cigar, a really thick cigar. And this is now our extended sample. And what I need is, I need the cross section of the sample A. And let's assume the length is l. This is diameter d. It's a cigar much, much larger than here.

And yes, we are talking about superradiance, we are talking about spontaneous emission. But if you see a long cigar with excited atoms, you think immediately about lasing action. The photon is emitted is amplified along the path. And of course, the preferred direction where you would expect the maximum effect to

happen is when the light is emitted along the long axis of the cigar. So you want to consider now preferential modes along the x-axis.

So if you now assume that you have many atoms, and they emit light, if an atom here and here would emit light in this direction, it may constructively interfere. But in another direction, it will destructively interfere. But let us now consider what is the solid angle into which all of the atoms can coherently emit.

Well, you know that from classical optics, the emission into a solid angle of lambda square over a can be coherent. Sort of similar to when you have a double slit and you ask, over what angle do the two slits emit inface. You get a bright fringe, and you get a dark fringe, you get the next bright fringe. The coherence, the angle over which the pass lengths differences do not add up to more than lambda. It's the deflection-limited angle which for a beam of size d is lambda over d. And if you take it to the second dimension, the solid angle is lambda squared over d squared. So that's what I'm talking about.

So if you would give all the atoms in your assemble just the right phase that they are coherent to emit into the x-axis, they will also coherently emit into a small, solid angle, and the solid angle is given by this number.

So the just of it is-- and I will not completely prove it to you, but I just want to give you a taste-- is that therefore, we still have a superradiant enhancement. We know the superradiant enhancement previously when we had the localized system was n. But now we have the n atoms act together, but they're not acting together for emission into 4 pi. They are acting together for emission into the solid angle.

And if I write the big n as density n times l times a squared, the a squared cancels out and I get n lambda square l. And if you remember that the cross section of an atom was lambda square for absorption, if the atom is excited, the cross section for amplification of light for stimulated emission is also lambda squared. So lambda squared is the gain cross section.

And what we find now as a superradiant enhancement factor is nothing else like

something which reminds us of a laser, which reminds us of optical gain. And actually, the lasing phenomenon in superradiance in extended samples has a lot of analogies. In some limits, it's even identical. When we are talking about spontaneous emission, we are not talking about stimulated emission. But if you have a system which is in some excited state superradiant Dickey states, and we are asking what are the spontaneously emitted photons coming out, to say different atoms emit into the same mode, and now you have to add up the feeds coherently, this is a language we have used so far.

Or if you use a language atom emits a photon and this photon gets amplified on its way out, those two language strongly overlap or in some limits are even identical. So the amplification of a photon on its way out, this is behind superradiance. But when the localize the atoms to lessen the wavelengths, well, the atoms pretty much emit as a whole and there is no pass lengths of the size of the optical wavelengths where you can say the photon propagates, gets amplified. So we have looked at just what comes out of it.

But in extended sample, you could even address this situation. How do the photons get amplified, magnified, augmented when they travel from the center to the edge? So you could actually ask, what is the light intensity as a function of the position within the cigar? For localized samples, you can't.

So let me just write that down. So this is analogous to optical amplification in an elongated, inverted medium. OK. So you can formally describe that. You can now define new Dickey states with respect to the preferred mode. And the preferred mode is the mode in the x direction.

So what I've done is-- remember, we have those atoms. Those atoms are now sitting at different positions, x1 and x2. And if I define Dickey states which have phase factors into the ik x1, into the ik x2, if now this atom emits a photon and this atom emits a photon, well, the second photon is x2 minus x1 ahead of this photon if you think of those atoms sitting aligned in a string. But the phase vector is exactly canceling the propagation phase for the first atom in such a way that if you are now

coupling these states to the electromagnetic field, the phase factors of the electromagnetic field in the mode cancel with those phase factors, and you again have the situation that each state here has an equal amplitude for emission. So now you have n possible contributions, and the normalization is 1 over square root n, and everything falls into place.

And you can define that for two excited atoms with two phase factors and so on. So you can use immediately the same formalism. And what happens is those phase factors for the interaction Hamiltonian-- and our interaction Hamiltonian is now different. It is di. And now in an extended sample, we have to keep track of the precision of the atom.

So for the coupling to the mode in the x direction, we have those phase factors. So all the phase factors cancel. And actually, I'm not telling you whether this is plus or minus in order to cancel. You pick the sign that they all cancel, and then you have superradiance. You have fully constructive interference.

Yes. So all this looks now the same as superradiance, but there are also things which are different, and this is the following. If the atom would emit now photons, not in the preferred mode, then-- remember, we had the Dickey ladder. We had the most superradiant ladder, little bit less superradiant ladder, and eventually we had the subradiant ladder in order of smaller and smaller total quantum number s.

Emission in the preferred mode stays in each letter, and we have the superradiant cascade. But emission into other modes is now coupling different s states. Because the operator or the phase factor into the IKR has broken to complete permutation symmetry between the sides. We have changed the symmetry. We have not the completely symmetric sum. We have a symmetric summation with phase factors.

So therefore, the phenomenon is somewhat different. But we still have superradiant cascade for the preferred mode. And the result is that we have an enhancement for the most symmetric for the superradiant states, which is given by that. And this is nothing else than the resonant optical density of your center.

So in experiments-- many of them go on in [INAUDIBLE] lab, where he uses collective spin and the storage of single photons in n atoms, the figure of [INAUDIBLE] of his samples is always the optical density, the number of atoms times lambda squared times the lengths.

Finally-- and sorry for keeping you three more minutes-- the form of superradiance which is very important is Raman superradiance. We don't have an excited state where we put a lot of excitations on, because the excited state would be very short lived. So what we instead do is, we have Rabi frequency omega 1.

We have a large [INAUDIBLE] delta. And then the spontaneous emission with the coupling constant g takes us down to the excited state. In the case that the Rabi frequency is much, much smaller than delta, we can eliminate the excited state from the description. And what we obtain is now a system which has an excited state. The widths of this excited state-- this is pretty much the virtual state here-- is the scattering rate which is the probability to excite the atom is Rabi frequencied over detuning squared. That's just perturbation theory. And then we multiply with gamma or gamma over 2. So this is the rate of spontaneous emission out of the virtual state.

And from this virtual state, we go now to the ground state. And the Rabi frequency, or the coupling for this virtual state, is the original coupling g between ground and excited state, but now pro-rated by the amplitude that we have mixed the excited state into the virtual state. So therefore, we have now obtained a superradiant system.

And for instance, we did experiments which became classic now because they are conceptually so clear-- we took a Bose-Einstein condensate, we switched on one strong of resonant laser beam, and then we had a system which was 100% inverted, because we had no atoms into the final state. The final state is a Bose-Einstein condensate but with a recoil kick. So by just having a Bose-Einstein condensation and shining this laser light on it, we had now in this picture a 100% inverted system, which is the ideal realization of a fully inverted Dicky state.

Everything is completely symmetric, and then we observed superradiant emission of light pulses. OK. So this has been the important realize-- so important experiments have been done via BECs in my group and with cold atoms in with laser code samples in [INAUDIBLE].

So why is superradiance so important? And this my last statement for this class and for the semester. So if you have extended sample superradiance, those samples are no longer coupling to the electromagnetic field with the coupling constant g. The coupling constant g is now multiplied by the optical density of your sample. And there is a lot of interest for current research for quantum computation, manipulation of photon states, and all that, to do cavity QED. And in cavity QED, we try to have very good mirrors, very small mode volume to have a very, very large g.

But this large g which we achieve in a cavity, if you put many atoms in it, gets enhanced by the optical density. So the cavity enhancement and the superradiant enhancement is multiplicative. And often, it's very favorable for single photon manipulation if you do Bose. You getting enhancement form the cavity and enhancement due to superradiance. And the person who has really pioneered work along this direction is [INAUDIBLE] here at MIT.

Anyway, yes, with five minutes delay, I finish the chapter on superradiance. Well, that's the end of this course. Let me thank you for your active participation. Sometimes as a lecturer, you learn as much as the students. And I think partially based on your questions and discussions, this is really true. I've learned a few new aspects of atomic physics. I hope you have learned something, too. And good luck in the future.