Assignment 5

Due: Monday, March 26, 2012

1. **Optical Traps and Scattering**

   The AC Stark effect has been used to trap neutral atoms at the focus of one or more laser beams. This obviates the necessity of magnetic traps, which can be bulky and power guzzling. Scientifically more important is the fact that many interesting phenomena occur between states that are not necessarily all magnetically trappable. This has led to studies of, for example, spin domains in spinor Bose-Einstein condensates (Sadler *et al.*, *Nature* **443**, 312 (2006)), among many other examples. In the following exercise, you will explore the proper laser power and wavelength needed to trap an ultracold atomic gas.

   Let us consider an alkali atom with resonance frequency ω_0 on the principal $nS \rightarrow nP$ transition. A sample of atoms in the ground state nS are exposed to monochromatic radiation of intensity I and frequency $\omega_L < \omega_0$ (red-detuned from resonance). Use the fact that essentially all of the oscillator strength out of the ground state comes from the $nS \rightarrow nP$ transition.

a) AC Stark Shift

   i. Calculate the AC Stark shift U_i using time-dependent perturbation theory for the dynamic polarizability $\alpha(\omega_L)$.

   ii. Now calculate the AC Stark shift U_{ii} using the rotating wave approximation *i.e.* $|\omega_L - \omega_0| \ll \omega_L + \omega_0$, and approximated for small field strengths *i.e.* Rabi frequency much less than the detuning.

   iii. Calculate the ratio U_i/U_{ii} as a function of ω_L. What is the ratio when $\omega_L \approx \omega_0$ and $\omega_L \approx 0$?

   So we see that if the intensity has spatial structure (which we have not considered so far), with the appropriate detuning, the AC Stark shift can have energy minima where the atoms can be trapped.

b) What is the probability of finding the atom in the excited state P_e using both pictures *i* and *ii*? Neglect rapidly oscillating terms and time-average the remaining time-dependent terms.

c) In this part you will calculate the photon scattering rate. Knowing the scattering rate for a particular atom and particular laser is important because it causes heating and may severely limit the holding time for an ultracold cloud. Use the fact that, from classical electromagnetism, the power P radiated by an oscillating dipole of dipole moment $\mathbf{d}(t) = \mathbf{d}\cos\omega t$ is $P = ck^4|\mathbf{d}|^2/3$.

   i. What is the photon scattering rate using *i* and *ii* as a function of ω_L?

   ii. Express the scattering rate for the case *ii* in terms of P_e that you found in part *b* and the natural scattering rate Γ and other factors. The significance of this will become clear if you think about the next optional part.

iii. (Optional) One can describe scattering as spontaneous emission from a virtual energy level. Draw the energy level diagram (including the ground, excited, and virtual state) including the frequency spacing. What is the lifetime of this virtual state in terms of the frequency spacing and other constants?

d) Finally, let us apply our formulae to the optical trapping of sodium. Do this problem just for the case *i* which includes the counter-rotating term. Atoms are to be trapped at the focus of a single laser beam. At the focal plane, the irradiance distribution has the form

$$I(r) = \frac{2P}{\pi w^2} \exp(-2r^2/w^2) \tag{1}$$

where $w = 6\,\mu$m is called the beam waist radius, r is the distance away from the center of the beam, and P is the laser power. Such a beam forms a trap for which the trap depth is given by the AC Stark shift at the maximum intensity $I(0)$.

We wish to trap a gas of sodium atoms with a temperature of about $1\,\mu$K. Thus, let us plan for a trap depth of $k_B \times 10\ \mu$K. The principal transition of sodium is at 590 nm. consider two types of lasers:

1. **Yellow Laser:** a detuning of 1.7 GHz *i.e.* $\omega_L - \omega_0 = -2\pi \times 1.7$ GHz (light at such a detuning is used for cooling sodium in the first place).
2. **Infrared Laser:** the light of a diode laser at 985 nm.

i. If you used the expressions derived from *ii*, *i.e.* the rotating wave approximation in the low field limit, for which type of laser would the expressions be better suited? Why?

ii. Calculate the required power and the scattering rate for each of the two types of lasers.

Many current experiments with Bose-Einstein condensates study phenomena that occur on the millisecond to second time scale, and so these considerations are important.

For reference:

Chu *et al.* "Experimental observation of optically trapped atoms," *Phys. Rev. Lett.* **57**, 314 (1986). First optical dipole trap for laser-cooled sodium atoms.

Stamper-Kurn *et al.* "Optical confinement of a Bose-Einstein condensate," *Phys. Rev. Lett.* **80**, 2027 (1998). First optical dipole trap for Bose condensed sodium atoms.

2. **Magic Wavelength Optical Trap**

The official time standard is currently based on a cesium microwave transition, but major research efforts are underway to improve on that by using an optical transition (see for example Ludlow *et al.*, Science **319**, 1805 (2008)). In particular, one path is to use neutral atoms trapped in optical lattices that have only one atom per lattice site. This can eliminate collisional effects that can limit the precision of such clocks. However, AC Stark shifts of the two states involved in the relevant transition due to the lattice lasers can cause systematic uncertainties. This can be alleviated by choosing a lattice laser wavelength where the AC Stark shift of the two states are exactly the same, thereby eliminating the systematic uncertainty. This wavelength is called the magic wavelength. You will find the magic wavelength for a model system in this problem.

Consider a laser of frequency ω and intensity I which creates a dipole trap for an atom.

a) Consider a two-level system with lower state $|S\rangle$ and upper state $|P\rangle$ with 'bare' energy separation $\hbar\omega_{PS}$. Also let the dipole moment be d_{PS}.

i. Find the AC Stark shift for the state $|S\rangle$ in terms of the dipole moment, the frequencies, and an overall constant factor. Do not make the rotating wave approximation!

ii. Find the AC Stark shift for the state $|P\rangle$ in the same way. What is the relationship between this and the AC Stark shift found in *i*?

iii. Consider the difference in transition frequency $E_P - E_S$ with and without the laser. Is there any way this can be zero with the current setup (while still having a trapping laser, of course)?

b) Now consider adding a third, higher level to the system to make it a three-level system with the third, highest state $|D\rangle$ with energy separation between $|D\rangle$ and $|P\rangle$ as $\hbar\omega_{DP}$. Also let the dipole moment be d_{DP}. Assume that $\omega_{PS} = f\omega_{DP}$ and $d_{PS} = nd_{DP}$, where f and n are some numbers, and that the states $|S\rangle$ and $|D\rangle$ do not couple directly.

i. Find the AC Stark shift for the state $|P\rangle$ taking into account both the higher state $|D\rangle$ and lower state $|S\rangle$.

ii. What is the laser frequency needed to make the $S \to P$ transition frequency independent of trap laser power? Express this in terms of ω_{DP} and d_{DP}. This is the so-called magic wavelength.

3. **Species-Dependent and Spin-Dependent AC Stark Shift**

Now consider how multiple species or multiple sub-states of atoms can be selectively manipulated using AC Stark shifts. We can consider putting more than one type of atom in the laser field. Also, in reality atoms do have sub-states, and the different sub-states can couple differently to the laser field. We will explore these situations in this problem. For these cases, the polarization of the light with respect to the quantization axis also begins to matter.

Consider a three-level system with fine structure, *i.e.* an alkali atom. The lowest level is $S_{1/2}$ and the upper P state splits into two levels, $P_{3/2}$ and $P_{1/2}$. Assume the state $P_{3/2}$ has higher energy than $P_{1/2}$. Let the energy difference be $\hbar\omega_1 = E_{P_{1/2}} - E_{S_{1/2}}$ and $\hbar\omega_2 = E_{P_{3/2}} - E_{S_{1/2}}$. (As a side note, the labels 1 and 2 for the frequencies correspond to the fact that the transitions are called the D1 and D2 lines, respectively.) Assume the atom is quantized along the z direction. You also need to know the dipole coupling of the form $\langle S_{1/2}, m_J|\hat{\epsilon}\cdot\mathbf{r}|P_{3/2,1/2}, m'_J\rangle$ between the relevant states, where $\hat{\epsilon}$ is the polarization vector of the electric field. The relevant dipole matrix elements are as follows:

$$|e\langle S_{1/2}, +1/2|\hat{\sigma}_+\cdot\mathbf{r}|P_{3/2}, +3/2\rangle|^2 = d_2^2 \tag{2}$$
$$|e\langle S_{1/2}, +1/2|\hat{\sigma}_-\cdot\mathbf{r}|P_{3/2}, -1/2\rangle|^2 = d_2^2/3 \tag{3}$$
$$|e\langle S_{1/2}, -1/2|\hat{\sigma}_+\cdot\mathbf{r}|P_{3/2}, +1/2\rangle|^2 = d_2^2/3 \tag{4}$$
$$|e\langle S_{1/2}, -1/2|\hat{\sigma}_-\cdot\mathbf{r}|P_{3/2}, -3/2\rangle|^2 = d_2^2 \tag{5}$$
$$|e\langle S_{1/2}, +1/2|\hat{\sigma}_-\cdot\mathbf{r}|P_{1/2}, -1/2\rangle|^2 = 2d_1^2/3 \tag{6}$$
$$|e\langle S_{1/2}, -1/2|\hat{\sigma}_+\cdot\mathbf{r}|P_{1/2}, +1/2\rangle|^2 = 2d_1^2/3, \tag{7}$$

where e is the electron charge. All other matrix elements are irrelevant for this problem (which should become clear if you consider the selection rules and the photon polarizations considered in this problem). You should also notice that the interference terms between the different polarizations vanish for this problem. Note that $\hat{\sigma}_\pm = (\hat{x} \pm i\hat{y})/\sqrt{2}$ correspond to $\sigma_\pm$ polarized light where absorbing a photon with such a polarization leads to $\Delta m_J = \pm 1$. The total AC Stark shift for a given sub-state is then just a sum over all of the AC Stark shifts due to the coupled states. Do not use the rotating wave approximation in this problem.

a) Consider a linearly polarized dipole trap laser with electric field polarization along the x direction propagating along the z direction.

i. At what laser wavelength will the atom in state $S_{1/2}$ not feel the dipole trap anymore? Use the following parameters to numerically find the wavelength specifically for rubidium: $\lambda_1 = 795$ nm, $\lambda_2 = 780$ nm, $d_1 = 3.0ea_0$, $d_2 = 4.2ea_0$, where a_0 is the Bohr radius.

ii. This has been used to create a system where one type of atom (^{41}K) feels a dipole trap, but the other type (^{87}Rb) does not, in order to demonstrate entropy exchange between the two species (Catani *et al.*, *Phys. Rev. Lett.* **103**, 140401 (2009)). They used a laser at a wavelength of 790 nm for this purpose. Does the wavelength you found agree with what was used in their paper? Is there anything that we neglected to take into account?

b) Now consider a σ_+ polarized light propagating along the z direction, with laser frequency much closer to the $S_{1/2} \rightarrow P_{3/2}$ transition than the $S_{1/2} \rightarrow P_{1/2}$ transition so that you can neglect coupling to the $P_{1/2}$ state.

i. What is the AC Stark shift for the two states $|S_{1/2}, m_J = +1/2\rangle$ and $|S_{1/2}, m_J = -1/2\rangle$?

ii. Rewrite the AC Stark shift you found in *i* in the form $U = U_S + U_V$, where U_S is the spin-independent, scalar light shift, and $U_V = -\mu \cdot \mathbf{B}_{\text{eff}} = g_J m_J \mu_B B_{\text{eff}}$ is the spin-dependent, vector light shift. Note $g_J(S_{1/2}) = 2$. What is $\mathbf{B}_{\text{eff}}$? We see that the spin-dependence of the AC Stark shift can be expressed in terms of a fictitious magnetic field acting on the spin of the atoms.

This configuration is relevant to situations where you want to induce spin-dependent interactions. This can be the used to generate entanglement for quantum information processing (Mandel *et al.*, *Nature* **425**, 937 (2003)), or realizing, for example, anti-ferromagnetic many-body ground states to study quantum magnetism (Duan *et al.*, *Phys. Rev. Lett.* **91**, 090402 (2003)).

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