8.321 Quantum Theory-I Fall 2017

Prob Set 4

1. Consider a free particle of mass m with a wave function at time $t = 0$ given by
$$\psi(x, t = 0) = A \exp\left(\frac{ip_o x}{\hbar} - \frac{x^2}{2a^2}\right) \tag{1}$$

   (a) Calculate $\psi(x, t)$.

   (b) Calculate the expectation values $\langle x\rangle, \langle p\rangle, \langle(\Delta x)^2\rangle, \langle(\Delta p)^2\rangle$ as a function of time t.

   (c) Repeat the calculation of these expectation values by using an alternate approach based on the Heisenberg equations of motion for the operators x and p. Solving these equations will enable expressing the operators $x(t), p(t)$ in terms of the corresponding operators at $t = 0$. The desired expectation values can then be calculated using the $t = 0$ wavefunction.

2. The state of a free particle is described at time t by a normalized wave function $\phi_0(p)$ in the momentum representation.

   (a) What is the momentum space wave function $\phi(p, t)$ at a later time t?

   (b) Write down an expression for the real space wave function $\psi(x, t)$ in terms of $\phi_o(p)$. Show that this wave function is normalized for all t.

   (c) For a wave packet
$$\phi_0(p) = \left(\frac{a}{\pi}\right)^{\frac{1}{4}} e^{-\frac{a}{2}p^2} \tag{2}$$
   find $\psi(x, t)$. How does it behave as $t \to \infty$?

3. **Sakurai Problem 2.7**

4. **Sakurai Problem 2.16**

5. A spin-1/2 system is placed in a magnetic field

$$\vec{B} = B_0 \hat{z} + B_1 \left(\cos(\omega t) \hat{x} + \sin(\omega t) \right) \tag{6}$$

The Hamiltonian is

$$H = -\gamma \vec{S} \cdot \vec{B} \tag{7}$$

Consider the general case where the driving frequency ω is not on resonance.

If at time $t = 0$ the system is in the eigenstate of S_z with eigenvalue $\frac{\hbar}{2}$, what is the probability that a measurement of S_z after a time t gives the same value? Use the result above to calculate the frequency of Rabi oscillations.