

Scattering, radiation of relativistic particles

Reading: Schwinger, Chaps. 35-38, 45.

1. Electromagnetic wave scattering.

Consider one or several electrons moving, driven by the field of an electromagnetic wave. The time-averaged radiation power can be expressed as the time-averaged energy flux in the wave times some area, called the scattering cross section. Note that, since both the radiation and the energy flux are proportional to the square of the field amplitude, the cross section is independent of the field intensity, and thus is a characteristic of electron dynamics only¹.

a) Find the scattering cross section for an electron bound on a spring of spring constant k in the presence of a circular polarized electromagnetic wave.

b) Find the scattering cross section for a small dielectric sphere in the presence of a linearly polarized electromagnetic wave. The sphere radius r is much smaller than the wavelength, the dielectric constant of the dielectric material is ϵ .

2. Impulsive radiation.

a) Consider a nonrelativistic electron colliding with an atom, with electron velocity changes from \mathbf{v}_1 before collision to \mathbf{v}_2 after the collision. Find the angle-resolved spectrum $dE/d\omega$ of electron radiation at $\omega\tau \ll 1$, where τ is the time of collision.

b) A nonrelativistic electron approaches a metal surface with velocity \mathbf{v} (normal incidence). Before collision, the electric field of charges induced in the metal can be approximately described by an image point charge moving with velocity $-\mathbf{v}$. After electron enters the metal its charge is totally screened. Find the radiation spectrum $dE/d\omega$ at small frequencies.

3. Radiation after nuclear reaction.

In a spontaneous breakup, a nucleus of charge Z and mass M_0 splits into two equal halves of charge $Z/2$ and mass $M_1 < M_0/2$, moving apart with *relativistic* velocities \mathbf{v} and $-\mathbf{v}$.

a) Find the angle-resolved spectrum $dE/d\omega$ of emitted radiation.

b) Integrate over angles to obtain the angle-averaged radiation spectrum $dE/d\omega$

4. Relativistic electron in a capacitor.

A relativistic electron flies through a parallel plate capacitor at a normal angle to the electric field. The electron path within the capacitor is short, and the field is weak, so that

¹The idea of scattering cross section, as well as the argument behind the formula $\sigma = P_{\text{scattered}}/S$, has nothing to do with radiation theory and even with E&M in general. Think of a beam of particles of uniform density n and constant velocity v , characterized by flux $J = nv$, bombarding a target that scatters the particles off. Then, if the number of particles scattered per unit time is $r = dN/dt$, the effective cross section area for scattering is $\sigma = r/J$.

The concept of scattering cross section is introduced as a simple way to separate the sheer number of scattered particles into the product of the number of incident particles J and a quantity σ which characterizes the scatterer. This is true in classical mechanics for particles, in EM for waves, and also in quantum mechanics for particles and waves. There are special cases (see Problem 5) where this simple factorization does not take place because the scattering intensity is enhanced by relativistic effects, which themselves depend on the incident wave intensity.

the deflection of electron velocity is small and the trajectory can be approximated by a slightly curved straight line. The electron velocity is \mathbf{v}_0 before entering the capacitor, and \mathbf{v}_1 after exiting ($|\mathbf{v}_1 - \mathbf{v}_0| \ll c$).

a) In the frame comoving with the electron, the radiation is given by the nonrelativistic Larmor formula. In this frame, relate the change of electron energy δE and momentum δp upon leaving the capacitor with the radiated energy and momentum.

b) Using Lorentz transformation for energy and momentum, find the radiated energy and momentum in the lab frame.

5. Relativistic circular motion.

a) Find the radiated power for cyclotron motion of a relativistic electron.

b) A relativistic electron is moving in the field of a circular polarized electromagnetic wave of amplitude E_0 . Find the radiated power and determine the scattering cross section of the wave. Compare to the nonrelativistic case, and note that the cross section now depends on the field intensity.