Assignment #7 **PHYSICS 8.284**

Due 11:04 am MONDAY 10 April 2006

- Reading: Clayton §4.3 through equation (4-62). Hansen & Kawaler §§6.3-5; Bohm-Vitense, Volume 2, Chapters 5 (Eddington-Barbier), 6 (Temperature Stratification) and 10 (Line Formation) as nedeed.

1. The solar neutrinos observed in the Homestake Mine experiment came from the PP III branch of the proton-proton chain. The limiting reaction is

$$\mathrm{Be}^7 + \mathrm{p} \rightarrow \mathrm{B}^8 + \gamma$$

which is followed almost immediately by the emission of a positron and a neutrino.

   a) Take the central temperature of the Sun to be 1.5×10^7K, so that $kT = 1.293$ kev. Calculate by how much (expressed as a percent or as a factor) the central temperature must drop to cause the rate of PP III neutrinos to drop by a factor of 3, which is the observed shortfall. *Hint:* Calculate the derivative of the natural log of the rate with respect to temperature and find the dT which changes the log of the rate by an appropriate amount.

   b) In the last problem set we adopted a scaling relation for the energy generation equation of the form $L = C'M\left(\frac{M}{R^3}\right)T_c^n$. While our expression for $< \sigma v >$ is clearly not a power law of temperature, it can be locally approximated as such by computing the value of $d\ln(< \sigma v >)/d\ln T$ at any point to give a effective exponent n. What is the effective n for part a)? This is similar to Clayton's problem 4-14.

2. In class we derived the functional dependence of photospheric temperature T_p on mass and radius for fully convective stars. We found an accidental cancellation in the R exponents, giving an amazingly weak dependence of T_p on R, which we offered as an explanation for the nearly vertical "Hyashi track" to which pre-main sequence stars, giants and asymptotic giants all adhere. For the sake of brevity we did not keep track of constants – G, k, ξ_1, m_p and even μ.

   a) Reconstruct the argument retaining the multiplicative constants, i.e. find an expression for T_p in terms of M and R appropriate to a fully convective star. Recall that we started with an expresion for the photospheric pressure, $P_p = g/\kappa_{H^-}$ where $g = GM/R^2$ and an approximate expression for the H^- opacity, $\kappa_{H^-} = 2.5 \times 10^{-31}(Z/0.02)\rho^{1/2}T^9$ cm^2/gm. We used the fact that the polytropic index for a fully convective star is n=1.5, and that for such a star $(\rho_c/\rho_p) = (T_c/T_p)^{3/2}$. We also used our expression for the central temperature T_c in terms of G, M, R, μ and ξ_1.

   b) Evaluate this expression for a star of solar composition with mass $M_\odot$ and radius $100R_\odot$.

3. The Eddington-Barbier Approximation:

   a) Show that the mean intensity seen by a distant observer (averaged over the disk of a star) is given by $2\int_0^{\pi/2} I_\nu(0,\theta)\cos\theta\sin\theta\, d\theta$.

   b) Show the flux, F_ν, emerging from a unit area of a star, is given by the expression $\int_0^{\pi/2} I_\nu(0,\theta) \cos\theta \sin\theta \, d\theta$.

c) In class we derived the following approximate expression for the monochromatic specific intensity at the surface of a star:

$$I_\nu(0,\theta) \approx B_\nu[T(\tau^*)] + (\cos\theta - \tau^*) \frac{dB_\nu}{d\tau} \bigg|_{\tau^*} + \ldots.$$

This approximation, obtained by expanding B_ν in a Taylor series about τ^*, is called the Eddington-Barbier approximation. Evaluate the integrals in parts a) and b), find the value of τ^* for which the coefficient of $dB_\nu/d\tau$ is zero, and substitute this value in the first term.

4. Suppose that the same star is eclipsed by a dark companion, with a diameter a factor of $\sqrt{3}/2$ smaller.

a) If the orbital inclination is 90°, for what value of τ^* does the coefficient of $dB_\nu/d\tau$ vanish (after integrating over that part of the star which is visible) at mid-eclipse?

b) Using the crude gray atmosphere presented in class, what is the ratio of the temperature observed in mid-eclipse to the temperature when not in eclipse?

c) What is the ratio of the flux (integrated over frequency) of the eclipsed star to that of the uneclipsed star?