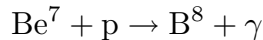


Due 11:04 am MONDAY 10 April 2006

- Reading: Clayton §4.3 through equation (4-62). Hansen & Kawaler §§6.3-5; Bohm-Vitense, Volume 2, Chapters 5 (Eddington-Barbier), 6 (Temperature Stratification) and 10 (Line Formation) as needed.

1. The solar neutrinos observed in the Homestake Mine experiment came from the PP III branch of the proton-proton chain. The limiting reaction is



which is followed almost immediately by the emission of a positron and a neutrino.

- a) Take the central temperature of the Sun to be $1.5 \times 10^7 \text{K}$, so that $kT = 1.293 \text{keV}$. Calculate by how much (expressed as a percent or as a factor) the central temperature must drop to cause the rate of PP III neutrinos to drop by a factor of 3, which is the observed shortfall. *Hint:* Calculate the derivative of the natural log of the rate with respect to temperature and find the dT which changes the log of the rate by an appropriate amount.
 - b) In the last problem set we adopted a scaling relation for the energy generation equation of the form $L = C' M \left(\frac{M}{R^3}\right) T_c^n$. While our expression for $\langle \sigma v \rangle$ is clearly not a power law of temperature, it can be locally approximated as such by computing the value of $d \ln(\langle \sigma v \rangle) / d \ln T$ at any point to give a effective exponent n . What is the effective n for part a)? This is similar to Clayton's problem 4-14.
2. In class we derived the functional dependence of photospheric temperature T_p on mass and radius for fully convective stars. We found an accidental cancellation in the R exponents, giving an amazingly weak dependence of T_p on R , which we offered as an explanation for the nearly vertical "Hyashi track" to which pre-main sequence stars, giants and asymptotic giants all adhere. For the sake of brevity we did not keep track of constants – G, k, ξ_1, m_p and even μ .
 - a) Reconstruct the argument retaining the multiplicative constants, i.e. find an expression for T_p in terms of M and R appropriate to a fully convective star. Recall that we started with an expression for the photospheric pressure, $P_p = g/\kappa_{H^-}$ where $g = GM/R^2$ and an approximate expression for the H^- opacity, $\kappa_{H^-} = 2.5 \times 10^{-31} (Z/0.02) \rho^{1/2} T^9 \text{ cm}^2/\text{gm}$. We used the fact that the polytropic index for a fully convective star is $n=1.5$, and that for such a star $(\rho_c/\rho_p) = (T_c/T_p)^{3/2}$. We also used our expression for the central temperature T_c in terms of G, M, R, μ and ξ_1 .
 - b) Evaluate this expression for a star of solar composition with mass M_\odot and radius $100R_\odot$.
 3. The Eddington-Barbier Approximation:
 - a) Show that the mean intensity seen by a distant observer (averaged over the disk of a star) is given by $2 \int_0^{\pi/2} I_\nu(0, \theta) \cos \theta \sin \theta d\theta$.
 - b) Show the flux, F_ν , emerging from a unit area of a star, is given by the expres-

sion $\int_0^{\pi/2} I_\nu(0, \theta) \cos \theta \sin \theta d\theta$.

- c) In class we derived the following approximate expression for the monochromatic specific intensity at the surface of a star:

$$I_\nu(0, \theta) \approx B_\nu[T(\tau^*)] + (\cos \theta - \tau^*) \left. \frac{dB_\nu}{d\tau} \right|_{\tau^*} + \dots$$

This approximation, obtained by expanding B_ν in a Taylor series about τ^* , is called the Eddington-Barbier approximation. Evaluate the integrals in parts a) and b), find the value of τ^* for which the coefficient of $dB_\nu/d\tau$ is zero, and substitute this value in the first term.

4. Suppose that the same star is eclipsed by a dark companion, with a diameter a factor of $\sqrt{3}/2$ smaller.
- If the orbital inclination is 90° , for what value of τ^* does the coefficient of $dB_\nu/d\tau$ vanish (after integrating over that part of the star which is visible) at mid-eclipse?
 - Using the crude gray atmosphere presented in class, what is the ratio of the temperature observed in mid-eclipse to the temperature when not in eclipse?
 - What is the ratio of the flux (integrated over frequency) of the eclipsed star to that of the uneclipsed star?