# Classical Mechanics III (8.09) Fall 2014
## Assignment 2

Massachusetts Institute of Technology
Physics Department
September 15, 2014

*Due September 22, 2014*
*6:00pm*

## Announcements

This week we will finish our discussion of problems with constraints, including the use of Lagrange multipliers, the topic of this problem set. Next up is the motion of Rigid Bodies.

## Reading Assignment for this week

- Systems with Constraints: D'Alembert's principle and systems with holonomic and non-holonomic constraints. Use of Lagrange multipliers to determine forces of constraint. The reading is **Goldstein**: sections 1.3-1.6, 2.4, 2.5, 8.5.

- Next up are Rigid Bodies which we'll likely start at the end of this week and continue next week (they are not on problem set #2). The main readings for this entire topic are **Goldstein** Ch.4, sections 4.1, 4.2, 4.4, 4.6, 4.9. Then read Ch.5, sections 5.1, 5.3–5.7. The Ch.4 reading is on rigid body kinematics where many physics topics will be familiar to you. Goldstein emphasizes vector notation and discusses rotations as matrices. Ch.5 is on rigid body dynamics. (Note that 4.3 is a linear algebra review. Other sections from Ch.4 and Ch.5 may be of interest, but the above are the most important ones.)

# Problem Set 2

These five problems involve friction, forces of constraint, and Lagrange multipliers. Hint: If you get stuck on a problem, think about what the conservation of $H$ implies in the coordinates you are using.

1. **Spherical Pendelum with Friction** [8 points]

   Consider a spherical pendelum, consisting of a mass $m$ attached to a massless rod of length $\ell$. The spherical mass is immersed in a viscous fluid, and it moves with a small velocity obeying Stokes law. It is acted upon by gravity.

   (a) [4 points] What is the dissipation function for this pendelum?

   (b) [4 points] What are its equations of motion? Include the effect of the viscous friction force.

   Note: A spherical pendelum means that the mass moves in three dimensions up to the contraint imposed by the rod. You do not need to use Lagrange multipliers for this problem.

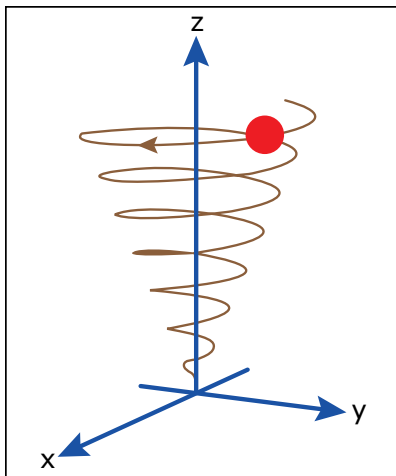2. **Bead Spiraling on a Helix** [12 points]



Image by MIT OpenCourseWare.

   Consider the motion of a bead of mass $m$ on the helix shown in the figure. In cylindrical coordinates the helix has a radius and angle described by $r = bz$ and $\theta = az$. At $t = 0$ let $\dot{z} = v_0 > 0$ and the height be $z = h$. There is no gravity in this problem.

   a) [8 points] By integrating the equation of motion find $\dot{z}$ as a function of $z$, and then determine the constraint torque $Z_\theta$ as a function of $a$, $b$, $m$, $z$, $v_0$, $h$.
   [Hint: $\int dt\, \ddot{z}/\dot{z} = \ln(\dot{z})$.]

   b) [4 points] Show that the particle's energy is constant, but that its angular momentum increases with increasing $z$.

3. **Hoop Rolling on a Cylinder** [12 points]

   Do Goldstein Ch.2 #14 starting with 3 generalized coordinates (try one radial and two angular coordinates). Use Lagrange multipliers for the two constraints. Note that up until the point that the hoop leaves the cylinder that these constraints are holonomic or semi-holonomic.

4. **An Infinite Rope** [8 points] (Adapted from Goldstein Ch.2 #9)

   Consider a rope of infinite length which passes freely over pulleys at heights $y_1$ and $y_2$ above the earth, where the pulleys are at horizontal distances $x_1$ and $x_2$ respectively. The rope has a uniform linear mass density and is affected by gravity. Demonstrate that the problem of finding the curve the rope assumes between the two pulleys is equivalent to a minimization problem. Using Euler-Lagrange equations or an equivalent method, solve to find the form of this curve. Write down the equations you would solve to fix the boundary conditions.

5. **A Falling Ladder** [20 points]

   Consider a uniform ladder of length $L$ and mass $M$ which has one end against a smooth horizontal floor and the other end against a smooth vertical wall. The ladder is initially at rest at an angle $\theta_0$ to the floor. We will be interested in studying the forces of constraints, so we will handle the constraints with Lagrange multipliers.

   (a) [3 points] Use the CM coordinates $(x, y)$ for the ladder, and angle $\theta$ that the ladder makes to the floor. The kinetic energy is given by a term for motion of the CM, plus a term for rotation about the CM. What is the moment of inertia of the ladder for rotations about the CM? Find the full Lagrangian for the ladder and the equations for any constraints.

   (b) [2 points] Derive the equations of motion using the method of Lagrange multipliers.

   (c) [4 points] Solve for the angle $\theta(t)$ of the ladder to the floor by finding a formula $t = t(\theta)$ (which may involve an integral that you leave undone).

   (d) [3 points] At what angle $\theta$ does the ladder leave the wall? What is the height of the upper end of the ladder at this angle?

   (e) [3 point] What are the equations of motion after the ladder loses contact with the wall?

   (f) [5 points] Show that the ladder never loses contact with the floor. (You can assume that when $\theta = 0$ it does not bounce. )

   (g) [0 points, not for credit] Consider solving the equations of motion in (a) and (d) numerically using the function NDSolve in mathematica to find $\theta(t)$, or the height of the ladder as a function of time.

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