

PROFESSOR: OK, so this is the picture you've studied so far. And now we will consider the next thing. So basis states. What's going to happen? We have to deal with basis states. We have electron spin. We have electron angular momentum.

What is the total angular momentum? The sum of them. We will have to deal with that. So we will have to deal with addition of angular momentum in the hydrogen atom. It's awfully important. That's the key of the matter. So let's see what we have.

Let's look at our basis states. So now the spin, the most important thing, we take it very seriously. Recall that when you have angular momentum-- in general, we use j and m , and those are the two quantum numbers, j^2 , the angular momentum eigenvalues, $\hbar^2 j(j+1)$. And j_z has eigenvalues, m .

That's a notation for angular momentum. So we have the electron spin. So what is the notation. It's s , and it's always equal to $1/2$, because the electron always has been $1/2$. The electron will have many orbital angular momentum, 0, 1, 3, but spin, it only has been $1/2$.

So s is always $1/2$. m_s can be plus minus $1/2$. So that's two states. For orbital electron, orbital, we have l and m . Those are the names, l and m is the quantum number.

So how do we define the uncoupled basis? The uncoupled basis is a set of states that enumerates the whole spectrum of the hydrogen atom, using those quantum numbers to distinguish all those states. So the uncoupled basis are those states that we have there. Uncoupled basis are all these states, and they're described by n , and the principal quantum number, l and m . This is the orbital.

And you could say, well, s and m_s , that would be a correct thing to do. But as we said, s is always So copying and copying again, something that is always the same value and doesn't have any new information is not worth it, so people don't include the s . And we put m_s .

And that's electron spin along the z direction, electron s_z . And it takes values plus minus $1/2$. So this is our uncoupled basis. For any electron in that table, you need to know uniquely that electron state. You need to give me all these numbers. You have to tell me where l am horizontally, after that, where l am vertically, I'm sorry, for m , where l am horizontally for l , within the multiplet, which is my value of m . And once you've done that, you should tell me up

or down.

So all those numbers are important. They're one to one correspondence to the basis state. But now, let's do the coupled basis. That's where things begin to get interesting. Coupled basis. So we'll consider the total angular momentum j , which is l plus s .

When we add angular momentum, we basically say, you know, you have states that are representations of orbital angular momentum and spin. But I want you to express those as eigenstates of the total angular momentum. That's all adding angular momentum means. It's recognizing that we want to re-express our basis states in terms of eigenstates of the total angular momentum. It's all you do.

So what do we do then? We have an l multiplet. What does an l multiplet mean? That's a set of vectors, a vector space. And we tensor it with a spin multiplet, and the result is equal to the sum of j multiplet because your states are nothing else but tensor products of these things, even though we never wrote it in the way of tensor product. Basically, the wave function has some expression having to do with l and m in here and has some value of the spin.

So a given state has all these properties. A given state lives in the tensor product, and we want to write it the sum of j multiplet. So I want to say a couple more things about this. When we do addition of angular momentum, what happens to the quantum numbers? That's, again, also a thing that's sometimes a little subtle, and I want to emphasize it.

So an l multiplet it is described by l and m . A spin multiplet is defined by s and m_s . But when you have a j multiplet, you have j and j_z —how do they call it? j_m . I think they call it. Yeah, j_m . But actually, you have a little more.

I claim that the states—so this is what I'm meaning by this. These states have two quantum numbers you can specify. You cannot specify any more. This is the z component of l , and you cannot specify the x or the y component. Here is all you can specify of these states.

You certainly can specify the value of j , and the value of j_m , the m component, the z component of j , but can you specify more? And the answer is yes. You can specify a little more. You can actually specify here all the states in here are eigenstates of l , of s , of j , and of j_m .

So you add two more. This may sound a little funny, but every state here had the same l eigenvalues. They were, for example l equal 3. So all the states have l equal 3. All the states

are eigenstates of L^2 , eigenstates. All these states here are L^2 eigenstates, and all the states here are S^2 eigenstates with the same eigenvalues.

So if you multiply them, and you rearrange them, because that's all this addition of angular momentum is just rearranging the states, you still have that all the states that are here have the same L^2 and the same S^2 . So L and S are good quantum numbers here. The things that are not good quantum numbers are m and m_s are not good.

They are not good quantum numbers of this state. They are not eigenstates of m or m_s . So this is something you've learned with addition of angular momentum. If this is a little fuzzy, it will be important that you review it and make sure this becomes clear. There will be [? stuffed ?] recitation about these things, and we'll do more with it.

But now, what is the application in the hydrogen atom for this? Well, our multiplets are multiplets of [INAUDIBLE] value of L that are being tensored with spin $1/2$. Orbital L , and when you have this, you know that the answer is $L + 1/2$ plus $L - 1/2$. Those are the two values of j , j_{\max} , and j_{\min} in this case.

So in the hydrogen atom, the notation is that this is the L state with j equal $L + 1/2$. This is the j value plus the L state with $L - 1/2$. So this is the capital L we were mentioning before, in this case L of L . And our notation is evolving. This is the spectroscopic notation in which we will describe states by an nLj .

So that's the spectroscopic notation. You put the principal quantum number, the capital L , that is for L equals 0, put an s , a p , a d , and the number which is the value of j here. So let's look at our spectrum again. We have to do that. It's an important thing.

So we'll do three cases here, n equal 1, n equals 2, and n equals 3. We have L equals 0, L equal 1, L equals 2. And that's s , p , and D . OK, let's begin. This state, ground state, what is the ground state? It's L equals 0, tensor with $1/2$ for the ground state. This is just the state $1/2$. So it will be a state with j equal $1/2$.

So it should be written as n , which is a 1, the capital L , which is 0. So it's s and the j value $1/2$. That's the state. $1s\ 1/2$. Go here. Well, that's still L equals 0. So this product source is $2s\ 1/2$, and here is $3s\ 1/2$.

That's the name of the multiplets in the coupled basis. What do we have here? We have L

equals 1. So with l equals 1, we have $1 \text{ tensor } 1/2$. So we will get j equals $3/2$ and $1/2$. So we must have 2.

What is the value of l ? Remember, the value of l is preserved. So if this was l equals 1, after you do the tensor product, l is still a good quantum number. So you have $2p_{3/2}$, and $2p_{1/2}$. Here, you would have two states, $3p_{3/2}$ and $3p_{1/2}$.

Finally, here l equals 2, you have $2 \text{ tensor } 1/2$ is $5/2$ plus $3/2$. So you would have $3d_{5/2}$ and $3d_{3/2}$. Those are your states. This is the notation we will use. So what is the uncoupled basis at the end of the day? The uncoupled basis is n . You still have n . What is the coupled basis now?

We have the uncoupled basis being this. The coupled basis is still n . Still l . We said l carries through. Still s would carry through, but we don't have it there. Now I cannot put m . m is no good when [INAUDIBLE], but I have j and j_m .

So these are your coupled basis states, coupled basis. And it's represented in this notation, the n , the little l is representative for whatever letter is here, the j is here, and well, the j_m is not said, but that's a multiplate. OK, so we've rewritten the stating the couple basis, because we will need those states in order to do perturbations theory.