PROFESSOR: We're going to be talking about adiabatic approximation. It's an interesting approximation that we can do when some physical parameters of your system change slowly. And sometimes, things don't change slowly. For example, if you have a uniform-- a non-constant magnetic field.

OK, that's spatial variation, no time variation. But suppose you send the particle in. The particle sees a changing magnetic field, because it's going from one region of low magnetic fields, say, to a region of high magnetic field. So if the particle is going at slow velocity, it's seeing a slowly-changing magnetic field, even though the magnetic field itself is not changing.

So there are several circumstances in which slow variations can be important. There can be cases where the magnetic field is slowly changing in time. You have a spin, and the magnetic field in which the spin is located is slowly changing, and then you want to know what happens.

So when a physical quantity in your system, in your Hamiltonian, changes slowly, you would expect that your dynamical variables, to some degree, will change slowly as well. And they will adjust themselves in a slow manner to these changes. And that's all true. And we kind of expect it. But there are some things that actually change very slowly, not just slowly, but very slowly.

And those are things that are adiabatic. Adiabatic changes correspond to things that change, due to some reason, even more slowly, perhaps, than you would have anticipated. And that's the kind of surprising thing that we need to understand. So we will begin with an example in classical mechanics to illustrate this point. And so our subject is adiabatic approximation.

And we go, then, to classical mechanics to get a hint of what can happen. And a typical example could be you're here, and you have a pendulum. And the pendulum is going this way. But you raise up and down your hand-- lift up and down your hand, so that the length of the pendulum varies. And therefore, the frequency of oscillation varies.

So this is a very simple system, in which you have a Hamiltonian that depends on x and p, and a frequency that depends on time. You've prescribed it. It's some frequency that depends on time. So here it is. p squared over 2m plus 1/2 m omega squared of t x squared.

OK, so omega is changing in time. And x and p-- this is classical motion, so it's all classical physics. This is going to be classical motion. Therefore, x and p are going to be functions of time. Well, let's see how things change. So let's calculate what would be the change of the Hamiltonian, or the total energy as a function of time.

Now this is a time-dependent Hamiltonian. There's no such obvious thing as conservation of energy. You're doing some work here that is changing omega. So the amount of energy that this system has will change. So time-dependent Hamiltonian like that, the energy of the system changes here.

So let's calculate the change in time. So x and p change in time. And therefore, this changes in time, x changes in time, omega changes in time, and the energy changes in time. So let's calculate how this energy changes in time.

So what should we do? We should do dH dx times the rate of change of x because H depends on x. dH dp times the change in time of p. We use dots for time derivatives. And then finally, we also have to differentiate H, with respect to time to take into account the variation of omega, which is a parameter here.

So all these things must be done. And let's assume these are functions of time because the system is doing physical motion. So we're trying to investigate how the energy changes if the system is doing physical motion. And here is the 0-th order result. If somebody would say, OK, the frequency is going to change slow, then you could say, OK, adiabatic result is that the energy is going to change slowly.

True, but not too interesting. We're going to do better than that, much better than that. So let's think a little more. If the motion is satisfying the equations of motion-- this is physical motion we're trying to understand, how the energy changes as this particle, its doing its motion-- we can use equations of motion of the system.

These are the Hamiltonian equations of motion that say dH dp is equal to x dot. And dH dx is equal to minus p dot. These are the Hamilton's equations of motion of classical mechanics.

It is a property of your education at MIT that you probably are less familiar with those than the quantum equivalent. So let me remind you of the quantum equivalent. If you have iH bar d dt of x, you remember that by Ehrenfest, this is xH commutator expectation value. That's Ehrenfest theorem.

And here, you also remember that x can be thought as iH bar d dp, such in the same way as p can be thought as H bar over i d dx. So this commutator, the way we compute it, is as iH bar d dp of H expectation value. And that's-- cancel the iH's. And here, you've got x dot is dH dp, the quantum version of that.

You have also seen that iH bar d dt of p is equal to expectation value of p with H. And that would be H bar over i dH dx using that p as that derivative. And that, canceling the H bars and noticing you have an i and a 1 over i here, gives you this equation with a minus sign.

So these are Hamilton's equations from classical mechanics that, in case you have not seen them, you know they're quantum analogs. And you probably believe that that is the case. Now, that has an important consequence on all of this, that the first two terms in this expression cancel, because dH dx gives you a minus p dot. So you get minus p dot x dot.

And from here, you get p dot x dot, so the two terms cancel. And we get that the dH dt is just dH dt. It's kind of nice because if there was no explicit time dependence in the Hamiltonian, the energy should be conserved. And therefore, it's nice that all that is left is just dH dt, which we can evaluate from the formula up there.

We just have to differentiate omega. So this gives you m omega omega dot x squared. It's a result that is of interest. So, so far so good. The energy will change.

And if omega changes slowly, the energy will change slowly. Nothing too dramatic. We need to do better. So let's think more precisely, what do we mean by adiabatic change?

So basically, it means that the time scale for change is much bigger than the time scale for an oscillation of your system. So this omega is going to change in time. But now you can ask-- given an omega there is a period, maybe a second. So probably, adiabatic change will hold if the change in omega is small over a second-- if the change in omega maybe happens over a year, with omega being about one second.

So this is a little like we had in the WKB approximation. I'll make the point here. So here is, for example, a typical graph that one draws. Here is omega of t.

And that's a constant, then it changes a bit. And it stabilizes after a while in some time, tau, for a finite change in omega of t from some initial value to some later value. In this case, the timescale from change, tau, should be much bigger than the time period of isolation, which is 2 pi over omega of t, which is the period.

But let's be a little more precise, like the way we did for the WKB case. What did we have in the WKB case? We said that the change in the Broglie wavelength over the Broglie wavelength was much smaller than to the Broglie wavelength. Or the change in energy over the Broglie wavelength was much smaller than the energy of the system.

So here, we can say the change in omega of t over a period is much smaller than omega of t. So how do we say that? Here is the rate of change of omega. This is, roughly, the change in omega over a period.

So rate of change in omega-- this is the period, so that's, roughly, the change in omega over a period-- must be much smaller than omega itself. So this is 2 pi over omega squared. d omega dt is much smaller than 1.

And we can write it in two different ways. This says, actually, that omega dot over omega squared is much less than 1. That's one way of writing it. Forget the 2 pi. And the other way is by saying that this is d dt of 2 pi over omega much less than 1.

That's correct, because when you differentiate 1 over omega, you get d omega dt times 1 over omega squared. And therefore, this is dT period dt is much less than 1, which might remind you of the WKB's d lambda dx was much less than 1. This was WKB, very, very analogous.