PROFESSOR: So one more thing let's do with this operator. So we're getting accustomed to these operators, and these permutation operators can also act on operators themselves. So that's important. So consider the action on operators.

So for example, an operator B that acts belonging to the linear operator some V, when acting on V tensor V, we define two operators, B1 and B2. And you define them in an obvious way, like B1 acting on Ui 1 tensor Uj 2.

OK, B as an operator knows how to act on every vector on the vector space capital V. So when you say 1, you're meaning that this operator acts on the first Hilbert space. So this is equal to B times Ui 1 tensor Uj 2. So it just acts on the first state. How does it act? Via B, that is an operator, in the vector space V.

Similarly, if you have B2 of Ui tensor Uj 2 you have Ui 1 tensor BUj 2 to OK. So these are operators that act either on the first state or in the second state. So the permutation operators can do things to these operators, as well.

So we can ask a question, what is P21B-- should I start with one? Yes, one-- P21 dagger. Remember, when you ask how an operator acts on an operator, you always have the operator that you're acting with come from the left and from the right. That is the natural way in which an operator acts on an operator.

You can think of this thing as your operator is being acted upon as having surrounded by a [INAUDIBLE] and a [INAUDIBLE]. And then when the states transform, one transforms with U, one transforms with U dagger. So always the action on an operator is with a U and a U dagger.

So if you ask how does the permutation operator act on B, you don't ask generally what's the product of P times B. You ask this question. This is the question that may have a nice answer. Then we'll see that there's other ways of doing this. So we want to investigate this operator.

So what I can do is let it act Ui, Uj. So what do we get? We get P21B1. Now, P21 dagger, we saw that it's Hermitian anyway, so it's just P21. And now it acts on Ui 1. I'll put the j here, and Uj 2. So I let up the P21 on that state, and that the moves the i's and the j's.

Now, B1 acts on the first Hilbert space. So now we have P21 and we have BUj 1 and tensor Ui

2. Now, P21 is supposed to put the second state in position one and the first state in position two. So this is Ui 1 BUj 2. I could put this thing-- BUj 2.

And then you see, oh, this term is here. So this is nothing else than B2 acting on the same state of the Ui Uj, which means-- I guess I could use this blackboard-- that P21 B1 P21 dagger is B2.

So it has moved you. The operator used to act on the first particle. Two and one changes the first particle with the second. It moved it into the other one.

Similarly, you could do this also. Would not be a surprise to you that P21 B2 P21 dagger is equal to B1. And you don't have to do the same argument again. You could multiply this equation by P21 from the left and P21 dagger from the right.

These things become one and one, and the operators remain on the other side and gives you this. So this second equation comes directly from the first. You don't have to go through the arguments.

So what is the use of this thing? You may have a Hamiltonian, and you want to understand what it means to have a symmetric Hamiltonian. And these operators allow you to do that.

So for example, you may have an operator O 1,2. What is an operator O 1,2? It's an operator build on things that act on one or act on two. So if you want to imagine it, it could be an O that depends on the operator A acting on the first label, an operator B acting on the second label, an operator C on the first label, an operator D on the second label. Could be a very complicated product of those operators acting on all kinds of labels.

Now suppose you act with P21 O 1,2 P21 dagger. Now, the great advantage of having a P and a P dagger acting on a string of operators is that it is the same as having a P and a P dagger acting on each one. Remember, if you have like P and P dagger, and it's a unitary operator on ABC, it's the same as PAP dagger, PABP dagger, PCP dagger. It's like acting on each one.

So when you have this P21 P21 dagger acting on this, it's as if each one of those is surrounded by a P21 P21 dagger. So each label one will become a label two, and each label two will become a label one. And therefore, this operation is going to give you O2,1 for an arbitrary operator acting on these two labels.

Now, it may happen that the operator is symmetric if O is symmetric. By that, we mean O 2,1 is equal to O 1,2. If that happens-- if that happens-- then from this equation you would have P21 O 1,2 P21 dagger is O1,2 is itself.

And you could multiply by a P21 from the right, giving you P21 O 1,2 equal O 1,2 You're multiplying by a P21 from the right that cancels this P21 dagger. P21. And there you see that an operator is symmetric if it commutes with the permutation operator.

So if always symmetric, this is true, and this is true, and then finally, P21 with O 1,2 commutator is 0. Oops. Too low. Let me see. It's a commutator. It's 0. So that's basically how you manipulate these operators on this Hilbert space.