PROFESSOR: I'm going to get started. And after a couple of minutes there will be an announcement. So today's lecture will begin by looking at the singlet state for a few minutes, a couple of its properties. And then we go into this Einstein-Podolsky-Rosen argument. We'll talk about what they said, what they wanted to criticize quantum mechanics for. And then we'll go through the answer given by Bell that actually demonstrated that Einstein, Podolsky, and Rosen were wrong. And after we'll do those Bell inequalities, we'll close this chapter on quantum states of spins and begin our treatment of angular momentum.

So I want to remind you of a couple of things. We've been discussing this so-called singlet state. Plus, minus. For the second particle, minus minus, plus. First and second particle. This state.

A few things we've discovered about this state--you've been working with it. We calculated its z-component of total angular momentum, the angular momentum of the first particle and the second particle. And it was 0. The x-component was 0. This y-component is 0. In fact it doesn't have any total angular momentum. So this state is rotationally invariant, we say, because it doesn't have angular momentum. You did very fine in the homework that the state is in fact rotationally invariant.

Now this state is a very interesting state. It's one of those entangled states that we discussed last time when we were talking about teleportation and Bell states. And apart from that, it's a state that is not hard to realize physically. In fact, it typically takes place, for example, in reactions in decays of certain particles. For example, pi 0 can decay into two photons. Then the two photos can be in the state of total angular momentum 0.

But more precisely, since we're talking spin-1/2 particles, if you have a meson called an eta 0, it's a interacting particle of strong interactions. A meson decays rather quickly into a mu-plus plus a mu-minus. Actually it decays into other things as well.

So it decays into two spin-1/2 particles. And this particle has 0 angular momentum. It's a scalar. It's not spinning. And therefore, if these two particles go into a state of 0 orbital angular momentum, conservation of angular momentum implies that these two particles are in the state of this form that has 0 spin angular momentum, total spin angular momentum.

So the realization of an entangled state like that is fairly common and fairly easy. So you have a decay of this form and you get particles that are entangled this way.

You showed that this state actually could be a written as one over square root of 2, n-plus, n-minus, 1, 2, minus n-minus, n-plus, 1, 2. Precisely because this invariant and the rotations, you could use instead of the plus-minus basis, any basis n. For any direction n, you have this state.

Now we'll talk about the probability that is of interest to us. We'll write to this symbol, probability to get a plus, say, b plus. And this means that probability to get particle-- let me see-- to find the first particle-- particle with spin along a so that this first particle to be in the state a plus. And the second particle-- particle will spin along-- along b. So in this state, b plus.

Now that looks-- this calculation of this probability-- So you're going to do some measurement. And you ask, what is the probability I find the first particle in this direction, second particle pointing in this direction? It may look like a somewhat non-trivial calculation. And it is.

But if you use the fact this the state over there-- because we're asking for this probability on this state. We're going to be talking about this state. So if you put the state and you write the state in the form, you pick one of the two vectors, say, a. Well, the state is a-plus, a-minus, 1, 2, minus a-minus, a-plus. Because I could choose n to be anything. So might this well choose one of the two vectors to be a.

And then you ask what is the probability to find first particle in this state and second particle in this state? Well, when somebody tells you, what is the probability to find a particle in a state, you put that state into-- you sandwich your psi with that state. And this overlap, which is a number, you square it. So we're going to do the same thing here.

So what is this probability? Probability to find a plus, b plus, would be the absolute value. And then we put here a plus in the first state. Tensor product, we could say. Well, b plus in the second state. And we should put-- because this is what we want to find, the state on the tensor product that we're looking for. And we put the psi here. So we must put the 1 over square root of 2 times the a-plus 1, a-minus 2, minus a-minus 1, a-plus 2.

So [? overlap ?] time, well, we go 1 with 1, 2 with 2. So well, a-plus with a-plus will give me 1. b-plus with a-minus, I don't know. Second term, a-plus with a-minus gives me 0. So this term is irrelevant. I just need this. So the a-plus with the a-plus gives me 1. I have the 1 over square root of 2 here. And I must close this and square. I forgot to write that.

So what is this probability? A-plus, b-plus is equal to-- the 1 over square root becomes 1/2. And then all we have left is b-plus with a-minus. In the second state space, in the particle 2 state space, the label doesn't matter at the end of the day now that we've disentangled the 1 and 2. So it's b-plus a-minus. I don't have to write that it's 2, 2. Or you can write it. And it's this squared.

So it's simplified a lot but not quite yet the answer. What we need here is the overlap between these two spin states. And I remind you that when you had any arbitrary spin states with n and n prime, long, long ago, homework three or four, something like that, you calculated the overlap between these two spin states. And the answer was that you would take cosine of 1/2 of the angle-- if there is an angle gamma. The overlap between the spin states squared was cosine squared of 1/2 the angle.

So here we have the vector a, see, the vector b. Here is the vector a-minus, a-minus direction. And if we call this theta ab, this is pi minus theta ab. So this should be 1/2 cos squared 1/2 of pi minus theta ab. 1/2 cosine square of half of the angle between the two relevant vectors. So this is cosine of pi over 2 minus theta over 2. That's sine squared of theta ab over 2.

So here it is. It's our calculation of this thing. It's a neat formula that we're going to need later today.

So one more comment before a little stop, if b is equal to minus a. Now in that case you should be in luck because precisely what's happening here is that if one spin is along the plus direction the other has to be along the minus direction, whichever you choose.

So in order to have that a be in plus and b be in plus, well, this first term would do it. a is in plus, the first particle. Here, no. And b, which is minus a, would be in plus. So the probably should be 1/2. So if b is minus a, the angle theta ab is equal to pi. And the probability of ab is 1/2, correctly. And it's 1/2 because half of the cases a is in plus. The other cases, a is in minus.

So the other case, for example, that could be interesting is, what is the probability that the first particle is in z-plus and the second particle is in x-plus? Well, these two vectors form 90 degrees. So you should have 1/2 of the sine of half that. So that should give you 1/2 of the sine squared of pi over 4. And it's 1/2, then it's 1 over square root of 2, that's another 1/2. So it's one quarter, for example.

OK, I went long enough for a moment. There's an announcement that [? Preshanth ?] wants to make so please listen to him.

[? PRESHANTH: Hi ?] I'm back for another announcement. So tonight, the MIT SPS is going to be holding its fall UROP lightning lectures. So if you don't have a UROP it's a great opportunity for you to come and see what other research your classmates are doing in physics. If you do have a UROP, I would encourage you to come as well because you can actually come and share your stories about the technical content of what you're doing in your UROP. It's 7:30 this evening in the PCR 8329. There will be free food, so please join us then. And see you all then. Thanks.

PROFESSOR: Thank you. All right. So, this was the introduction to what we really need to do today. So before we get started, are there any questions on what we've done so far? On these properties of this entangled state?

So this is a measurement of an entangled state. These two particles could have flown away a big distance. Two observers, one tries to see what is the probability. The first observer sees the spine pointing in some direction. The other observer sees the spin pointing in another direction. It's a the natural question which can be done experimentally. And we've calculated that answer.

Now so let's begin with this EPR story. Now, you've seen some of EPR last semester in 804. The only complication with that is that you really needed to have these mathematics to appreciate it completely. So this second look at EPR should be fairly complete in that we won't leave almost anything out of the story.

There are many ways of doing EPR and essentially these Bell inequalities, which is the really non-trivial thing that comes after that. So some are in the homework, some elaborations. And probably in recitation later in the course we'll see a little more.

But it all begins with a strange thing, the kind of thing that you wouldn't expect people in physics to discuss. And it's the point of this so-called-- Einstein, Polosky, and Rosen wrote a paper. And they talked about local realism. Now, that sounds like philosophy. And for awhile people thought, well, this is interesting, but undecidable. Can't really do anything with it.

So what is local realism? Now, again, not being exactly physics, it's not all that easy to say what it is. And people discussed that. But some notion of it is fairly clear. The notion is that this reflects something-- it's basically two assumptions about measurement results.

So you measure something and obtain a number. And the first assumption, one, is that these measurement results correspond to some aspects of reality. Just said like that it seems a little funny. That you measure something, if get some numbers, because that was something real about this object, it had this property. And so measurement corresponds to some aspect of reality. So measurements-- assumptions about measurement results. So measurements. m, correspond to some aspect of reality.

Two, the measurements that you do in you lab are not affected whatsoever by the measurements that somebody else is doing at the same time at the moon. There's no time for the information of what that result in the moon has given to reach you. So at that instant of time, what they are doing at the moon doesn't affect the result of your experiment. So it's measurement is independent of actions performed at a distant location at the same time.

Now to Einstein and Polosky and Rosen-- but Einstein was very vocal-- Physics must satisfy that. It's kind of sad I think, actually. The person that managed to see through and discover how nature works at so many deep levels-- the photoelectric effect, special relatively, general relativity-- somehow became convinced that this had to be true. And unfortunately, he was wrong. Or fortunately, I guess. It's not worth trying to qualify that. But these two things that seem just so reasonable are just not true.

This one, measurements correspond to some aspect of reality-- you see, you have a Stern-Gerlach apparatus, you throw a spin, it goes up. You say it ended up with spin up. Well, Einstein would say it always had spin up. It was a reality about that object, at it had spin up. You just didn't know. You did the experiment to you discovered it.

So the thing that people try to do in order to understand this concept, that it corresponds to something having to do with the reality, is that you admit that half of the particles go up and half go down. But you say, actually, there's something about these particles you don't know. And if you knew that, you would just be able to tell. This is a particle that has spin up. And it will go up.

But in quantum mechanics, we have abandoned that. We've said these particles here are a superposition of a state up and a state down. And there's nothing definitely up about this particle or definitely down. So the way people do this, to correspond to this aspect of reality that you don't know, is by introducing what is called hidden variables. Some things that allow you-- there's something hidden about this spin particle that you don't know. But if you knew it you would know exactly how it's going to come out through this Stern-Gerlach experiment. And you say, well, that sounds fairly untestable. But the fact is that it's not.

Now, so this is implemented, this assumption is-- when people try to modify quantum mechanics, they use what is called hidden variables. Some things that you don't know about the particle, but if you knew, you would see that in fact this particle has spin up.

This second is in some ways even more disturbing because we got accustomed to the idea that, locally, simultaneous things that cannot be reached-- events that cannot talk to each other via the exchange of light cannot effect each other. So simultaneous things that's happened far away can't effect each other. So this also sounds very reasonable. But that's also wrong.

And there's the obvious question, so if this is wrong, can you send information faster than the speed of light? And people looked at it in many, many ways. And it's very interesting. And we could discuss that. But it would take us long, so I will leave it to, maybe, the recitations, maybe other forums. But here, actually, there's no contradiction, no way of finding real information going faster than the speed of light, even though things far away at the same time can affect you.

So two very interesting things that seemed very dangerous to discard but turned out to be wrong. So this is what EPR did. And they made some thought experiments that we're going to review to some degree and see if we can discard these assumptions. So that's what we're going to do now. We're going to try to understand that.

Now, if you're interested in what hidden variables are, my discussion will not use hidden variables. Although they are kind of implicit, you will see, as I state some things. I'm basically going to be explicit on the fact that things, [? the ?] [? real ?] [? facts, ?] as Einstein would like you to think. And we'll try to see if those real qualities about particles get us in trouble.

So let's begin and try to discuss this first experiment. You see observer one, Alice and Bob again, if you want. Alice is going to measure about the z-axis. Bob is going to measure about the z-axis. And therefore, if Alice finds spin up, Bob finds spin down. If Alice finds spin down, Bob finds spin up. And that's a correlation. And it's very interesting.

But Einstein, Bell, and Polosky would say, look, it's not all that interesting. There's nothing all that mysterious happening here. EPR would says, the pairs that you've built, the so-called entangled pairs, are pairs of particles with definite spins-- spin directions, spin vectors. So what you have built, EPR would say, if you have here particle 1 and particle 2-- I'm going to list the properties.

Suppose you have a particle 1, you've created. Einstein would say some particle 1s with spin up in z and particle 2 with spin down in z. Or you've created particles 1 with spin down in z and particle 2 with spin up in z.

And you've created [INAUDIBLE] 50% of the particles are of this type, of the pairs. And 50% of the pairs are of this kind. So don't tell me all this superposition hocus pocus. Half of your pairs, one particle has spin up, one particle has spin down. The other 50% of your pairs, particle one is down, the other particle is up.

No wonder they're correlated. You get plus, gets minus. Get minus, get plus. What is the probability that you get plus? 50%. What is the probability you get minus? 50%. Everything is reproduced. Mystery over. No quantum superpositions. OK?

There's no mistake here. No mathematical mistake. You're saying that the particle has a definite spin. You maybe not know it. But we say, look, your particles in fact have a definite spin, z-plus and z-minus and those things.

And this is where hidden variables would come along. You would say, well, if you have the particle 1, it's spin is a function of some hidden variable that you don't know. But if you knew it, you would know what the spin is because it has a definite spin. You don't know what is the hidden variable. But as a function of the hidden variable the spin is known, definite, not a superposition, nothing like that.

It's a very aggressive attack on quantum mechanics and something that troubled people. And in fact fascinates people even up to now because the idea, these kind of things are really absolutely wrong. It's very shocking.

Perhaps the second even more shocking because, by now may be you're accustomed to all kinds of variables that are a little more detached from reality. You have electromagnetic fields. And they forced you to learn about potentials that seem to be a little more abstract. And similarly here.

So no problem at all. So people say, OK, this is a simple situation. But it may be that we're going to do more measurements. And we're going to consider two directions that are different.

Maybe Alice has two Stern-Gerlach machines, one that measures about z and one the measures about x. And Bob has also to Stern-Gerlach machines, one of the measures about z and one that measures about x. And they are going to ask different questions. Because we know the spin, how it transforms. So getting those results right with two directions is going to be a little more interesting.

So we're going to try measuring in two possible directions. Both A and B, Alice and Bob, can measure in two directions, z and x. And Einstein would say, look, you can measure in z and in x.

To avoid confusion, let's not talk about one measurement after another. Particle comes, you measure in z or you measure in x. And realism says that you don't know what you will get because maybe there are hidden variables. But each particle has a definite answer if you ask what is the z direction of the spin and has a definite answer if you ask what is the x-component of the spin. Definite answers, real answers, realism again.

So I'm going to label the particles. For example, this is a label for a particle z-plus, x-minus. This particle labeled like that would be such that if you measure its spin in the z direction, it's plus. If you measure the spin in the x direction, it's minus. So measured in z, it is up. Measured in x, it is down.

These are the kind of particles that EPR would say exist. It's not that you got a particle out and it's in some strange entangled state. These particles are flying away. They're not talking to each other anymore.

And this particle, since you can measure in two directions, there's some reality, and the measurements correspond to reality, so there are attributes. And this particle is classified by having these attributes. If you measure z, plus. If you measure x, minus.

So how about this situation. Well, let me make a list now. Particles 1 and particle 2. And this would be the list that EPR would do for you. EPR comes along and says, look, here's what you're doing. Particle 1, suppose it's a z-plus, x-plus.

Well, in your beams, actually, when particle 1 is that, your other particles are z-minus, x-minus. So if particle 1 is of this type, particle 2 is of this type. That way, EPR protect themselves because they say, look, if you measure z-plus and you measure z-minus, you get correlation. If you get plus and plus, you get correlation as well.

Now there can be particles in z-plus, x-minus. This will go with a z-minus, x-plus. There could be a particle z-minus, x-plus, and this as z-plus, x-minus. And there could be particle z-minus, x-minus, z-plus, x-plus. So there are four cases, four types of particles they say have been produced. And 25% of pairs are of this form. 25% of pairs are of this, 25 of this, and 25 of this.

So you could ask some questions to EPR. What is the probability that you get z-plus for 1 and z-minus on 2. Well, z-plus on 1, it's these two cases. And z-minus on 2, well, those two cases. So this is 50% of the times. So it's 1/2 that probability. It's correct. That's what we would predict from an entangled state viewpoint.

But let's ask something mixed. Let's see if we get in trouble. P of z-plus on 1, and let's say x-plus on 2. Well, z-plus on 1 and x-plus on 2, that's not it. This case. z-plus on one, x-plus on 2. 25% of the times, one 1/4. This was the probability we calculated.

So you tried with one direction. You don't need quantum mechanics to produce a result. You try with two directions, you don't need quantum mechanics to get the result. So people got stuck and they said, well, maybe it's undecidable. Maybe this is philosophy. Maybe this is something. And people had to wait for Bell. He said I'm going to try three directions.

Now, three directions makes a big difference. This is the first time it really goes wrong. So it's kind of surprising, perhaps, at some level. But this is subtle stuff. So it takes a while before you find something wrong. You're talking about showing Einstein it's wrong, so that's not so easy.

So three directions. So particles are going to be of types-- EPR would say, look, I'm going to use the same strategy. I'm going to say that particles have three attributes now. They're all physical. They correspond to reality. Because if you measure in either of three directions, they have to have an answer for that.

So here's a label for a particle. And for example, a-plus, b-minus, c-plus. So if you measure, it would give spin in a direction plus h bar over 2. Spin in the b direction minus-- well, in the b direction would give you minus h bar over 2. Spin in the c direction would give you plus h bar over 2 .

So we're not measuring simultaneously. We're just asking, well, you take a particle, do a measurement, and see what you get. And we're always going to be asking for probabilities of this kind, probably that the first particle is doing this and the second is doing that.

Well, EPR would start now with particles again, particle 1, particle 2. And populations. So let's list quickly the particles. a-plus, b-plus, c-plus. Then you will go a-plus, b-plus, c-minus. Then you've done the c-plus, c-minus here. You go for two more. A-plus, b-minus, c-plus. a-plus, b-minus, c-minus.

I was supposed to fit four more there. Can I? Well, I will try. a-minus. Now you've done all the four a-pluses so you need four a-minuses. Then you have b-plus, b-plus, b-minus, b-minus, c-plus, c-minus, c-plus, c-minus. I got all, I think.

And particle two of course is correlated. You would say, well, I don't need to write it. But it helps seeing what's going on so I'll write it. a-minus, b-minus, c-minus, a-minus, b-minus, c-plus, a-minus-- let's use bars here-- a-minus, b-plus, c-minus, a-minus, b-plus, c-plus, a-plus, b-minus, c-minus, a-plus, b-minus, c-plus, a-plus, b-plus, c-minus, a-plus, b-minus, c-- this one is minus so it's plus here.

We're done. Lots of labels. And you could say, well, maybe you want to put 1/8 in each of them. But actually the argument is more interesting. It doesn't need you have to try to put fractions. So let's consider that there's a total number of particles N, which is N1 plus up to N8. And here are N1 of this, N2 of this, N3 of this, N4 of this, N5, N6, N7, and N8. Let's see how--

All right, so we have that. Well, it takes imagination to see how you're going to run into some contradiction. So what is the basis for the contradiction? Somehow this formula, which is really quantum mechanical must eventually go wrong with all these attempts to deny that the world is quantum mechanical.

So we could split again those particles into equal fractions. But there's no need to do that. And it's clearer if you don't. So you try to combine the three directions into one equation. So one way to do that would be to say, OK, what is the probability that you get a-plus and b-plus?

So this is for the particle number 1. And this is particle number 2. So then you must look at the table and say which one's do that. Well, you need a-plus in the first so it's one of the first four rows. And b-plus in the second. So actually, it's these two cases, N3 and N4, over N.

We want to involve three directions. So let's go for another one. P of a-plus with c-plus. Let's see how much is that. Again, this is for the first particle. This is for the second particle. So I must look at the first four rows. And see that you have an a-plus. First part is in a-plus for the first four rows. But the second should be in c-plus and that is cases N2 and N4. So we get N2 plus N4 here over N.

Well, we've involved this a with b, a with c. How about involving b with c? So I'll put c-plus, b-plus, for example. OK c-plus, b-plus. I must look at c-pluses, and b-pluses, no. C-plus and b-plus, yes. N3 is there, which is good because it already was there. And which else? c-plus and b-plus. c-plus, no. c-plus here b-plus, yes. N7.

Now N3 plus N4 is less than N3 plus N7 plus N4 plus N2. You see you have N3 and N4. And now I add whatever N7 and N2 are. And that's then an inequality because it's going to be more cases. Now I divide by N. So you obtain an inequality that P of a-plus, b-plus is less than or equal than N3 plus N7 and N4 plus N2. I'll right the second first, P of a-plus, c-plus plus P of c-plus, b-plus.

So I didn't put specific populations 1/3, 1/4. But in general whatever populations you choose, this inequality must hold. So that's the more clever strategy. Because suppose you choose some populations and you don't get in trouble, well, maybe with some other populations you would get in trouble. Maybe it's not so easy to get the relative factors. So here is something that must be true whatever populations you choose.

And now that is Bell's inequality. So the achievement of Bell is to somehow translate this assumption of realism into an inequality. And now quantum mechanics has a formula for these things, for this probability. So we can test whether this is true.

So let's do that. This is the so-called Bell inequality. So if quantum mechanics is true, the following should hold. Let's see that. If QM is true, well, there should be a problem with this inequality. So let's see what happens. Let's see if it's true, this inequality. Is it really true?

Well, the left hand side would be, given the formula that we had, 1/2 of the sine squared of theta ab over 2. So let me just emphasize, this was derived using local realism. Local realism gives that. So you do the experiment, get these probabilities.

And if realism is true, this should hold.

Let's see what quantum mechanics has to say. Let's plug-in the values that you get from quantum mechanics. Now we calculated this probability. We put the first term. Here is sine squared of theta ac over 2 and 1/2 of sine squared theta bc over 2.

So does that work? Does that always work? Can I orient this axis in such a way to disprove EPR? And in fact, it turns out to be quite easy to do that. So you choose three vectors like this. ac, bc, so c here, a here, and b here, I believe. Yep.

So put an angle theta here. An angle theta here. And then what do you have? Theta ab would be 2 theta. Theta ac would be equal to theta bc equal to theta. That's a pretty nice simple choice of angles.

If you choose these angles now, let's see what happens with our inequality. So you get 1/2 sine squared theta ab over 2 would be theta. Is it less than or equal to this? Well, these two become the same. So you get sine squared of theta over 2.

Violated or not violated? Is it true or false, this, for all theta? What do you say? Yes?

STUDENT: If theta is less than pi, that's not true.

PROFESSOR: Close. It's not true for a small theta. So if you're this, and you're desperate to know, the thing you have to do is assume theta is very, very small. See if you get in trouble.

How much is this? 1/2 theta squared. Let's see. Half of this one? Yeah. And how much is this? This is theta squared over 4. Sorry, I was not seeing it.

Sine theta for small theta is roughly theta. So here is theta over 2 squared. But here is 1/2 theta squared. And it's false. The 1/2 theta squared is not smaller than one quarter theta squared.

And in fact, for theta equal pi over 2, I think this is an equality. Because theta equal pi over 2, you get on the left hand side 1/2 less than or equal than sine of 45 degrees, which is correct. So it's an equality of pi over 2. Fails below.

So it was a shock. That if you could do an experiment in quantum mechanics and experiment with correlated, entangled particles, that you could to measure these probabilities, these correlations. And you would obtain a result that actually contradicts for certain alignments of your experiments the assumptions of local realism.

So it was a great result of Bell to show that quantum mechanics is in contradiction with local realism. There's no way to keep the ideas of quantum mechanics and put hidden variables and assume that there's real values for things and that there's no effect at a distance. It would all be contradicted by an experiment.

That was done later and the definite version of the experiments around 1980 or '82 by Alan Aspect and others. Very clever experiments worth reading and understanding. But confirmed that the quantum mechanical result is really true by measuring correlated pairs. And this inequality is violated. Yes?

STUDENT: So what about David Bohm's theory of hidden variables quantum mechanics? So--

PROFESSOR: David Bohm's theory of what?

STUDENT: His hidden variable quantum mechanics theory allegedly reproduces the same results as quantum mechanics but it's still a hidden variables theory.

PROFESSOR: I don't think there's any hidden variable theory that works. David Bohm, I think, actually was credited by rewriting EPR, who essentially talked about position and momenta to talk about spins. And he might have been the first one that began to try to do hidden variable theory.

But no hidden variable theory works at this moment. And this shows it. So people say that actually this assumes that there's local hidden variable theories and there's non-local hidden variable theories and all kinds of strange things. But it's more and more unnatural. So it doesn't seem to do something very interesting.

Yes, there are many questions. Aaron, maybe you want to say something.

STUDENT: Let's see, I think Bohm has a non-local hidden variable theory solution. It's kind of awful looking. But I guess violates the second principle rather than the first one. It also doesn't extend to-- this doesn't really work for everything. We don't know how to make it work for a spin-1/2 particle to find [? a dimension. ?] So [INTERPOSING VOICES] really believe to be a true theory of [? equivalence. ?]

PROFESSOR: Thank you. More questions. Steve?

STUDENT: In the case with EPR, is it a problem that we can have scenarios where the spin is greater than a spin-1/2 particle would have? If we had the state a-plus, b-plus, c-plus, we would have 3 h bar over 2?

PROFESSOR: No. The statement that is done here is not that-- well, this is a label for a particle. EPR just assumed that if you measured a you would be able to get this. If you measure along b, you would get this. And if you measured along c, you would get this. So this is a single particle. Any measurement gives some results this side of the list. There's no sense in which these are added.

STUDENT: But even when you measure one or the other, the other values still exist for those measurements.

PROFESSOR: Well, I believe there's no need to discuss that. So they don't talk about the statement of doing subsequent measurements within this statement. You just take this particle and you decide. You measure a or measure b or measure c. You don't try to measure simultaneously. You don't try to measure one after another. You just do one measurement.

And that already, which is the minimum you can do, gets you in trouble. So I'm not sure how EPR would phrase subsequent measurements after they've done the first measurement or things like that. But they're not necessary for this stage.

OK, so look, it's a very interesting thing. There's lots to discuss here, but it's best if you read it. James had a question. Just let's take one quick question.

STUDENT: I was just wondering if there was any other extension beyond three directions for

Bell's inequalities. Is there a N-direction of Bell's inequalities or some type of form of it? Or is there [INAUDIBLE]

PROFESSOR: There are other forms of Bell inequalities. I'm not sure if it's popular with four directions or anything. But certainly Weinberg, for example, discuss other ways. There are alternative ways to phrase it. I've talked about here probabilities to observe results.

There is a more, perhaps, [? common ?] way talking about expectation values or correlation functions. This is something you'll do in the homework. The sort of game that is done in the homework that was a suggestion by Aaron to put it in the homework-- this game in which with quantum strategy and entangled pair you beat the system-- is yet another formulation of the Bell inequalities as well.

So lot to do. But I think it's better now that we stop and talked about angular momentum from now until the end of the semester. This was about spins, but we now have to put together angular momentum which is orbital and spin and all the various kinds.

So we'll begin with angular momentum. So there are notes on the web on that that I wrote and modified a little this time. And lots of little exercises. So what I want to do now is guide you through the things that happen there so that you get a view of what we're going to do. We're going to work with this thing in a elegant way using vector notation for operators is going to help us understand things better.

So we've seen angular momentum before. And let me summarize simple things that we know about it. If you have angular momentum, and let's begin with orbital. This is what we have when we take Lx to be y Pz minus z Py. Ly equals z Px minus x Pz. And Lz to be equal to x Py minus y Pz. These are the angular momentum, orbital angular momentum operators.

Now, it's better for many things to use labels like x, y, and z, those operators. Call them x1 hat, x2 hat, x3 hat. And Px, Py, Pz, P1, P2, P3. In that way you can write commutation relations like xi, Pj equal i h bar delta ij. xi with xj equal pi with pj equal

0.

So it allows you to write things more quickly. In fact, the angular momentum operators become also a little simpler. It's sort of x2 P3 minus x3 P2. And you'll have the x, y, z labels and all that.

So we want to use vector notation. Now, vector notation, you can do it in two ways. You can talk about triplets. Those are vectors. Or you can form the vectors themselves. Now if you form the vectors, you get objects that sometimes are a little disconcerting. But we try that they not be so.

So here is the r vector operator. You could think of it as the triplet of x, y, and z. But let's call it like this, x thing times the first basis vector plus y operator times the second basis vector plus z operator times the third basis vector.

And we've done things like that. And we understand that these basis vectors really don't talk with these operators. They can be moved across each other. The basis vectors are things that help you write expressions, rather, at talking about triplets.

Same thing for momentum. Let's do it this way. P1 e1 plus P2 e2 plus P3 e3. And finally, well, angular momentum, the vector operator-- you've done a lot of the angular momentum vector operator for spin. So here you would put Lx or L1 e1 plus L2 e2 plus L3 e3.

So those are ways to write equations that carry all the operators and treat them as vectors, even though they're operators. So they're unusual vectors. They're vectors whose components are operators, are not numbers.

So the obvious question is, what changes then? So we're going to define dot product and cross products as we had before. But we have to be a little aware that when we write these things we could make a mistake unless we're not careful.

So here are two vector operators. What is the dot product of these two vector operators? Well, you know it's supposed to be the first component of this and the first component of that, second second, third third. So it should be ai bi, summed.

Repeated in this is our [? summed. ?]

Now, I should not write bi ai with a reverse order because this thing, the components, are now operators. And maybe they don't commute. So I've defined this once and for all to be this.

And a cross b, the i-th component of this thing is going to be defined once and for all to be epsilon ijk aj bk. Definition. The a to the left of the b.

And with this, we can check our usual rules of manipulation of operators. So one more definition. a squared is going to be aa, and it's going to be ai ai.

Simplest calculation is a dot b equals to b dot a. Yes or not? No, they're operators. So let's calculate the difference. Let's get a little practice calculating differences.

So I write a dot b is equal to ai bi, its sum. So then I say, that's ai bi plus bi ai. You see the commutator is ai bi minus bi ai. And I add it back. But this thing, bi ai is b dot a.

So here I've got a formula. a dot b is equal, actually, to b dot a plus this commutator, ai bi. And now you've got a new formula for operator vector analysis. a dot b and b dot a are not the same but they differ by this thing.

And a very important corollary, very famous corollary, what is r dot p? Is it the same as p dot r? If you're working quantum mechanics you maybe tempted to, oh r dot p and p dot r are the same, but r and p don't commute, so what is the difference?

R dot p is equal to p dot r plus xi pi. And how much is that?

[INAUDIBLE]

Sorry?

STUDENT: i h bar.

PROFESSOR: i h bar?

STUDENT: 3.

PROFESSOR: 3 i h bar. Yes, don't forget the sum. This is supposed to be summed. So it says x1 commutative, p1, x2 going to 3 i h bar. So here's a famous formula. rp differs from pr plus three h bar i-- i h bar, people write. i h bar.

Another formula that you would be curious to know. Well, the dot product was supposed to be symmetric. It's not. The cross product is supposed to be antisymmetric. Is it or not? a cross b sub i is equal to epsilon ijk aj bk.

What do I do next? I want to move the a and b's around. So I'm going to replace this by a commutator plus the other ordering, ijk. And I put here-- well, this would be a parentheses-- aj bk commutator plus bk aj.

Now, what do we get? Well, you have to look at the second term. Let's put the first term here because that's-- pretty much, we're not going to be able to do much with it. aj bk. And the first term, I would write it like this. Minus epsilon, flip these two, ikj bk aj.

If you do it like that, then it sort of fits nicely with the definition of the cross product. Because in the cross product, the first label those here, the second label goes with the last labeled of the epsilon. So this thing is minus b cross a. And that was a cross b. Plus epsilon ijk aj bk.

So that's your formula for of how the cross product. Now fails to be antisymmetric. It's not necessarily antisymmetric unless you're lucky.

So here is a property that you should try to think about. How about r cross r? Is it 0 or not 0? r cross r. You could say, what can I do here? Well, r cross r minus r cross r-- two r across r's should be equal to this. But both components are x's. So r cross r is 0. p cross p, that's also 0.

And therefore, say, L cross L, is it 0? Maybe. Is that right? No. L cross L, we'll see it a little later, but it's not 0 because one L with another L don't commute.

Lights, high. OK.

So L cross L is actually not 0 because this thing is not 0. We'll talk about it a little later. But actually this is a very famous one. L cross L is proportional to L with an h bar. It's a lovely formula.

So another interesting thing. Well, what is r cross p? r cross p, from this formula we would minus p cross r. And how about the other term? Is it 0 or not 0? Is r cross p equal to minus p cross r, or does that fail?

Well, r and p don't commute. Actually this formula-- I should have-- somebody should have complained. This should be the i-th component, the i-th component, and here is the i-th component. So i-th component the i-th component.

But then let's look at here. Epsilon ijk xj pk. But xj and pk is i h bar delta jk. Now, this delta jk is symmetric. This is antisymmetric.

You should get accustomed to the idea that that is 0. If this is antisymmetric and this is symmetric, this is 0. Let me do it one more time. Maybe this is not completely obvious.

But epsilon ijk delta jk. The intuition is also obvious. Epsilon must have the three numbers different and this forces them to be the same. But it's more general. If this is antisymmetric and this is symmetric, they should be 0.

And the way you do that is you say relabel j and k. Whatever was called j call it k. Whatever was called k call it j. So this is epsilon ikj delta kj.

And then we use the symmetry properties. So when you exchange this back you get a minus sign when you exchange this back you don't get a sign. So minus ijk delta jk. So you have shown that this thing is equal to minus itself. And therefore it's 0. Something that's equal to minus itself is 0. So this is 0.

And you've got that r cross p is really minus p cross r. And that's what we call the angular momentum, r cross p. But now you know that it's also equal to minus p cross r.

Let's see, one other one, for example, that is classically true. Let's see if it's quantum mechanically true. r dot L. The angular momentum L is supposed to be perpendicular to r and perpendicular to p because this is the cross product. Is that true quantum mechanically or not true?

Well, maybe I didn't say-- well, we said L is r cross p. So this is r dot r cross p. So this would be ri times epsilon ijk-- no, x-- xj pk.

So what is this? It's epsilon ijk xi xj pk. Well, this is 0 because these two x's are operators but they commute. Therefore this object is symmetrical in i and j. This is antisymmetric. This is 0. So r dot L is actually 0.

How about p dot L. Well, there's two ways of doing that, one that looks pretty obvious, one that looks a little harder. Let's do the one that looks a little harder. This would be pi epsilon ijk xj pk.

So here is the temptation. Must be 0. Because there is a pi pk. Symmetric in i and k. And here's antisymmetric in i and k. But it's this wrong.

These are not obviously symmetric unless you can move them across. And there's an x in the middle laughing at you and saying, beware. This could go wrong. So you have to be a little more careful.

So let's be careful. ijk and you put pi xj pk, all our operators. But pi with xj the commutator would be a delta ij and that vanishes.

So this p actually can be moved across the x and therefore show that the p is on the other side. Because the epsilon is there. So it's a lucky thing. So pi xj pk is not symmetrical in i and k. But if there's an epsilon, you're in better shape.

So here the p can be moved. ijk xj pi pk. And now nobody can stop you from saying this is symmetric. They're next to each other, they can be moved across each other. It's really symmetric in i and k. And therefore i and k antisymmetric, this is 0. p dot L is equal to 0 as well.

So this was a little bit hard way. If you had to used that L is actually equal to p cross r, then the p would have been next to the p from the beginning. And you would have saved a little time. The same time that it took you to realize that this equality is true.

So those are true equalities. p dot L is 0. And it's also equal to L dot p. It's not obvious that L and p commute. But in this case L dot p is 0.

If you use this formula for L, it's obvious. r dot L is 0. It's also equal to L dot r. Doesn't make a difference. It's also true as well.

So that's roughly what goes on. Look, I would like you to just read those pages. It's a continuation of this. It's about eight pages. I leave exercises to be done. That's in the homework. They're of this type, playing with it. And this is a good thing that you get accustomed to it.

And the material that you need just for the last problem of the homework will be covered on Monday. And we can talk about it in recitation tomorrow. So that's it for today.