

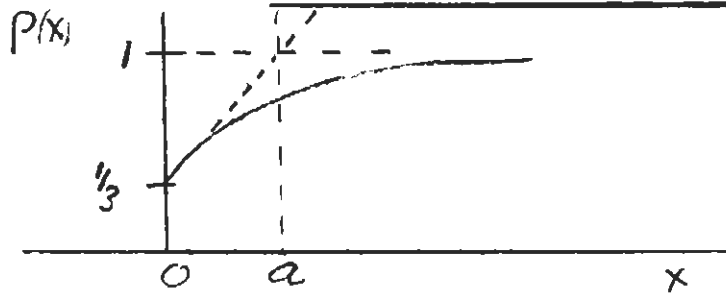
8.044 SOLUTIONS PRACTICE EXAM # 1

1. a/  $P(x) = \int_{-\infty}^x p(x') dx' = \underline{\underline{0}}$  for  $x < 0$

$$= \frac{1}{3} + \frac{2}{3} \int_0^x e^{-x'/a} d(x'/a)$$

FROM  $\delta$  FUNCTION  $\int_0^{x/a} -e^{-\xi}$

$$= \frac{1}{3} + \frac{2}{3} (1 - e^{-x/a}) \quad x \geq 0$$



b/ Prob.  $(x > a) = \int_a^{\infty} p(x) dx = \frac{2}{3} \int_1^{\infty} e^{-\xi} d\xi = \underline{\underline{\frac{2}{3} e^{-1}}}$

c/  $\langle x \rangle = \int_{-\infty}^{\infty} x p(x) dx = \frac{2}{3} a \int_0^{\infty} \xi e^{-\xi} d\xi = \underline{\underline{\frac{2}{3} a}}$

$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 p(x) dx = \frac{2}{3} a^2 \int_0^{\infty} \xi^2 e^{-\xi} d\xi = \frac{4}{3} a^2$

$\text{Var}(x) = \langle x^2 \rangle - \langle x \rangle^2 = (\frac{4}{3} - \frac{4}{9}) a^2 = \underline{\underline{\frac{8}{9} a^2}}$

d/  $p(d) \approx \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(d - \langle d \rangle)^2}{2\sigma^2}}$  by CLT

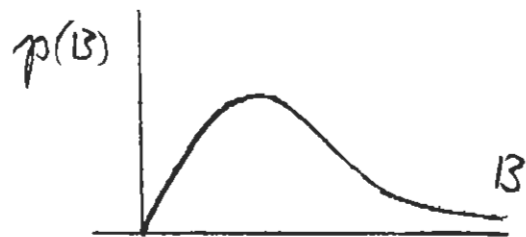
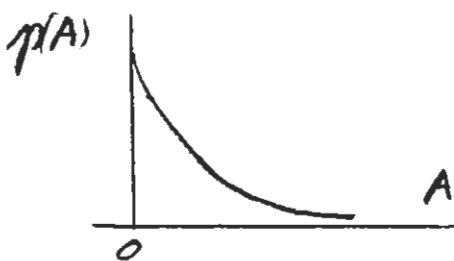
with  $\langle d \rangle = 36 \times \langle x \rangle = \underline{\underline{24a}}$

$\sigma^2 = 36 \times \text{Var}(x) = \underline{\underline{32a^2}}$

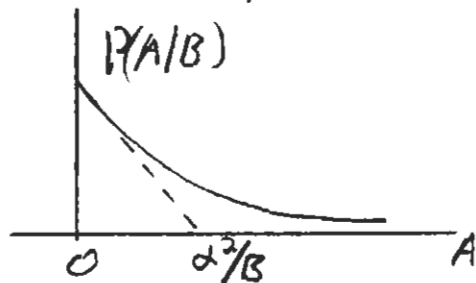
(2)

$$\begin{aligned}
 2. \dots a/ \quad p(A) &= \int_{-\infty}^{\infty} p(A, B) dB = \underline{0 \text{ if } A < 0} \\
 &= \frac{\gamma^2}{\alpha^6} \int_0^{\infty} B^2 e^{-B(A+\gamma)/\alpha^2} dB \\
 &= \frac{\gamma^2}{\alpha^6} \left[ \frac{\alpha^2}{(A+\gamma)} \right]^3 \underbrace{\int_0^{\infty} \xi^2 e^{-\xi} d\xi}_2 = \underline{\underline{\frac{2\gamma^2}{(A+\gamma)^3} \quad A > 0}}
 \end{aligned}$$

$$\begin{aligned}
 p(B) &= \int_{-\infty}^{\infty} p(A, B) dA = \underline{0 \text{ if } B < 0} \\
 &= \frac{B^2 \gamma^2}{\alpha^6} e^{-B\gamma/\alpha^2} \underbrace{\int_0^{\infty} e^{-BA/\alpha^2} dA}_{\frac{\alpha^2}{B} \int_0^{\infty} e^{-\xi} d\xi} \\
 &= \underline{\underline{\left(\frac{\gamma}{\alpha^2}\right) \left(\frac{B\gamma}{\alpha^2}\right) e^{-B\gamma/\alpha^2} \quad \text{if } B \geq 0}}
 \end{aligned}$$



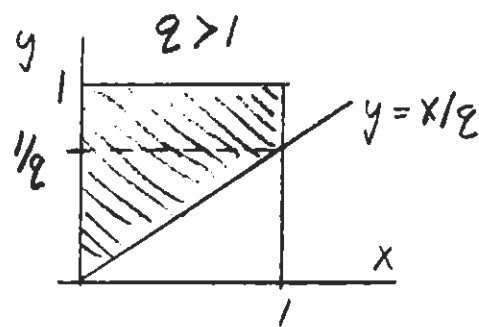
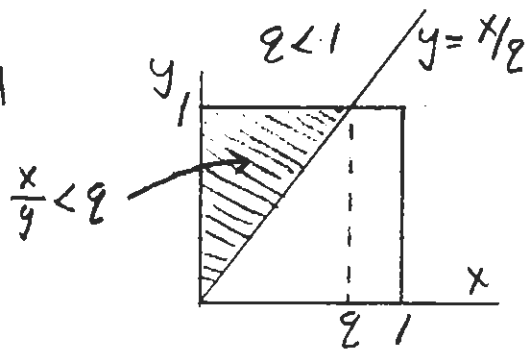
$$\begin{aligned}
 b/ \quad p(A|B) &= p(A, B) / p(B) = \frac{B}{\alpha^2} e^{-A(B/\alpha^2)} \quad A \geq 0 \\
 &= 0 \quad A < 0
 \end{aligned}$$



c/ A & B are not S.I., because  $p(A, B) \neq p(A) p(B)$

(3)

3. Step A



Note: geometry differs for  $q < 1$  and  $q > 1$

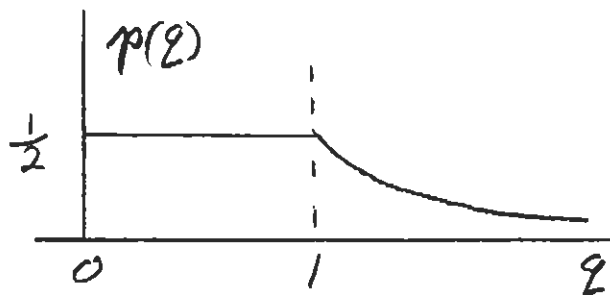
Step B  $p(x,y) = 1$  in square,  $= 0$  outside

$$q < 1 \quad P(q) = \text{area of shaded region} = \frac{1}{2}q$$

$$q > 1 \quad P(q) = \text{area of shaded region} \\ = 1 - (\text{unshaded area in square}) \\ = 1 - \frac{1}{2}\left(\frac{1}{q}\right)$$

$$\text{Step C} \quad p(q) = \frac{d}{dq} P(q) = \underline{\underline{\frac{1}{2} \quad 0 < q < 1}}}$$

$$= \underline{\underline{\frac{1}{2}\left(\frac{1}{q^2}\right) \quad q > 1}}$$



$$\int_0^{\infty} p(q) dq = \frac{1}{2} + \int_1^{\infty} \frac{1}{2} \frac{dq}{q^2} = \frac{1}{2} + \frac{1}{2} \int_1^{\infty} -\frac{1}{q} = \frac{1}{2} + \frac{1}{2} = \underline{\underline{1}}$$

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8.044 Statistical Physics I  
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