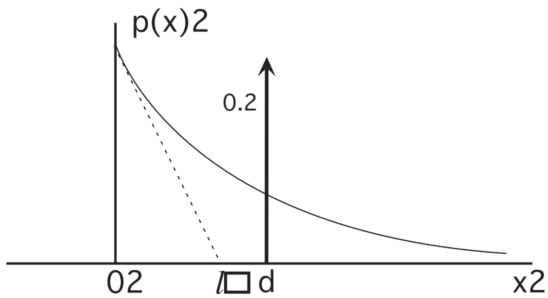
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2004

Exam #1

Problem 1 (30 points) Doping a Semiconductor



When diffusing impurities into a particular semiconductor the probability density $p(x)$ for finding the impurity a distance x below the surface is given by

$$\begin{aligned} p(x) &= (0.8/l)\exp[-x/l] + 0.2\,\delta(x-d) && x \geq 0 \\ &= 0 && x < 0 \end{aligned}$$

where l and d are parameters with the units of distance. The delta function arises because a fraction of the impurities become trapped on an accidental grain boundary a distance d below the surface.

a) Make a carefully labeled sketch of the cumulative function $P(x)$ which displays all of its important features. [You do <u>not</u> need to give an analytic expression for $P(x)$.]

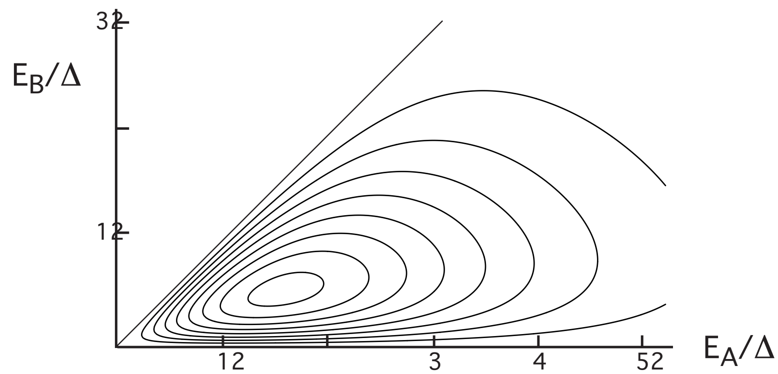
b) Find $< x >$.

c) Find the variance of x, $\mathrm{Var}(x) \equiv < (x- < x >)^2 >$.

The contribution to the microwave surface impedance due to an impurity decreases exponentially with its distance below the surface as $e^{(-x/s)}$. The parameter s, the "skin depth", has the units of distance.

d) Find $< e^{(-x/s)} >$.

Problem 2 (40 points) Collision Products



A certain collision process in high energy physics produces a number of biproducts. When the biproducts include a pair of elementary particles A and B the energies of those particles, E_A and E_B, are distributed according to the joint probability density

$$
\begin{aligned}
p(E_A, E_B) &= \frac{4E_B(E_A - E_B)}{\Delta^4} \exp[-(E_A + E_B)/\Delta] \qquad && \text{for } E_A > 0 \text{ and } E_A > E_B > 0 \\
&= 0 && \text{elsewhere}
\end{aligned}
$$

Δ is a parameter with the units of energy. A contour plot of $p(E_A, E_B)$ is shown above. Note that the energy E_A is always positive and greater than the energy E_B.

a) Find $p(E_B)$. Sketch the result.

b) Find the conditional probability density $p(E_A \mid E_B)$. Sketch the result.

c) Are E_A and E_B statistically independent? Explain your reasoning.

The collisions are statistically independent random events that occur at some uniform rate in time. The pair A and B only occurs in a fraction f of the collisions. When the pair is produced, it is detected with 100% efficiency. When the pair is not produced, there are no competing background events.

d) If the overall collision rate is 10^6 per hour, how long must one run the experiment in order that the uncertainty in the determination of f is of the order of one part in 10^4 of the value of f measured in that run? Note: one does not need the answers to a), b), or c) to answer this question.

Problem 3 (30 points) Equipment Failure

A graduate student begins an experiment which depends on two critical pieces of apparatus: a dilution refrigerator and a sophisticated laser system. Each is prone to failure, the failures are statistically independent, and a failure of either one ends the experimental run. The probability of failure after a time t for the refrigerator is given by

$$\begin{aligned} p(t_r) &= (1/\alpha)\exp[-t_r/\alpha] && t_r \geq 0 \\ &= 0 && t_r < 0 \end{aligned}$$

and for the laser by

$$\begin{aligned} p(t_l) &= (1/\beta)\exp[-t_l/\beta] && t_l \geq 0 \\ &= 0 && t_l < 0 \end{aligned}$$

We want to find the probability density for the duration of an experimental run T; that is, we want to find the probability density for $T \equiv \mathrm{Min}(t_r, t_l)$.

a) Find an analytic expression for the cumulative function $P(T)$. No short cuts here; do the integrals. [Hint: a bit of thought beforehand can decrease the work considerably.]

b) Find the probability density $p(T)$ and sketch the result.

Integrals

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2\,e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2-2ax+2)$$

$$\int \frac{dx}{1+e^x} = \ln\left[\frac{e^x}{1+e^x}\right]$$

Definite Integrals

For integer n and m

$$\int_0^\infty x^n e^{-x}\,dx = n!$$

$$\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx = \sqrt{\pi}$$

$$(2\pi\sigma^2)^{-1/2}\int_{-\infty}^{\infty} x^{2n} e^{-x^2/2\sigma^2}\,dx = 1\cdot 3\cdot 5\cdots(2n-1)\,\sigma^n$$

$$\int_0^\infty x\,e^{-x^2}\,dx = \frac{1}{2}$$

$$\int_0^1 x^m(1-x)^n\,dx = \frac{n!m!}{(m+n+1)!}$$