PROFESSOR: Let me discuss, for a second, effective potential. And-- actually, let me ask you a question to get you started. We're going to look at high n. N-- so n's, say, 100. And then l will go from 0 up to 99. Very high l's. What's going to be different from them?

Well here is one possibility, and that's a possibility that happens. The orbits are going to go from being circular to being elliptical as you change l. And I ask you-- which l would correspond to circular orbit, and which l would correspond to the most elliptical orbit?

So let's take a poll/ so most circular orbit-- is it l equals 0 or l equals 99? Who votes for l equals 0 for the most circular orbit? I have about half [INAUDIBLE], a little bit more than half. OK. Who votes for l equals 99? [INAUDIBLE].

Which is the most [INAUDIBLE]? OK, this is funny, because we all associate most circular with l equals 0, spherically symmetric. But it's wrong. This is the most circular.

AUDIENCE: What if l equals 100?

PROFESSOR: You cannot reach l equal 100. Actually-- and this is most elliptical. and you can have the intuition for that. Let's avoid the calculation and be intuitive. And the intuition it is kind of interesting, because we said at effective potential. And indeed, I want to discuss-- that's what I want to understand. Effective potential.

So maybe I'll draw this line a little lower. And what is the effective potential? We've written it already many times. It's h squared l times l plus 1 over 2mr squared minus e squared over r. So the minus e squared over r is here. Maybe I should draw it in a way that gives me more room. Like that. OK.

And then there is the energy level that I'm looking at. It's some high energy level. So I'll put it near here, not too near that I don't see it, but near here. Here is-- my en is here.

OK. So the answer is actually already there. Not too clearly, but here it is. What does a particle do in this potential? It goes with lots of kinetic energy, very fast here, it goes from here to here. Here to here. But what does that mean? That the radius is changing from r to some other value of r. From r equals 0 to some other value of r.

So what looks like an orbit? If you have a planet going around the sun, and at some point it seems to reach the sun, and at some point it goes far-- here is the sun-- then the orbit must be like this. That's likely to be the orbit.

On the other hand, as soon s is equal to 1, you have the part of the contributions that affect the potential. You have a 1 over r square like this. And then the total potential, which is the original term plus that, will have something that looks like this, and go like that. And then this orbit is going to go from some small value of r, but different from 0, to another one.

So those are all elliptical orbits, because the radius is not constant. And then this orbit, it goes like that. Now look at it-- if you have some elliptical orbit-- so then after that time, it just becomes a little more like this, in which the minimum radius for, say l equal 1 is here, and the maximum radius has been reduced. l equal 2, l equal 3, they all produce different ones. My graph is not perfect. Different ones. Different ones.

And finally, one that just touches it. And that one, the radius is fixed. The particle is moving like that. And that is your spherical orbit. And if you increase l a little more, no more solution. So it's the top l that produces-- this is already almost circular, and this is perfectly circular. So then it goes like this, and then eventually goes like this, and finally at the end it goes like that. And it's circular at the last case.

For the most l, you finally get your circular orbit. And also the intuition is clear. If you have a circular orbit, you have lots of angular momentum-- r cross p is very good. When two vectors are orthogonal, the cross-product of angular momentum is large. But look here. Here is r, for the shortest orbit, and p is in that direction. R cross p almost like that.

This is a very low angular momentum orbit. So l equals 0 is the most elliptical orbit. L equal very high is the most circular orbit. There is one thing-- a calculation-- maybe I'll use 10 minutes next time-- is to show that there's two turning points here-- an r plus and an r minus.

So there is an r plus for one solution, and an r minus, and it goes like that. So the r pluses and the r minuses corresponds to an ellipse that-- here is the center. That's where the proton sits. And you go r plus out is the maximum, and r minus is here. So the two turning points that define the maximum r and minimum r tell you about the lips on these r plus plus r minus.

And when you find those critical points and do the calculation, you in fact can show that r plus plus r minus is indeed equal to n squared a 0. I think that number comes out exactly there. R plus plus r minus over 2. The average is exactly that. So the roots, the two roots, are such that their average is always the same.

So this is not-- I'm not sure I get it right with my graph, but it should be that if you take the average of that intersection points, they're always at the same point. And so that's how it happens under an analog in-- so you get the intuition here. Here are your orbits.

And these are equal energy orbits of different eccentricity as you go in there. And even Kepler knew that, because Kepler figured out that the time that it takes the planet to go around depends only on the sum of these two things, the major diameter. So all these solutions over here-- if they were planets-- would take the same time to go around the sun.

So they all, in quantum mechanics, have the same energy, and they're therefore degenerate. So the mystery of the quadratic potential that makes all of these periods to be the same is the mystery that is making the energies the same in all of these orbits. So we'll finalize this story next time and do some other fun things.