

**PROFESSOR:** What did we have last time? We had a constant copper that reflected the energy so a unit free version of the energy, was greater than 0, because we were looking at bound states, so the energies were negative. And we found that this [INAUDIBLE] this quantized  $1/2$  copper was  $N + l + 1$ -- something that we call  $n$ . And  $n$  was the principal quantum number. So this is the principle quantum number  $n$  was the degree of a polynomial in the solution.

And  $l$  was an important quantum number, because it gave you the amount of angular momentum the system had. And given that we think of it as principal quantum number coming first, once you have  $n$  and you fix it because the energy just depends on  $n$ -- Once you fix  $n$ , you'll have that  $l$  can go from 0 up to  $n - 1$ , and those corresponds to the various values of capital  $N$ . But we don't have to focus on it up to  $n - 1$ . And at the same time in terms of quantum numbers,  $m$  goes from minus  $l$  up to  $l$ .

So the order of thinking is fixed co-- at little  $m$ , and the principal quantum number. Then fix an  $l$  can go from up 0 to  $n - 1$ . Once you fix a little  $l$ , you fix the  $m$ . And  $m$  can go from my  $l$  to  $l$ . And those are the states of the hydrogen atom. The energies in terms of  $n$  are  $m_e z^2 / (2 \hbar^2 n^2)$ . And the solutions--  $\psi_{nlm}$  that depend on  $r$   $\theta$  and  $\phi$  were of the form of a normalization constant,  $A r^l$  that you can isolate. That's a behavior for small  $r$ , then a polynomial  $n r$  over  $a_0$  of degree, capital  $N$ , which is  $n - l + 1$  times an exponential decay with radius, which goes like  $e^{-z/r}$ . It has to be dimensionless, the argument inside the exponential. And it turns out to depend on  $a_0$ , in fact  $n a_0$  and finally, the spherical harmonic. So that was the total solution.

We didn't investigate the polynomial in detail, because it takes time. It's not necessary for many things. Only very detailed calculations require this polynomial. And it's a Laguerre polynomial.

And if you needed-- if you needed to construct the quadratic polynomial, it would be a fight between looking up some table of Laguerre polynomials and spending time checking that the conventions they use are the same than the ones you choose versus taking the recursion relation and building it up yourself of the third coefficient. So that's what it is. Now the hydrogen atom, there is a classical description, a diagram, for the hydrogen atom and in fact, for any central potential.

So if you're looking at bound states, the way we do bound states and represent them for central potentials is by a diagram in which you put the energy on the vertical line. It's a negative energy. So you can put 0 in here.

The way the hydrogen atom works is better, of course, as usual to use something dimensionless here. And the thing we put is minus 1 over n squared, the integer the principle quantum number n minus 1 over n squared, because this ratio is, in fact, equal to the energy divided by this dimensionless quantity  $z^2 e^2 / 2a_0$ . So there is n squared, and the levels go like 1 over n squared, indeed.

So there is minus 1 here, and then goes to minus 1/4 for n equals 2 minus 1/9 minus 1/16. And they crowd here. That's why I don't do everything in scale. I cut the scale here, otherwise the diagram is very long. And you can put say the minus 1/4 here, though, 1/9 would be here. minus 1/9 minus 1/16-- somewhere here.

Those are places where you have energy levels, and here it comes. Let's look at what we can get. Well, for n equals 1, you're in here. So what are we going to plot in this axis.

The idea is to plot a quantum number. So actually to say here is l, but we don't do it in marking the values of l. We'll put here l equals 0 and list all the states that we get. So it's like a histogram or something like that, l equal 1, l equals 2, equals 3. And then we put dashes here, and each dash is a state. If you look at it and you say, what's the value of l?

So this corresponds to the idea that you already know that if you're solving a central potential problem, you have to solve a radial equation for different values of l. Each time another l, l equals 0, l equals 1, l equals 2. You go on with them. So m, so when n is equal to 1, l can be only 0.

So you have 1 states here. l is equal to 0. And is equal to 1. And capital N is it's equal to 0.

So the only thing that I cannot read immediately, I know that n is equal to 1, because l'm here. I know that l is equal to 0, because l'm here. But I put the extra information, the capital N equal to 0 here. And that's it for this level.

So this is that little n equal 1 level. Then we go to that little n equals 2 level. And when little n is equal to 2, l can be 0, or it can be 1. When l is equal to 0, capital N would have to be 1 so that capital N, which is 1 plus little l, which is 0 plus 1 is equal to 2. And this is a level n, little n equals to 2, principal quantum number equal to 2.

So here we have  $n$  equals to 2. Since  $l$  is 0, capital  $N$  is 1. Here you have capital  $N$  is equal to 0. And then we go to the next level little  $n$  equals to 3.

Well, little  $l$  can now go from  $l$  equals 0,  $l$  equals 1 and  $l$  equals 2. Since little  $l$  plus 1 plus capital  $N$  is equal to the principal quantum number, here you'll get  $N$  equal to 2,  $N$  equals 1, and it goes down,  $N$  equals 0. These are the states.

Let's do one more. Little  $n$  equals to 4. And, yes, there's a state for  $l$  equal 0, 1 for equal 1, 1 for equal 2, and this time we get to  $l$  equal 3. Each time you get one more  $l$ , because  $l$  can go up to  $n$  minus 1.

And what is capital  $N$ ? Here is 3, 2, 1, and 0. So this is your diagram. This is a very nice diagram. And it has actually is some sort of mystery in this diagram.

Let me emphasize first one point that is not mysterious. It's this growth from an equal 0 to an equal 1 to  $n$  equal 2 to  $n$  equal 3. There seems to be a pattern here. Here  $n$  equals 0, 1, 2. And it will go up.

So why is that necessary? Why did that happen? Remember, that we were solving a radial equation, which was like a one-dimensional potential. And if you are solving a one-dimensional potential, the node theorem works.

So there should be no nodes for the ground state. And the fact that the wave function vanishes at  $r$  equals 0 is not a node, because that's the end of the world at  $r$  equals 0. But then you're solving this radial equation. And let's look at this polynomial.

Well, you're not going to get a 0 of the wave function, because of this factor. It's an overall factor. And the exponential never vanishes just at infinity, but never doesn't vanish at the point. So all the 0's of the wave function have to arise from this polynomial. And there shouldn't be any 0 for the ground state.

So good.  $N$  equal 0 means no nodes here, no nodes for the-- this is a different problem. This is an  $l$  equals 1. It's again a new potential. So you solve the radial equation again. Yes, the ground state, when  $l$  is equal to 1 must have no nodes.

So you should, remember, you're solving the radial question. One's here for one potential, here for another potential, because the effective potential depends on  $l$ , here for another

potential. There for another potential.

So each time, it's a new one-dimensional problem, which must have a ground state and a state with 1 node, which is possible, because a degree 1 polynomial has one 0. A degree two polynomial can have two 0's.

Why it has another degree  $n$  polynomial may have  $n$  0's But it may have less 0's if the 0's are complex. But it better be that this polynomials don't do that, because they would violate the known theorem.