PROFESSOR: Scattering states are energy eigenstates that cannot be normalized. And when you say this cannot be normalized, so what's the use of them? They don't represent particles.

Well, it's like they e to the ipx over h bar, those infinite plane waves. Each one by itself cannot be normalized, but you can conserve wave packets that are normalized. So the whole intuition that you get with scattering states is based on the idea that we're going to construct energy eigenstates.

This time we cannot think of them as states of a particle. Bound states, yes. We can think of them, they're normalizable. But this energy eigenstates and bounded scattering states are not states of one particle. So we definitely have to go back and produce wave packets.

But the intuition from those energy eigenstates is very valuable. So scattering states. And we call them sometimes scattering states because they look like the process of scattering. This will be non-normalizable energy eigenstates. And you've played a little with some of them. And we'll now study one case in detail. We'll try a couple of cases between today and next lecture.

So the step potential. And the step potential is a potential that is 0 up to x equals 0. Here's the x-axis, and then suddenly there's a step at v0. And here is the potential. But then the wave, this is here, goes up. It's a step.

And any energy eigenstates here has to be bigger. The energy has to be bigger than the lowest point of the potential. You know that, you kind of have an energy that is like that less because this would have decayed exponentially for infinite distance. It just, all over it would have to decay exponentially. It's impossible.

So all the energy states, eigenstates here, must have positive energy. So we have actually qualitatively two possibilities. The energy may be less than v0, might be greater than v0. It would look like you have to solve the problem two times. Happily, we'll solve one, then let the other happen by analytic continuation.

So here is the energy. I'll take the energy greater than v0. But whatever is the energy, even if it's less than v0, the solution over here is going to be an exponential or a cosine and a sine, a non-decaying function, and therefore can't be normalized because it's non-decaying forever and ever. So it cannot be normalized.

So how do we write the solution for the energy eigenstate? It's a psi of x. Well, I should write two formulas: a formula for what's happening on the left side, and the formula for what's happening on the right side.

Now I have a choice actually here. There's two ways of visualizing this. I can visualize it as a wave that is coming from the left, moving here. Or a wave that is coming from the right. So let's visualize this solution as a wave that's coming from the left. It will be a little easier.

So I will write it. A e to the ikx. OK. Why is it coming from the left? Because if you put the energy-- that I will not put it, it's in stationary state, presumably this is a state with some fixed energy. You will have a factor e to the minus iEt over h bar. And when you see kx minus Et, you know that that's a wave that is moving to the right. So this A e to the ikx is moving to the right.

And then what will happen? Now it's a matter of finding a solution of Schrodinger's equation. So you can try to find the solution of Schrodinger equation, but you have to write some answers for what's happening on the right. I will write an answer here, that we'll put C e to the i k bar x. And another k. Well, we'll see now what those k's are.

I say the following. Here, the energy is bigger than the potential so it has to be a wave. But here they energy is still bigger than the potential so it also must be a wave. But a wave with different kinetic energy, different momentum, therefore different de Broglie wavelength and different k. But we know from Schrodinger's equation what that should be. This wave is also moving to the right, because probably if I have a wave moving to the right here, it produces some transmitted wave to the right. But then, you could try solving the Schrodinger equation with this. It won't be enough because physically you would expect the wave bouncing back as well from here.

So I will put a B e to that minus ikx. That's a wave moving towards the left with an unknown coefficient. And, now let's get those constants. I'll finish in two minutes.

What is k? Well if you have energy E, you know that the energy is h squared k squared over 2 m. You can look at the Schrodinger equation with 0 potential over there. And therefore, k squared is also 2mE over h squared. It's a combination you've been seeing quite a bit. The intuition for k bar should be that k bar squared is 2m times the kinetic energy, so it should be e minus v0 over h squared.

So these are k and k bar. And the wave function must be continuous at x equals 0. That gives you A plus B equal to C. At 0, all the exponentials vanish. And the derivative must be continuous at x equals 0. And the derivative being continuous because there's no delta function anywhere here. So you have ikA, that's the derivative of the first term, minus ikB, the same k in that region of course, is equal to i k bar C. So from this you get A minus B is equal to k bar over kC.

Two equations and two unknowns. And that's OK, even though there are three coefficients, because the way to think of this is that you're sending in some wave and you're going to get some reflection and some transmission. So in some sense, A is the input. You could want to call it 1 or whatever. So what we're looking for is what is B over A? And what is C over A?

And these two equations, it's a one line computation. I'll write the answer. B over A is k minus k bar over k plus k bar. Do it for fun. And C over A is 2k over k plus k bar.

B gives you a sense of how much is reflected. C, how much is transmitted. But this is the beginning. Because this is not a particle coming in. So we'll have to build the packet and send it in and see how this relations tell you what's going to happen. So this is a nice story that we will develop next time.